TFY4305 solutions exercise set 21 2014

Problem 11.3.7

a) and b) See Fig. 1.

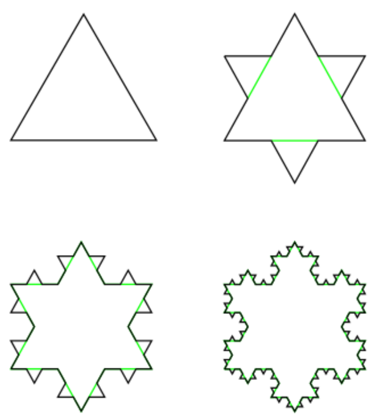

Figure 1: The von Koch snowflake at various stages.

c) At each stage, the length is increased by $\frac{4}{3}$, i. e. $L_n = \frac{4}{3}L_{n-1}$ and therefore $L_n = (\frac{4}{3})^n L$, where L is the original length of the line. Hence

$$L(S_\infty) = \lim_{n\to\infty} L_n = \underline{\underline{\infty}} \,. \tag{1}$$

d) Let A_0 be the area of the triangle. In the first step, one adds three triangles with total area $3\frac{1}{9}A_0 = \frac{1}{3}A_0$, in the second step one adds 12 triangles with total area $12\frac{1}{9^2}A_0 = \frac{1}{3}\frac{4}{9}A_0$. At the nth step, one adds $3\,4^n$ triangles of total area $3\,4^{n-1}\frac{1}{9^n}A_0 = \frac{1}{3}(\frac{4}{9})^{n-1}A_0$. The total area added then becomes

$$\Delta A = \frac{1}{3}A_0 \sum_{n=1}^{\infty} \left(\frac{4}{9}\right)^{n-1}$$

$$
\begin{aligned}
&= \frac{1}{3}\frac{1}{1-\frac{4}{9}}A_0 \\
&= \frac{3}{5}A_0 \ . \qquad (2)
\end{aligned}
$$

The total area is then $\underline{\underline{A = A_0 + \frac{3}{5}A_0 = \frac{8}{5}A_0}}$.

e) If we scale the length by a factor 3, we need 4 times as many sticks to cover the next figure in the sequence. Hence

$$
d = \underline{\underline{\frac{\ln 4}{\ln 3}}} \ . \qquad (3)
$$

Now you can imagine that the dimension of the coastline of Norway is larger than one. In fact it is measured to be 1.55.

Problem 11.4.2

We need 8 squares of side $\frac{1}{4}$ to cover S_1 (see textbook). We need 8^n small squares with sides $\epsilon = \left(\frac{1}{3}\right)^n$ to cover S_n . We then have $N(\epsilon) = 8^n$ for $\epsilon = \left(\frac{1}{3}\right)^n$ and thus

$$
\begin{aligned}
d &= \lim_{\epsilon\to 0}\frac{\log N(\epsilon)}{\log(1/\epsilon)} \\
&= \lim_{n\to\infty}\frac{\log 8^n}{\log 3^n} \\
&= \underline{\underline{\frac{\ln 8}{\ln 3}}} \ . \qquad (4)
\end{aligned}
$$