A flavor of two-color QCD

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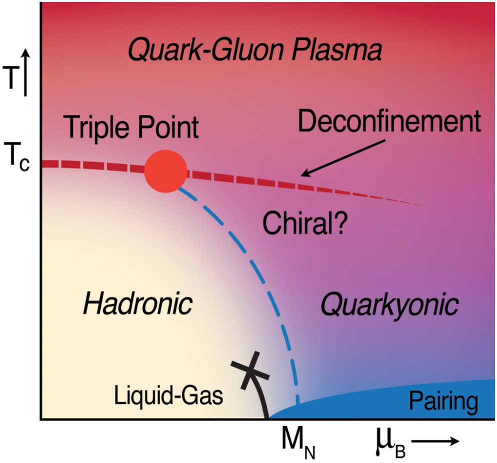
[1] Colloborators: Tomas Brauner. arXiv:1001.5168, Phys. Rev. **D81**, 096004 (2010).

Outline

Phase diagram of QCD

- Phase diagram of QCD



QCD Lagrangian and symmetries

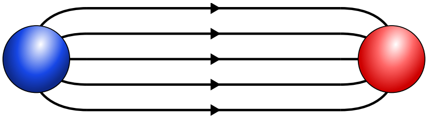
$$\mathcal{L} = \bar{\psi} i\gamma^\mu(\partial_\mu - igT_a A_\mu^a - m_0)\psi - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

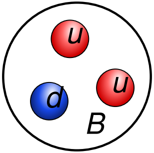
- Local $SU(N_c)$ gauge symmetry. Real world $N_c = 3$.
- Global $SU(N_f)_L \times SU(N_f)_R$ symmetry if $m_0 = 0$.
- Global $U(1)_B$ symmetry.
- Global $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. Gives rise to Goldstone bosons (pions).
- $N_f = 2$:

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

Confinement and chiral symmetry breaking

- All observed particles are color singlets (confinement) - no long-range forces
- Global symmetries are broken - Goldstone bosons (pions)



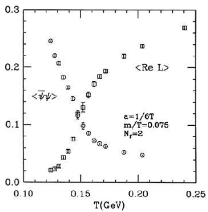


Confinement and chiral symmetry breaking

- Chiral symmetry breaking:

$$\langle \bar\psi\psi \rangle \neq 0 \,.$$

- Confinement and the Polyakov loop (global Z_N symmetry) :



$$\begin{aligned} L &= \mathrm{Tr} P \exp\left(\int_0^\beta igA_0(x,\tau)d\tau\right) \,, \\ L &\rightarrow zL \,, \\ \langle L\rangle &= e^{-\beta\mathcal{F}_q} \,, \\ \langle L\rangle &= 0 \,, \qquad \text{confinement} \,, \\ \langle L\rangle &\neq 0 \,, \qquad \text{deconfinement} \,. \end{aligned}$$

Baryon chemical potential

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}(i\not{D} - m_0 + \mu_B\gamma^0)\psi - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} \\
\mathcal{Z} &= \int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-\int \mathcal{L}_E} \\
&= \int \mathcal{D}A\mathcal{D}e^{-\frac{1}{2g^2}\mathrm{tr}(F_{\mu\nu}F_{\mu\nu})}\det[\not{D} + m_0 - \mu_B\gamma_0] \,,
\end{aligned}
$$

$$
\begin{aligned}
\det[\not{D} + m_0 - \mu\gamma_0] &= \begin{pmatrix} m & iX - \mu_B \\ iX^\dagger - \mu_B & m \end{pmatrix} \\
X &= D_0 + i\tau \cdot \mathbf{D}
\end{aligned}
$$

- Positive determinant only if $\mu_B = 0$.

Isospin chemical potential

- Third component of isospin

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - igT_aA_\mu^a - m_0 + \frac{1}{2}\mu_I\gamma^0\tau_3)\psi$$

- Dirac determinant with μ_I:

$$\begin{aligned} D &= \gamma_\mu(\partial_\mu + iT_aA_\mu^a + \frac{1}{2}\mu_I\gamma^0\tau_3)\ , \\ D^\dagger &= \tau_1\gamma^5 D\tau_1\gamma^5\ . \end{aligned}$$

- Determinant positive
- Simulations with finite isopin density possible

D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **86**, 592 (2001).

