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Phase diagram of QCD in a strong magnetic field

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Outline

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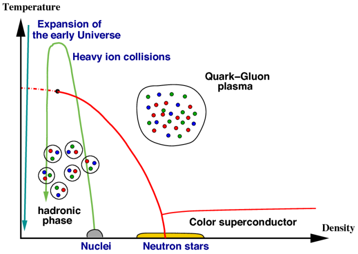
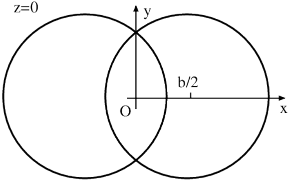
Models, DS, and FRG

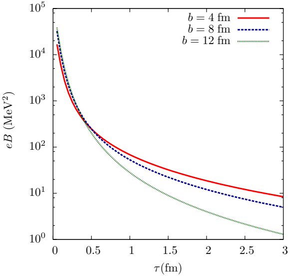
Phase diagram of QCD



Hadronic matter in strong magnetic fields

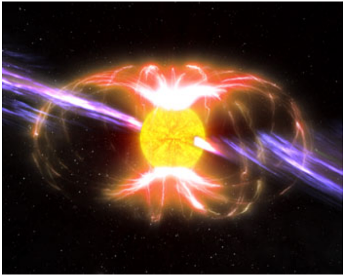
1. Noncentral heavy-ion collisions [1]





[1] Kharzeev, McLerran, and Warringa (2008), Skokov, Illarianov, and Toneev (2009)

2. Magnetars $B = 10^{14} - 10^{15}$G.[2]



3. Electroweak phase transition $B = 10^{23}$ G. [3]

[2] Duncan and Thompson

[3] Vachaspati, Enqvist and Olesen. Rummukainen et. al.

Conclusions

1. Chiral models and lattice predict magnetic catalysis at $T = 0$: Quark condensate is an increasing function of B.
2. Chiral models show magnetic catalysis at temperatures around T_c: T_c is an increasing function of B.
3. Lattice simulations show inverse magnetic catalysis for physical quark masses at temperatures around T_c: T_c is a decreasing function of B.
4. Critical temperature and order of phase transition depends on treatment of vacuum fluctuations - take them seriously.

[4]

[4] JOA+William Nalyor+ Anders Tranberg arXiv:1411.7176

Chiral models

Quark-meson model

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\Big[\gamma_\mu\partial_\mu + g(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\Big]\psi + \frac{1}{2}\Big[(\partial_\mu\sigma)^2 + (\partial_\mu\boldsymbol{\pi})^2\Big] \\ &\quad + \frac{1}{2}m^2\Big[\sigma^2 + \boldsymbol{\pi}^2\Big] + \frac{\lambda}{24}\Big[\sigma^2 + \boldsymbol{\pi}^2\Big]^2 - h\sigma\ .\end{aligned}$$

1. $O(4)$-symmetry broken to $O(3)$ by chiral condensate. Three Goldstone bosons (pions).
2. Magnetic field breaks $SU(2)$ isopin symmetry. Only the neutral pion is a Goldstone mode.
3. $\sigma^2 + \boldsymbol{\pi}^2 \to (\sigma^2 + \pi_0^2) + 2(\pi^+\pi^-)$ with two separate couplings.

Mean-field approximation

1. Background and quantum field

$$\sigma \quad \to \quad \phi + \tilde{\sigma} \; .$$

2. Omit bosonic quantum and thermal fluctuations

3. Effective potential:

$$\begin{aligned} \mathcal{F}_{0+1} &= \frac{1}{2}m^2\phi^2 + \frac{\lambda}{24}\phi^4 - h\phi - \mathrm{Tr}\log S^{-1} \; , \\ \mathrm{Tr}\log S^{-1} &= \sum_f \mathrm{Tr}\log[i\gamma_\mu(P_\mu + q_f A_\mu) + m_q] \; , \\ m_q &= g\phi \; . \end{aligned}$$

One-loop contribution

$$\begin{aligned}
\mathcal{F}_1 &= -\sum\!\!\!\!\!\!\!\!\int_{\{P\}}^{B} \ln\left[P_0^2+p_z^2+M_B^2\right] \\
&= -\frac{|q_fB|}{2\pi}\sum_{s,k,P_0}\int\frac{dp_z}{2\pi}\ln\left[P_0^2+p_z^2+m_q^2+|q_fB|(2k+1-s)\right] \ . \\
\mathcal{F}_1^{T=0} &= -\frac{|q_fB|}{2\pi}\sum_{s=\pm1}\sum_{k=0}^{\infty}\int\frac{dp_z}{2\pi}\sqrt{p_z^2+m_q^2+|q_fB|(2k+1-s)}
\end{aligned}$$

