

TFY4305 solutions exercise set 22 2014

Problem 11.3.2

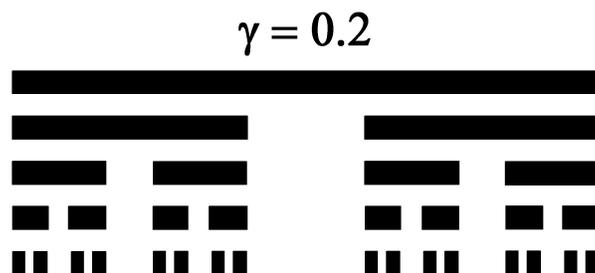


Figure 1: A generalized Cantor set with $\gamma = a = 0.2$.

If we scale the original segment by a factor of $(1/2 - \gamma/2)$, we need two segments to cover the next iterate. Thus the fractal dimension is

$$d = \frac{\ln 2}{\ln [2/(1 - \gamma)]}$$

Problem 11.3.8

a) See Fig. 2 below.

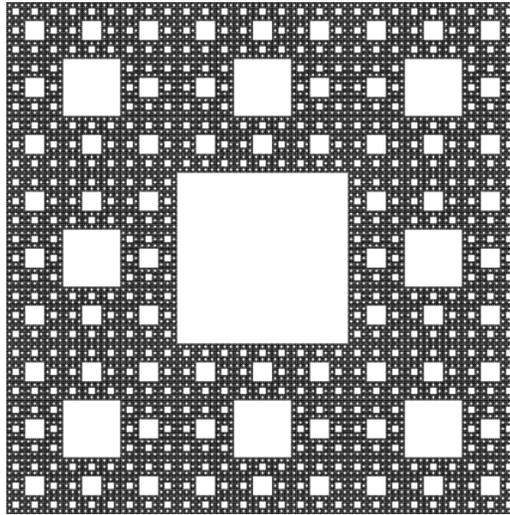


Figure 2: The Sierpinski carpet.

b) If we scale by a factor of three, we get eight copies of the original figure. Thus $r = 3$ and $m = 8$ and

$$d = \frac{\ln 8}{\ln 3}. \quad (1)$$

c) At every stage, we remove $1/9$ of the area, i.e. $A(S_n) = \frac{8}{9}A(S_{n-1})$. Thus

$$A(S_n) = A(S_0) \left(\frac{8}{9}\right)^n. \quad (2)$$

Taking the limit $n \rightarrow \infty$, we obtain $A(S_\infty) = 0$.