Two-color QCD and baryon chemical potential

$$\mathcal{L} = \bar{\psi} i\gamma^{\mu}(\partial_{\mu} - igT_a A_{\mu}^{a} - m_0 + \mu_B \gamma^0)\psi \ .$$

- Dirac determinant with μ_B:

$$\begin{aligned} D &= \gamma_{\mu}(\partial_{\mu} - igT_a A_{\mu}^{a} - m_0 + \mu_B \gamma^0) \ , \\ D^* &= \gamma^5 C T_2 D \gamma^5 C T_2 \ . \end{aligned}$$

- Determinant positive
- Simulations with finite baryon density possible

Two-color QCD and $SU(4)$ symmetry

$$\mathcal{L} = i\bar{\psi}\gamma_\mu D_\mu \psi$$

- Introducing four-component spinor

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix}$$

- Conjugate quarks

$$\tilde{u}_R = \sigma_2 T_2 \psi_R^* \,, \qquad \text{etc}$$

J. B. Kogut, M. A. Stephanov, and D. Toublan, Phys. Lett. **B464**, 183 (1999).

- $B(q) = +1$, $B(\tilde{q}) = -1$, but $A(q) = A(\tilde{q})$
- Manifest $SU(4)$ symmetry in chiral limit:

$$\mathcal{L} = \begin{pmatrix} \psi_L^* & \tilde{\psi}_R^* \end{pmatrix} \begin{pmatrix} \sigma_\mu D_\mu & 0 \\ 0 & \sigma_\mu D_\mu \end{pmatrix} \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix}$$

$$\begin{aligned} D &= \gamma_\mu(\partial_\mu + iT_a A_\mu^a + \mu_B \gamma_0) , \\ D^* &= \gamma_5 C T_2 D \gamma_5 C T_2 . \end{aligned}$$

- Determinant positive
- Simulations with finite baryon density possible

Two-color QCD and $SU(4)$ symmetry cont'd

- $SU(4) \sim SO(6)$ sextet of σ, $\pi^{\pm}$, π^0, and (anti)Δ in chiral limit.
- Mass term breaks $SO(6)$:

$$\bar{\psi}\psi = \frac{1}{2}\bar{\Psi}\sigma_2 T \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Psi + \mathrm{h.c.}$$

- Chiral condensate breaks $SO(6)$ to $SO(5)$:
  - Five pseudo-Goldstone bosons.
  - 3 quark-antiquark states and two diquarks.

Two-color QCD and $SU(4)$ symmetry cont'd

- Baryon chemical potential:

$$
\begin{aligned}
\mu_B \bar{\psi}\gamma_0\psi &= \mu_B \Psi^\dagger \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Psi \\
SO(6) &\rightarrow SU(2)_L \times SU(2)_R \times U(1)_B
\end{aligned}
$$

- “Baryons” are bosons
- Finite baryon density is Bose-Einstein condensation of diquarks
- Superfluid rather than color superconductor

Two-color QCD and $SU(4)$ symmetry cont'd

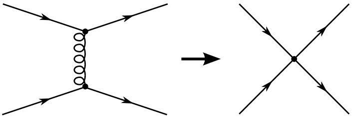
- Isospin chemical potential:

$$\mu_I\bar{\psi}\gamma_0\tau_3\psi = \mu_I\Psi^\dagger\left(\begin{array}{cc}\tau_3 & 0\\ 0 & -\tau_3\end{array}\right)\Psi$$

- Symmetry: charge conjugation of *d* quark and $\mu_B \leftrightarrow \mu_I$
- Phase diagram symmetric in μ_B-μ_I plane

NJL models

- QCD notoriously difficult theory.
- Local four-fermi interaction from one-gluon exchange.



- No gauge fields. Only global symmetries.

NJL models

- The NJL model is not asymptotically free
  - Coupling has negative dimension.
- The NJL model is not confining
- The NJL model is “nonrenormalizable”.
  - Normally hard three-dimensional cutoff Λ used - can use any regulator (?).
- The NJL model can still be used to study chiral symmetry breaking and condensation.