1. Zero-temperature part is regularized using dimensional regularization and zeta-function regularization

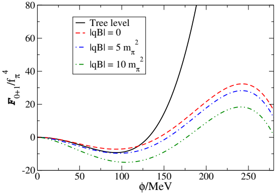
$$
\begin{aligned}
F_1^{T=0} &= \frac{(q_fB)^2}{2\pi^2}\left(\frac{e^{\gamma_E}\Lambda^2}{2|q_fB|}\right)^{\epsilon}\Gamma(-1+\epsilon)\left[\zeta(-1+\epsilon,x_f)-\frac{1}{2}x_f\right] \\
&= \frac{1}{(4\pi)^2}\left(\frac{\Lambda^2}{2|q_fB|}\right)^{\epsilon}\left[\left(\frac{2(q_fB)^2}{3}+m_q^4\right)\left(\frac{1}{\epsilon}+1\right)\right. \\
&\quad \left. -8(q_fB)^2\zeta^{(1,0)}(-1,x_f)-2|q_fB|m_q^2\ln x_f+\mathcal{O}(\epsilon)\right], \\
x_f &= \frac{m_q^2}{2|q_fB|}\,.
\end{aligned}
$$

2. Must renormalize g and B.
3. Parameters tuned to reproduce vacuum physics, i.e. pion mass etc.

Renormalized one-loop effective potential

$$
\begin{aligned}
\mathcal{F}_{0+1} &= \frac{1}{2}B^2\left[1+N_c\sum_f\frac{4q_f^2}{3(4\pi)^2}\ln\frac{\Lambda^2}{|2q_fB|}\right]+\frac{1}{2}m^2\phi^2+\frac{\lambda}{24}\phi^4 \\
&\quad -\frac{N_c}{2\pi^2}\sum_f(q_fB)^2\left[\zeta^{(1,0)}(-1,x_f)+\frac{1}{2}x_f\ln x_f-\frac{1}{4}x_f^2\right. \\
&\quad \left.-\frac{1}{2}x_f^2\ln\frac{\Lambda^2}{2|q_fB|}\right]-\frac{N_c}{8\pi^2}\sum_f|q_fB|K_0^B(\beta m_q)\,. \\
K_0^B(\beta m) &= 8\sum_{s,k}\int_0^\infty\frac{p_z^2\,dp_z}{\sqrt{p_z^2+M_B^2}}\frac{1}{e^{\beta\sqrt{p_z^2+M_B^2}}+1}\,.
\end{aligned}
$$

Magnetic catalysis at $T = 0$



1. One-loop effective potential unstable.

2. By subtracting one-loop effective potential at $B = 0$, one avoids the instability.

Free energy:

$$\mathcal{F} = \frac{M^2}{2G} + \frac{1}{(4\pi)^2}\left[\frac{1}{2}\Lambda^4 - 2\Lambda^2 M^2 + \frac{1}{2}M^4 + M^4 \ln\frac{M^2}{\Lambda^2}\right]$$

Gap equation in NJL model

$$M\left[\frac{4\pi^2}{G} - \Lambda^2 + M^2 \ln\frac{\Lambda^2}{M^2}\right] = 0 \,.$$

Nontrivial solution for

$$G > G_c = \frac{4\pi^2}{\Lambda^2}$$

Gap equation for nonzero B

$$\frac{4\pi^2}{G} - \Lambda^2 + M^2 \ln\frac{\Lambda^2}{M^2} - |2q_f B| \Big[\zeta^{(1,0)}(0, x_f) + x_f - \frac{1}{2}(2x_f - 1)\ln x_f\Big] = 0\,.$$

Solution for $G < G_c$

$$M^2 = \frac{|qB|}{\pi}\exp\left[-\frac{1}{|q_f B|}\left(\frac{4\pi^2}{G} - \Lambda^2\right)\right]\,.$$

Dynamical symmetry breaking. Same form in LLL approximation. Dimensional reduction $3+1 \to 1+1$ dimensions. [5]

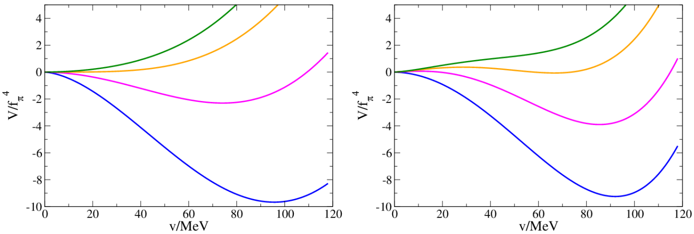
[5] Gusynin, Miransky, and Shovkovy (1995)

Solution for $G > G_c$

$$M^2 = M^2(B=0)\left[1+\frac{1}{2}\beta\frac{q_f^2B^2}{M^4(B=0)}+...\right]$$

Governed by the QED β-function.

