The phase diagram of neutral quark matter: Pseudoscalar diquark condensates revisited

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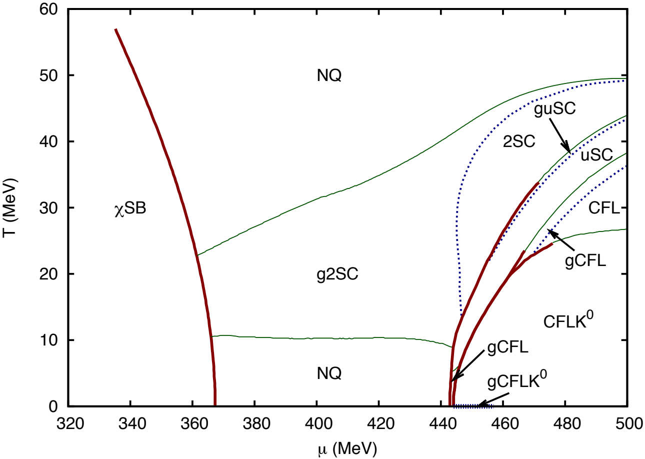
arXiv:0912.3411

HIC for FAIR Helmholtz International Center

HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

The object of interest:
The phase diagram of neutral quark matter



Particles and charges

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Particles

Quarks:

- up ($m_u = 5.5$ MeV)
- down ($m_d = 5.5$ MeV)
- strange ($m_s = 140.7$ MeV)

Leptons:

- electrons (massless)
- muons ($m_\mu = 105.7$ MeV)

Charges:

- Total quark number n
- electric charge n_Q
- two color charges:
  - $n_3 = n_r - n_g$
  - $n_8 = (n_r + n_g - 2n_b)/\sqrt{3}$

Chemical potentials:

$$\hat{\mu}_q = \mu + \mu_Q Q + \mu_3 \lambda_3 + \mu_8 \lambda_8 \quad (1)$$

$$\mu_e = -\mu_Q \quad (2)$$

$$\mu_\mu = -\mu_Q \quad (3)$$

Lagrangian

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$$\mathcal{L} = \bar{q}(i\not{\partial} - \hat{m} + \gamma_0 \hat{\mu})q + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq} + \mathcal{L}_6 \,, \tag{4}$$

with:

$$\mathcal{L}_{\bar{q}q} = G \sum_{a=0}^{8} \left[(\bar{q}\tau_a q)^2 + (\bar{q} i\gamma_5 \tau_a q)^2 \right], \tag{5}$$

$$\mathcal{L}_6 = -K \; \{\det{}_f \left[\bar{q}\,(1+\gamma_5)\, q\right] + \det{}_f \left[\bar{q}\,(1-\gamma_5)\, q\right]\} \tag{6}$$

and

$$\begin{aligned} \mathcal{L}_{qq} = H \sum_{A,B=2,5,7} \Big[& \left(\bar{q} i\gamma_5 \tau_A \lambda_B C \bar{q}^T\right) \left(q^T C i\gamma_5 \tau_A \lambda_B q\right) \\ & + \left(\bar{q}\tau_A \lambda_B C \bar{q}^T\right) \left(q^T C \tau_A \lambda_B q\right) \Big] \end{aligned} \tag{7}$$

Condensates
Mean field approximation

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Scalar and pseudoscalar diquark condensates:

$$\Delta_{AB}^{(s)} = 2H\langle q^T C\gamma_5\tau_A\lambda_B q\rangle \quad (8)$$

$$\Delta_{AB}^{(p)} = 2H\langle q^T C\tau_A\lambda_B q\rangle \qquad A, B \in \{2, 5, 7\} \quad (9)$$

with λ_a the Gell-Mann matrices in color space
and τ_a the Gall-Mann matrices in flavor space.

