Crossover to Cluster Plasma in the Gas of Quark-Gluon Bags

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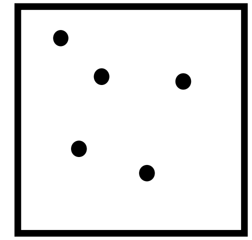
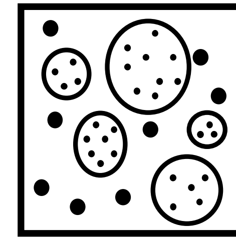
V.B., M.I. Gorenstein, W. Greiner, J. Phys. G. (2009)

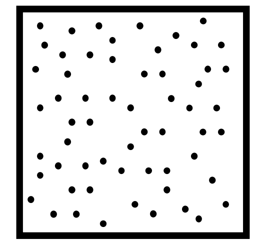
Some history of the Bag model

Chodos, Jaffe, Johnson, Thorn, Weisskopf,
Bag model (1974)



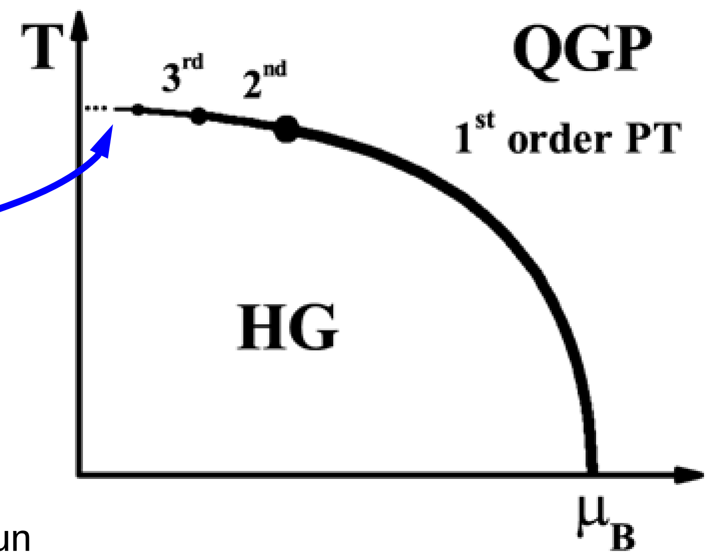


Gorenstein, Petrov, Zinovjev,
Excluded volume (1981)



Rischke, Gorenstein, Schafer, Stocker, Greiner,
Clusters (1992)

Gorenstein, Gazdzicki, Greiner,
Critical Line of PT (2005)



Ferroni, Koch,
Crossover (2009)
(no excluded volume)

Gas of quark-gluon bags

Bag spectrum

$$Z(V,T) = \sum_{N=0}^{\infty} \frac{1}{N!} \prod_{i=1}^{N} \int dm_i dv_i \, \rho(m_i, v_i) \, \phi(T, m_i)$$

$$\times \left(V - \sum_{j=1}^{N} v_j \right)^N \theta\left(V - \sum_{j=1}^{N} v_j \right)$$

Excluded volume

particle number density

$$\phi(T,m) \equiv \frac{1}{2\pi^2} \int_0^{\infty} k^2 dk \; \exp\left[- \frac{(k^2+m^2)^{1/2}}{T} \right] = \frac{m^2 T}{2\pi^2} K_2\left(\frac{m}{T}\right)$$

$$\equiv \frac{\overline{N}^{id}}{V}$$

Gorenstein, Petrov, Zinovjev, Phys. Lett. B (1981)

Gorenstein, Petrov, Shelest, Zinovjev,
Theor. Math. Phys. (1982)

Mass-volume spectrum

$$\rho(m,v) = \rho_H(m,v) + \rho_B(m,v)$$

hadrons

$$= \sum_j g_j\, \delta(m - m_j) \cdot \delta(v - v_j)$$

$$+\ C\ v^{\gamma} (m - Bv)^{\delta} \exp\left[\frac{4}{3}\ \sigma_Q^{1/4}\ v^{1/4}\ (m - Bv)^{3/4}\right]$$

bag filled with non-interacting massless quarks and gluons

Minimal volume, mass

$$v > V_0, \qquad m > B\,v + M_0$$

$$V_0 \approx 1\ \mathrm{fm}^3, \quad M_0 \approx 2\ \mathrm{GeV}, \quad B \approx 400\ \mathrm{MeV/fm}^3, \quad \sigma_Q = \frac{95\pi^2}{60}$$