Chiral transition



1. Order of phase phase transition depends on treatment of vacuum fluctuations [6]

2. First-order vs crossover at the physical point.

[6] Khan+JOA (2012)

Coupling to the Polyakov loop

1. NJL model describes chiral symmetry breaking but is not confining

2. Couple the model to the Polyakov loop which the order parameter for confinement [7]

[7] Fukushima (2003)

Thermal Wilson line L and Polyakov loop

$$\begin{aligned} L &= \mathcal{P}e^{i\int_0^\beta d\tau A_0(x,\tau)} \,. \\ l &= \frac{1}{N_c}\mathrm{Tr}\langle L\rangle \end{aligned}$$

Temperature behavior

$$\Phi = \langle l\rangle \sim 0 \,, \text{confinenment at low } T \,,$$

$$\Phi = \langle l\rangle \sim 1 \,, \text{confinenment at high } T \,.$$

Adding the Polyakov loop by introducing a constant background A_0

$$\partial_\mu \quad \to \quad \partial_\mu - i\delta_{0\mu}A_\mu \,.$$

Distribution function

$$N_F \quad \to \quad \frac{1+2\Phi e^{\beta E_q}+\Phi e^{2\beta E_q}}{1+3e^{\beta E_q}+3\Phi e^{2\beta E_q}+e^{3\beta E_q}}\;.$$

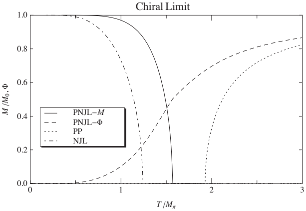
Limits

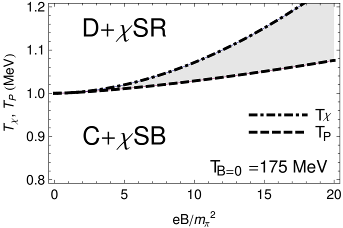
$$N_F \quad \to \quad \frac{1}{e^{3\beta E_q}+1}\,, \qquad\qquad \Phi=0\;.$$
$$N_F \quad \to \quad \frac{1}{e^{\beta E_q}+1}\,, \qquad\qquad \Phi=1\;.$$

Pure-glue potential

$$\frac{\mathcal{U}}{T^4} \quad = \quad -\frac{1}{2}a(T)\Phi^2+b(T)\ln[1-6\Phi^2+8\Phi^3-3\Phi^4]\;.$$

$a(T)$ and $b(T)$ determined to reproduce pure-glue QCD thermodynamics.





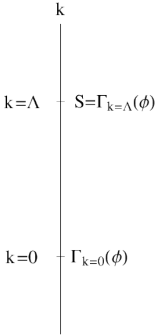
Chiral and deconfinement transitions at $B = 0$ (left) and as functions of eB/m_π^2 (right) [8]

[8] Amador and JOA (2013). Gatto and Ruggieri (2010)

Functional renormalization group

1. Method to implement Wilson's idea of integrating out momentum shell successively. [9]

2. Average effective action $\Gamma_k[\phi]$ that interpolates between classical action and full quantum action



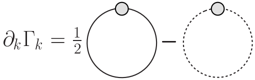
[9] Wetterich, *Nucl. Phys. B* **352** (1991) 529, Berges, Pawlowski, Schaefer and Wambach, Skokov, von Smekal...

Exact flow equation for $\Gamma_k[\phi]$

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\left[\partial_k R_k(q)\left[\Gamma_k^{(2)} + R_k(q)\right]^{-1}\right] + \text{term for fermions} \,.$$

$R_k(q)$ is a regulator function that implement the renormalization group ideas.