$$C = i\gamma^2\gamma^0 \quad (10)$$

The antiquark-quark condensates

$$\phi_f =< \bar{q}_f q_f >, \quad f \in \{u, d, s\} \quad (11)$$

Phases

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	$\Delta_{22}^{(s)}$	$\Delta_{55}^{(s)}$	$\Delta_{77}^{(s)}$	$\Delta_{25}^{(p)}$	$\Delta_{52}^{(p)}$	$\Delta_{27}^{(p)}$	$\Delta_{72}^{(p)}$	$\Delta_{57}^{(p)}$	$\Delta_{75}^{(p)}$
NQ	-	-	-	-	-	-	-	-	-
2SC	$\times$	-	-	-	-	-	-	-	-
uSC	$\times$	$\times$	-	-	-	-	-	-	-
p2SC	$\times$	-	-	$\times$	-	-	-	-	-
CFL	$\times$	$\times$	$\times$	-	-	-	-	-	-
CFL+K^0	$\times$	$\times$	$\times$	$\times$	$\times$	-	-	-	-
CFL+$\pi^{\pm}$	$\times$	$\times$	$\times$	-	-	$\times$	$\times$	-	-
CFL+$K^{\pm}$	$\times$	$\times$	$\times$	-	-	-	-	$\times$	$\times$

CFL $\rightarrow$ CFLK^0

CFL condensates break:

$$SU(3)_{\text{color}} \times SU(3)_V \times SU(3)_A \times U(1) \quad \Rightarrow \quad SU(3)_{\text{color}+V} \tag{12}$$

$$q \rightarrow \exp\left(i\theta_a\left(\tau_a - \lambda_a^T\right)\right) q \tag{13}$$

Goldstone theorem $\Rightarrow$ degenerate ground states

$$q \rightarrow \exp\left(i\theta_a \frac{\tau_a}{2}\gamma_5\right) q \tag{14}$$

Rotates the condensates (a=6,7)

$$\Delta_{22}^{(s)} \rightarrow \Delta_{52}^{(p)} \tag{15}$$

$$\Delta_{55}^{(s)} \rightarrow \Delta_{25}^{(p)} \tag{16}$$

T. Schäfer, Phys. Rev. Lett. 85. 5531 (2000)

CFLK^0

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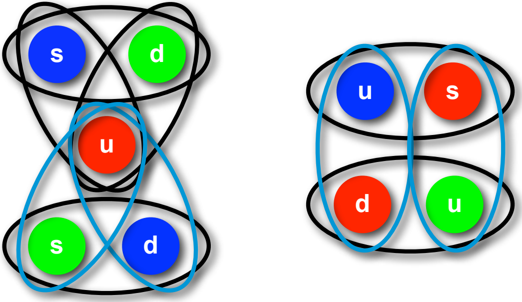
Diquark condensates

$$\Delta_{22}^{(s)}, \Delta_{55}^{(s)}, \Delta_{77}^{(s)}, \Delta_{25}^{(p)}, \Delta_{52}^{(p)} \tag{17}$$

- K^0 lightest Goldstone boson
- less strangeness than CFL



Thermodynamic potential I (Mean field)

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$$\Omega = \Omega_q + \underbrace{\Omega_e + \Omega_\mu}_{\text{ideal gas}} \tag{18}$$

$$\mathcal{L}^{MF} = \bar{\Psi} S^{-1} \Psi - \mathcal{V}, \qquad \Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} q \\ C\bar{q}^T \end{pmatrix} \tag{19}$$

$$\mathcal{V} \ = \ 2G\left(\phi_u^2 + \phi_d^2 + \phi_s^2\right) - 4K\phi_u\phi_d\phi_s + \frac{1}{4H} \sum_{A,B=2,5,7} \left({\Delta_{AB}^{(s)}}^2 + {\Delta_{AB}^{(p)}}^2 \right) \tag{20}$$

inverse propagator:

$$S^{-1} = \begin{pmatrix} \not{p} + \hat{\mu}\gamma^0 - \hat{M} & \sum \left(\Delta_{AB}^{(s)} \gamma_5 \tau_A \lambda_B + \Delta_{AB}^{(p)} \tau_A \lambda_B \right) \\ \sum \left(-\Delta_{AB}^{(s)} \gamma_5 \tau_A \lambda_B + \Delta_{AB}^{(p)} \tau_A \lambda_B \right) & \not{p} - \hat{\mu}\gamma^0 - \hat{M} \end{pmatrix} \tag{21}$$

$$M_a = m_a - 4G\phi_a + 2K\phi_b\phi_c \tag{22}$$

Thermodynamic potential II

$$\Omega_q(T,\{\mu_i\}) = -T\sum_n \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\mathrm{Tr}\ln\left(\frac{S^{-1}(i\omega_n,\vec{p})}{T}\right) + \mathcal{V} \tag{23}$$