Two-color QCD and the NJL model

- NJL-model Lagrangian:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}[i\gamma^\mu\partial_\mu - m_0 + \tfrac{1}{2}\gamma_0(\mu_B + \tau_3\mu_I)]\psi + \mathcal{L}_1 + \xi\mathcal{L}_2 \\ \mathcal{L}_1 &= G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2 + |\bar{\psi}^C\gamma_5 T_2\tau_2|^2] \\ \mathcal{L}_2 &= G[(\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}\boldsymbol{\tau}\psi)^2 + |\bar{\psi}^C T_2\tau_2|^2]\end{aligned}$$

- $\mathcal{L}_1 + \mathcal{L}_2$ $U(4)$ invariant, $\mathcal{L}_1 - \mathcal{L}_2$ $SU(4)$ invariant.

$$\begin{aligned}\mathcal{L} &= \bar{\psi}[i\gamma^\mu\partial_\mu - m_0 + \tfrac{1}{2}\gamma_0(\mu_B + \tau_3\mu_I) - \sigma - i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi} - \xi\boldsymbol{\pi}\cdot\rho]\psi \\ &\quad + \tfrac{1}{2}(\Delta^*\bar{\psi}^C\gamma_5\sigma_2\tau_2\psi) + \mathrm{h.c} - \frac{1}{4G}(\sigma^2 + \boldsymbol{\pi}^2 + \xi\rho^2 + |\Delta|^2)\end{aligned}$$

Mean-field approximation

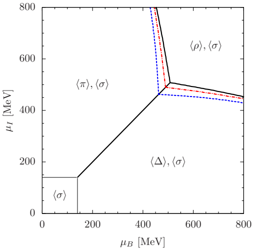
- Bilinear in fermion fields. Integrate them out.
- Effective potential in mean-field approximation:

$$\begin{aligned}\frac{\Omega}{V} &= \frac{1}{4G}\left(\sigma^2+\xi\rho^2+\Delta^2\right)\\ &\quad -T\int\frac{d^3k}{(2\pi)^3}\mathrm{Tr}\log\left[1+e^{-\beta H}\right]\end{aligned}$$

- H is 16×16 matrix. Eigenvalues found numerically.

Phase diagram

- Phase diagram at $T = 0$



- Phase diagram symmetric in μ_B and μ_I [a]
- Diquark condensation for $\mu_B > m_\pi$
- Competition between pion and diquark condensate
- Stress on condensates towards diagonal

[a] K. Splittorff, D.T. Son, and M. A. Stephanov, Phys. Rev. **D64**, 016003 (2001). JOA and T. Brauner, Phys. Rev. D **81**, 096004 (2010).

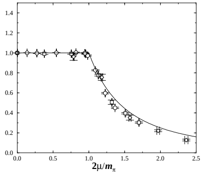
Condensates

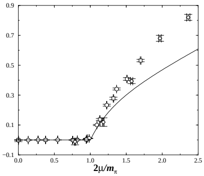
- Predictions from chiral perturbation theory and the $SO(6)$-symmetric scalar field [2]

$$
\begin{aligned}
\frac{\langle\bar{\psi}\psi\rangle}{\langle\bar{\psi}\psi\rangle_0} &= \left(\frac{\mu_c}{\mu_B}\right)^2 \\
n_B &= \mu_B\Delta^2 \\
\Delta &= f_\pi\sqrt{1-\left(\frac{m_\pi}{\mu_B}\right)^4+2\frac{\mu_B^2-m_\pi^2}{m_\sigma^2-m_\pi^2}}
\end{aligned}
$$

[2] J. B. Kogut, M. A. Stephanov, and D. Toublan, Phys. Lett. **B464**, 183 (1999), K. Splittorff, D.T. Son, and M. A. Stephanov, Phys. Rev. **D64**, 016003 (2001). JOA and T. Brauner, Phys. Rev. D **81**, 096004 (2010).

- Chiral perturbation theory vs lattice simulations [3]
  - Chiral condensate
  - Baryon density





[3] S. Hands *et al*, Eur. Phys. J **C17** 285 (2000)

Collective excitations

- Inverse propagators:

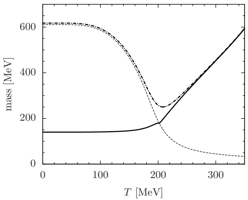
$$
\begin{aligned}
D_\sigma^{-1}(\omega) &= \frac{1}{2G} - 4\sum_{e=\pm}\int d^3k\,\frac{k^2}{\epsilon_k^2}\left[\frac{1-f(e\xi_k^{e})-f(e\xi_k^{-e})}{\omega+2e\epsilon_k}\right.\\
&\quad\left.+\frac{1-f(e\hat{\xi}_k^{e})-f(e\hat{\xi}_k^{-e})}{\omega+2e\epsilon_k}\right]\,,\\
\epsilon_k &= \sqrt{k^2+M^2}\,,\\
f(x) &= \frac{1}{e^{\beta x}+1}\,,\\
\xi_k &= \epsilon_k+\frac{e}{2}(\mu_B+\mu_I)\,,\\
\hat{\xi}_k &= \epsilon_k+\frac{e}{2}(\mu_B-\mu_I)\,.
\end{aligned}
$$

