NTNU
Fakultet for Naturvitskap og Teknologi
Institutt for Fysikk

Exam TFY4305 Nonlinear dynamics
Fall 2012

Lecturer: Professor Jens O. Andersen
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Friday desember 7 2012
09.00h-13.00h

You may bring:
Approved calkulator
Rottmann: Matematisk Formelsamling
Rottmann: Matematische Formelsammlung
Barnett & Cronin: Mathematical Formulae

The problem set is three pages. Read carefully. Ask if in doubt. Viel Glück!

Problem 1

The equation for an anharmonic oscillator with damping can be written as

$$\ddot{x} + b\dot{x} - kx + x^3 = 0 , \tag{1}$$

where b and k are real parameters.

a) What is the interpretation of the sign of b og k? For which values of b og k is the system conservative?

b) Show that equation (1) can be written as

$$\dot{x} = y \,, \tag{2}$$
$$\dot{y} = -by + kx - x^3 \,. \tag{3}$$

c) Find the fix point and the associated Jacobian matrix for equations (2)-(3).

d) Find the eigenvalues of the various fixed points that you found in c).

e) Let $b \neq 0$ and $k \neq 0$. Classify the various fixed points as functions of b and k.

f) Consider the special case $b = 0$. Classify the fixed points as functions of k.

g) Consider the special case $k = 0$. Classify the fixed points as functions of b. Find the critical value b_c for b at which the system undergoes a bifurcation. What type of bifurcation is this? Hint: Do not linearize, use physical intuition.

Problem 2

Given the set of equations

$$\dot{x} = x - y - x^3 \,, \tag{4}$$
$$\dot{y} = x + y - y^3 \,. \tag{5}$$

a) Find the fixed points of the equations given above. Hint: The polynomial $x^8 - 3x^6 + 3x^4 - 2x^2 + 2$ does not have zeros on the real axis.

b) Show that the system has at least one periodic solution Hint: The Poincare-Bendixon theorem and $\frac{1}{2} \leq \cos^4 x + \sin^4 x \leq 1$.

Problem 3

Consider the tent map which is defined by

$$t(x) = \begin{cases} rx \,, & 0 \leq x \leq \frac{1}{2} \,, \\ r(1-x) \,, & \frac{1}{2} \leq x \leq 1 \,, \end{cases} \tag{6}$$

where $0 \leq r \leq 2$ is a real parameter and $x \in [0, 1]$.

a) For which values of r is the fixed point $x = 0$ stable? For which values of r is $x = 0$ globally stable? Hint: Use a cobweb.

b) Show that the points $(p, q) = (\frac{r}{1+r^2}, \frac{r^2}{1+r^2})$ form a period-2 cycle and find the values of r for which they exist.

c) Is the periode-2 cycle in b) stable? For which values of r does the tent map exhibit chaos?