Feynman diagram for flow equation



Ansatz for effective action

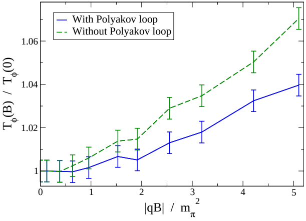
$$\Gamma_k[\phi] = \int_0^\beta d\tau \int d^3x \left\{ \frac{1}{2} \left[(\partial_\mu \sigma)^2 + (\partial_\mu \boldsymbol{\pi})^2 \right] \right. \\ \left. + U_k(\phi) + \bar{\psi}\gamma_\mu \partial_\mu \psi + g_k \bar{\psi} \left[\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} \right] \psi \right\} \;,$$

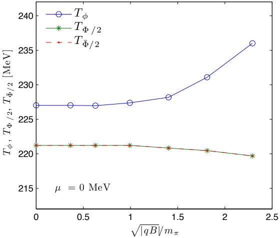
Flow equation for effective potential $U_k(\phi)$ with boundary condition

$$\Gamma_{k=\Lambda}[\phi] = S_{\rm classical} \;,$$
$$S_{\rm classical} = \int d^4x \left[\frac{1}{2} m_\Lambda^2 \phi^2 + \frac{\lambda_\Lambda}{24} \phi^4 \right] \;.$$

m_Λ^2 and λ_Λ tuned to reproduce vacuum physics $T = B = 0$ for $k = 0$.

Transition temperatures as functions of $|qB|/m_\pi^2$





1. Less magnetic catalysis at finite temperature with the Polyakov loop
2. Deconfinement transition hardly affected by finite B [10]

[10] JOA+Naylor+Tranberg

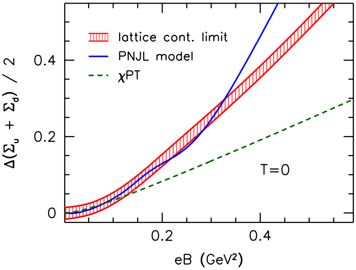
Lattice calculations

Partition function

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x \int_0^\beta d\tau \bar{\psi}(\not{D}(B)+m)\psi} \\
&= \int \mathcal{D}A_\mu e^{-S_g \det(\not{D}(B)+m)} . \\
S_g &= \frac{1}{4}\int d^3x \int_0^\beta d\tau \mathrm{Tr}[G_{\mu\nu}G_{\mu\nu}] , \\
\not{D}(B) &= \begin{pmatrix} 0 & iX \\ iX^\dagger & 0 \end{pmatrix} , \\
iX &= D_0 + i\sigma\cdot\mathbf{D} \\
\det(\not{D}+m_f) &= \det\left[X^\dagger X + m_f^2\right] .
\end{aligned}
$$

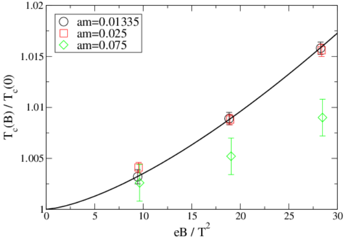
Manifestly positive fermion determinant - no sign problem

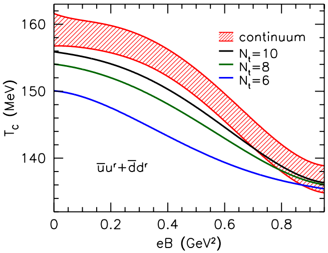
Quark condensate at $T = 0$ [11]



[11] Lattice results by Bali et al (2012).

Transition temperature[12]





Inverse magnetic catalysis for physical quark masses and magnetic catalysis for large quark masses.

12 D'Elia et al 2010 (left) and Bali et al 2012 (right).

Partition function and quark condensate:

$$
\begin{aligned}
\mathcal{Z}(B) &= \int d\mathcal{U} e^{-S_g} \det(\not{D}(B)+m) \; , \\
\langle\bar{\psi}\psi\rangle &= \frac{1}{\mathcal{Z}(B)} \int d\mathcal{U} e^{-S_g} \det(\not{D}(B)+m) \mathrm{Tr}(\not{D}(B)+m)^{-1} \; ,
\end{aligned}
$$

Valence contribution:

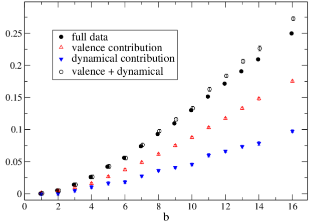
$$
\langle\bar{\psi}\psi\rangle^{\mathrm{val}} = \frac{1}{\mathcal{Z}(0)} \int d\mathcal{U} e^{-S_g} \det(\not{D}(0)+m) \mathrm{Tr}(\not{D}(B)+m)^{-1} \; .
$$

Sea contribution:

$$
\langle\bar{\psi}\psi\rangle^{\mathrm{sea}} = \frac{1}{\mathcal{Z}(B)} \int d\mathcal{U} e^{-S_g} \det(\not{D}(B)+m) \mathrm{Tr}(\not{D}(0)+m)^{-1} \; .
$$

Relative increment of valence and sea contributions to the quark condensate.[13]

$$r = \frac{\langle\bar{\psi}\psi\rangle(B)}{\langle\bar{\psi}\psi\rangle(0)} - 1\,.$$

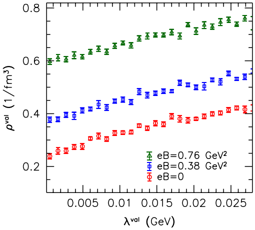


[13] D'Elia et al (2010).