- Meson masses:

$$D^{-1}(\omega^2 = m_\sigma^2) \;=\; 0\,.$$

- Two-particle continuum:

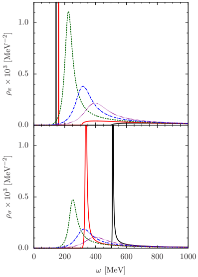
$$\omega = 2M\,.$$



Collective excitations

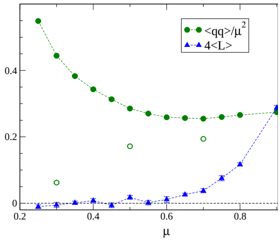
- Spectral function:

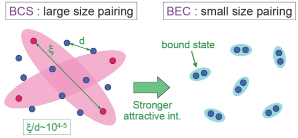
$$\rho(\omega) \;=\; \frac{\mathrm{Im}D^{-1}(\omega+i\epsilon)}{[\mathrm{Re}D^{-1}(\omega+i\epsilon)]^2+[\mathrm{Im}D^{-1}(\omega+i\epsilon)]^2}$$



Deconfinement

- Deconfinement for $\mu_B a \simeq 0.65$.
- Transition from confined nuclear matter (BEC of diquarks) to degenerate fermionic matter [4]





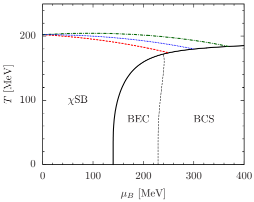
[4] S. Hands, S. Kim, J-I Skullerud, Eur. Phys. J. **C48**, 193 (2006); Phys. Rev. **D81**, 091502(R) (2010).

BEC-BCS crossover

- BEC-BCS crossover determined by

$$M = \frac{1}{2}\mu_B \, .$$

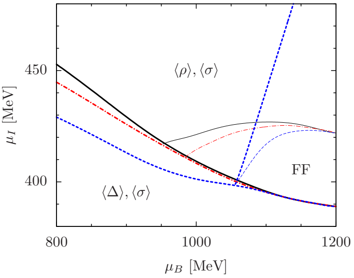
- Not a phase transition



FF phase

- Diquark condensate

$$\langle qq \rangle = \Delta e^{2i\mathbf{q}\cdot\mathbf{x}}$$



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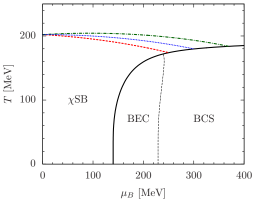

Thermodynamics

- Pressure from thermodyanmic potential, $\Omega = -\mathcal{P}V$
- Normalizing to Stefan-Boltzmann

$$P_{\rm SB} = \frac{\mu_B^4}{48\pi^2} + \frac{\mu^2 T^2}{6} + \frac{7\pi^2 T^4}{45}$$

- n_B and $\mathcal{E}_B$ follow from thermodynamic identities
- Chiral pt via linear sigma model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)^2 + \frac{1}{2}M^2\Phi^2 + \frac{1}{4}\Phi^4 - H\Phi$$

Thermodynamics

- Linear sigma model

$$
\begin{aligned}
\frac{\mathcal{P}}{\mathcal{P}_{\rm SB}} &= 24\pi^2\left(\frac{f_\pi}{m_\pi}\right)^2\left(1-\frac{1}{x^2}\right)^2\left(\frac{1}{x^2}+\frac{1}{\tau^2-1}\right) \\
\frac{n}{n_{\rm SB}} &= \frac{12\pi^2}{x^2}\left(\frac{f_\pi}{m_\pi}\right)^2\left(1-\frac{1}{x^4}+2\frac{x^2-1}{\tau^2-1}\right) \\
\frac{\mathcal{E}}{\mathcal{E}_{\rm SB}} &= 8\pi^2\left(\frac{f_\pi}{m_\pi}\right)^2\frac{x^2-1}{x^4}\left(1+\frac{3}{x^2}+\frac{3x^2+1}{\tau^2-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
x &= \frac{\mu_B}{m_\pi} \\
\tau &= \frac{m_\sigma}{m_\pi}
\end{aligned}
$$