$$= -\int \frac{d^3p}{(2\pi)^3}\sum_{j=1}^{18}\left(|\epsilon_j| + 2\ T\ \ln\left(1+e^{-\frac{|\epsilon_j|}{T}}\right)\right) + \mathcal{V} \tag{24}$$

$$\frac{\partial}{\partial x}\Omega(T\{\mu_i\}) = -\frac{T}{2}\sum_n \int \frac{\mathrm{d}^3p}{(2\pi)^3}\sum_{i=1}^{72}\frac{1}{i\omega_i+\epsilon_i}\left(U^\dagger\frac{\partial M}{\partial x}U\right)_i + \frac{\partial \mathcal{V}}{\partial x} \tag{25}$$

where ϵ_i are the positive eigenvalues of S^{-1}, $M = p_0 - \gamma_0 S^{-1}$ and $U/U^\dagger$ diagonalizes S^{-1}.

Gap equations and parameters

- gap equations:

$$\frac{\partial\Omega}{\partial\phi_f} = 0, \quad \frac{\partial\Omega}{\partial\Delta_{AB}^{(s)}} = 0, \quad \frac{\partial\Omega}{\partial\Delta_{AB}^{(p)}} = 0 \qquad (26)$$

- neutrality conditions:

$$n_Q = -\frac{\partial\Omega}{\partial\mu_Q} = 0, \quad n_3 = -\frac{\partial\Omega}{\partial\mu_3} = 0, \quad n_8 = -\frac{\partial\Omega}{\partial\mu_8} = 0. \qquad (27)$$

Parameters:

$$m_u, m_d, m_s, G, K, \Lambda \qquad (28)$$

fitted to vacuum meson masses and decay constants by

P. Rehberg, S.P. Klevansky and J. Hufner, Phys. Rev. C 53, 410 (1996)

$$H = \frac{3}{4}G \qquad (H = G) \qquad (29)$$

Phase diagram
$H = \frac{3}{4} G$

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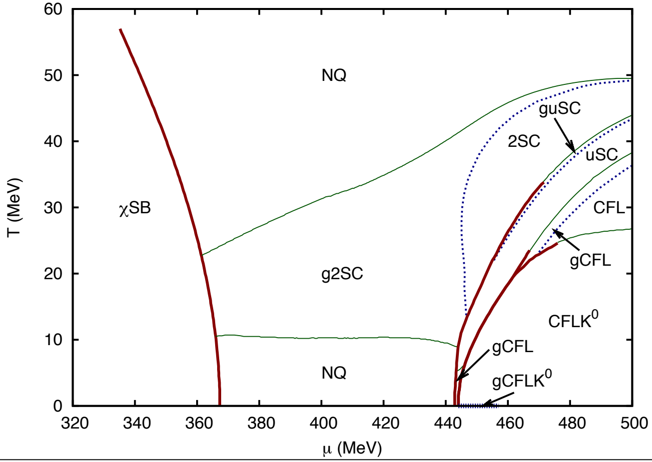


Comparison with previous results

Without pseudoscalar condensates:

S.B. Rüster, V. Werth, M. Buballa, I.A. Shovkovy, D. Rischke
Phys. Rev. D 72, 034004 (2005)

✓

With pseudoscalar condensates:

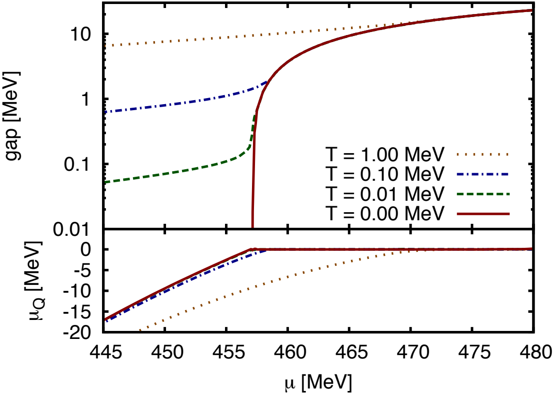
Harmen J. Warringa
arXiv:hep-ph/0606063

$\approx$ ✓

Gapless CFLK^0 phase: The gap

Gapless CFLK^0 phase:

- only at very low T
- shortly before CFLK^0 becomes disfavored



$\Rightarrow$ Compatible with the results for the CFL phase

M. Alford, C. Kouvaris and K. Rajagopal, Phys. Rev. D71 054009 (2005)

gCFL window lower μ than CFLK^0

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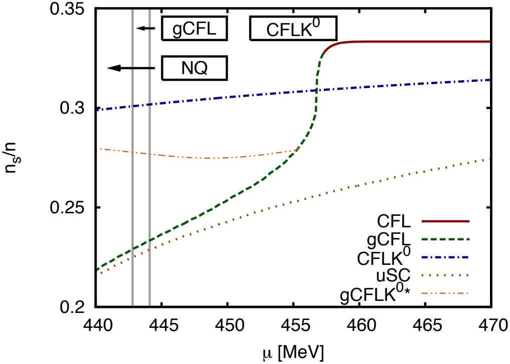
The strange quark fraction:



p2SC phase: Observations

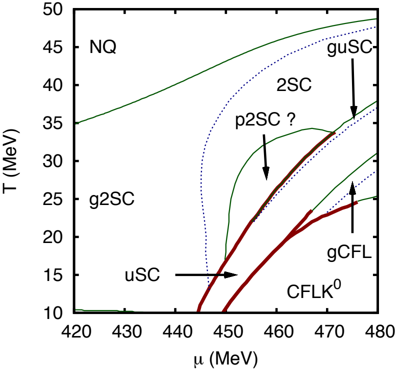
Numerically we (and arXiv:hep-ph/0606063) find a p2SC phase: $\Delta_{22}^{(s)}, \Delta_{25}^{(p)}$.

axial color rotation

$$q \rightarrow \exp\left(i\theta \frac{\lambda_7}{2}\gamma_5\right) q \qquad (30)$$

good numerical agreement with

$$\left(\Delta_{22}^{(s)}\right)^2_{\text{p2SC}} + \left(\Delta_{25}^{(p)}\right)^2_{\text{p2SC}} = \left(\Delta_{22}^{(s)}\right)^2_{\text{2SC}} \qquad (31)$$



p2SC phase: Axial color rotations

No Goldstone bosons in the 2SC phase to explain p2SC.

Axial color rotations are;

- not a symmetry of QCD
- not a symmetry of our Lagrangian $\mathcal{L}$
- a symmetry of Ω_{MF} in the case $M_u = M_d = 0$

So, in the chiral limit in mean field approximation; 2SC can be rotated into p2SC.

In out case:

- small Masses $(10 - 20\ \mathrm{MeV})$
- approximate symmetry

p2SC phase: rotating away the color charge

In the 2SC phase: $q \rightarrow \exp\left(i\theta_a \frac{\lambda_a}{2}\right) q$ rotates n_8 into n_1, n_4, n_6.

M. Buballa and I.A. Shovkovy, Phys. Rev. D 72, 097501 (2005)

With axial color rotations

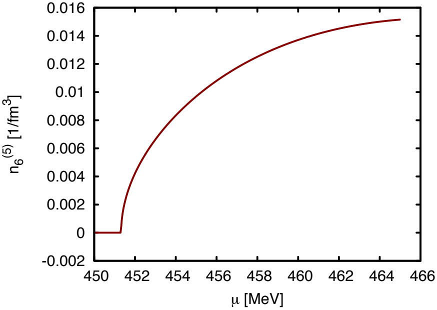
$$n_a^{(5)} = \langle q^\dagger \gamma_5 \lambda_a q \rangle \qquad (32)$$

$$n_8\big|_{\text{p2SC}} = \frac{1}{4}(1 + 3\cos\theta)\, n_8\big|_{\text{2SC}}\,,$$

$$n_3\big|_{\text{p2SC}} = \frac{\sqrt{3}}{4}(1 - \cos\theta)\, n_8\big|_{\text{2SC}}\,,$$

$$n_6^{(5)}\Big|_{\text{p2SC}} = -\frac{\sqrt{3}}{2}\sin\theta\, n_8\big|_{\text{2SC}}\,. \qquad (33)$$

$T = 25$ MeV



Summary

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Model:

- NJL type model
- including pseudoscalar condensates
- mean field approximation

Results:

- Phase diagram mostly agree with previous results
- Better insights of the situation at small temperatures
- $gCFLK^0$ phase at very small temperatures
- p2SC phase probably an artifact of the mean field approximation