Banks-Casher relation

$$\lim_{V\to\infty}\lim_{m\to 0}\langle\bar{\psi}\psi\rangle \quad \sim \quad \lim_{V\to\infty}\lim_{m\to 0}\rho(\lambda) \; .$$

Spectral density for different values of B [14]



[14] Endrodi et al (2013).

Average

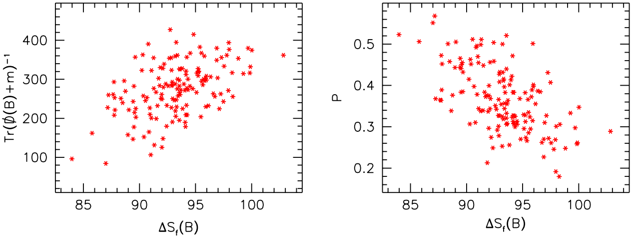
$$\begin{aligned} \langle \mathcal{O} \rangle_B &= \frac{\langle e^{-\Delta S_f(B)} \mathcal{O} \rangle_0}{\langle e^{-\Delta S_f(B)} \rangle_0} \,, \\ -\Delta S_f(B) &= \log\det(\not{D}(B) + m) - \log\det(\not{D}(0) + m) \end{aligned}$$

Valence and full condensate:

$$\begin{aligned} \langle \bar{\psi}\psi \rangle^{\rm val} &= \langle \mathrm{Tr}(\not{D}(B) + m)^{-1} \rangle_0 \\ \langle \bar{\psi}\psi \rangle &= \frac{\langle e^{-\Delta S_f(B)} \mathrm{Tr}(\not{D}(B) + m)^{-1} \rangle_0}{\langle e^{-\Delta S_f(B)} \rangle_0} \,, \end{aligned}$$

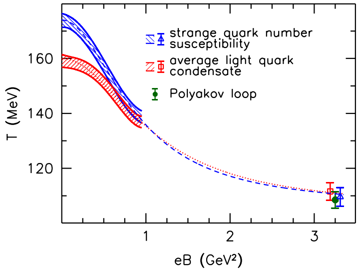
Phase diagram of QCD in a strong magnetic field

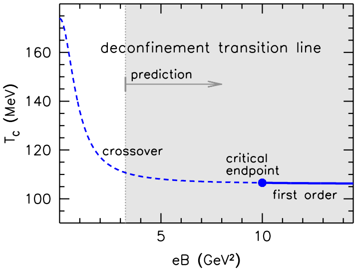
Scatter plot of quark condensate and the Polyakov loop as a function of $\Delta S(B)$[15]



[15] Endrodi et al (2013).

Very large magnetic fields[16]



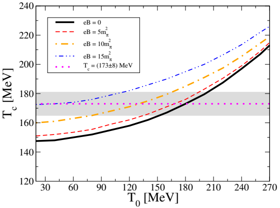


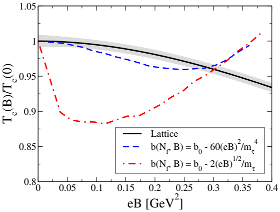
Anisotropic pure-glue theory in the limit $B \to \infty$.

[16] Endrodi (2015)

Models, DS, and FRG

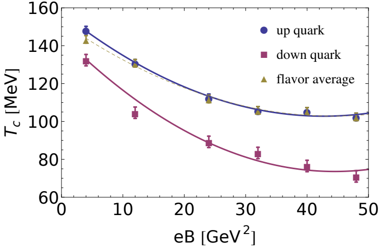
- Assume B-dependent coupling G constant to fit to lattice data
- Assume B-dependent T_0 in pure-glue potential[17]





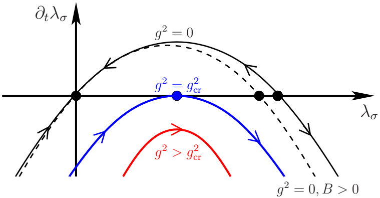
[17] Fraga et al (2014)

Dyson-Schwinger [18]



[18] Braun et al (2014) and Muller Pawlowski (2015).

Fixed-point analysis[19]



Critical coupling increases with T, signalling chiral symmetry restoration, and decreasing with B, signalling inverse magnetic catalysis.

[19] Braun et al (2014).

Thanks for your attention!