Laplace transform

$$\hat{Z}(T,s) \equiv \int_0^\infty dV \exp(-sV)\ Z(V,T) = [\ s - f(T,s)]^{-1}$$

$$f(T,s) = \int_0^\infty dm\, dv\ \exp(-vs)\ \rho(m,v)\, \phi(T,m)$$

In thermodynamic limit $V \to \infty$

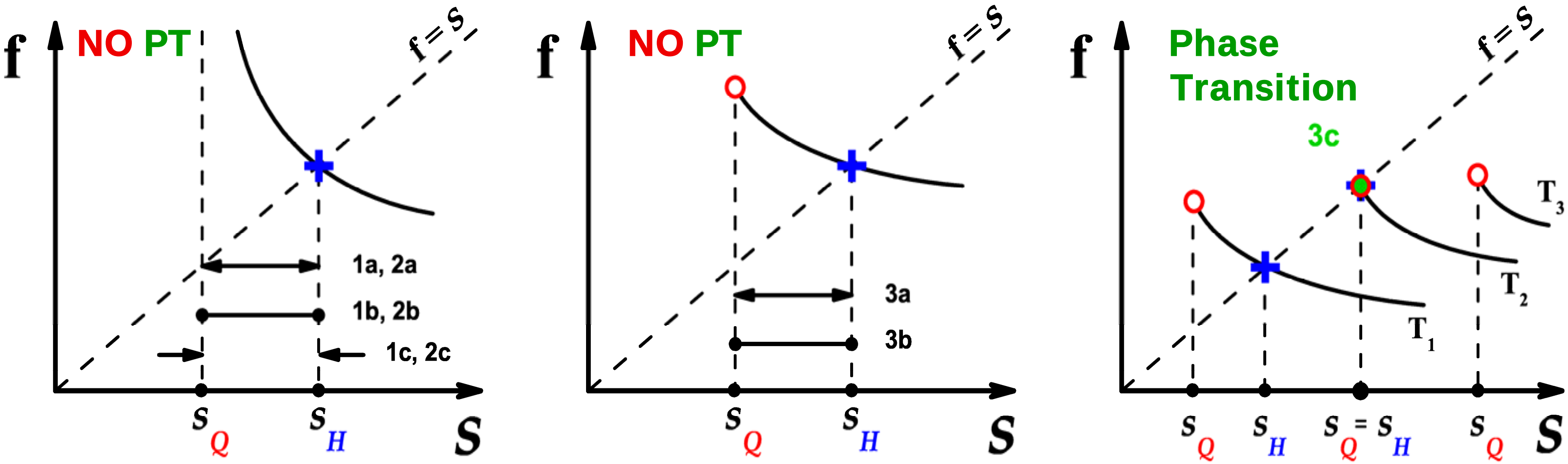
partition function behaves as $Z(V,T) \cong \exp\left[p\, V/T\right]$

The pressure equals the pole singularity s_H **or the singularity** s_Q **of** $f(T,s)$ **itself.**

$$p(T) = T \lim_{V\to\infty} \frac{\ln Z(V,T)}{V} = T\ s^*(T)$$

$$= T \cdot \max\{s_H(T), s_Q(T)\}$$

The classification



$$s_H = u(T)\int_{V_0}^{\infty} dv\ v^{2+\gamma+\delta}\ \exp(-v\,\Delta s) \propto \boxed{T^{10+4\delta}\ (\Delta s)^{-a}\ \Gamma(a, V_0\Delta s)}$$

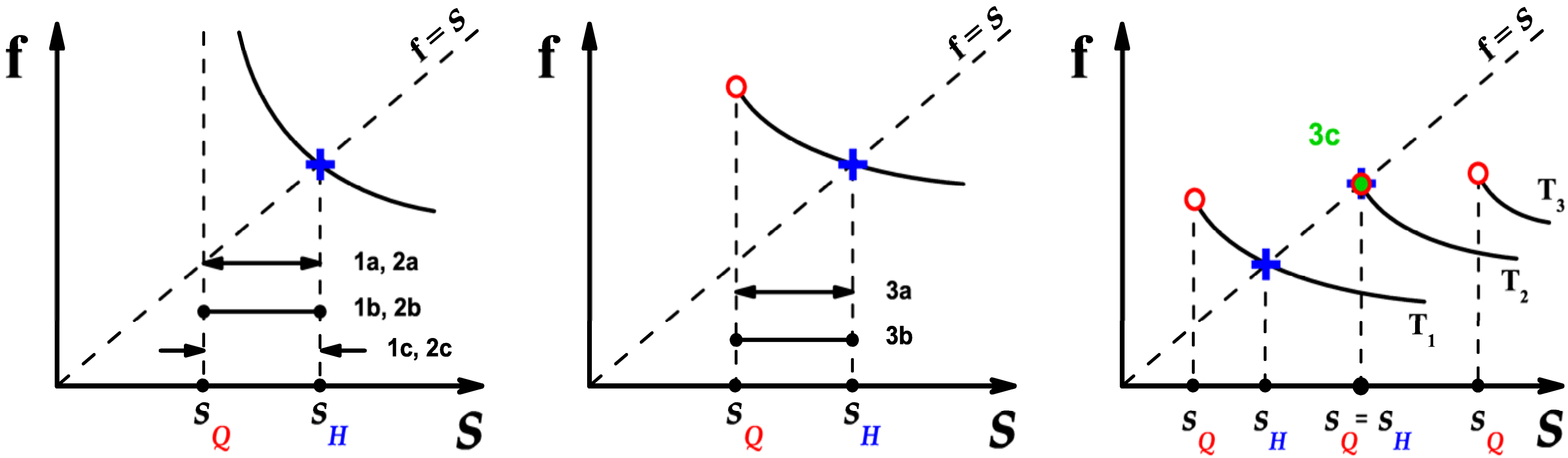
$$a \equiv \gamma + \delta + 3, \qquad \Delta s \equiv s_H - s_Q$$

1) $\gamma + \delta > -3$, 2) $\gamma + \delta = -3$, 3) $\gamma + \delta < -3$

a) $\delta > -7/4$, b) $\delta = -7/4$, c) $\delta < -7/4$

Average volume

$$\overline{v}(T) = \frac{\int dvdm\ v\ \rho(m,v)\ \phi(T,m)\ \exp(-s^* v)}{\int dvdm\ \rho(m,v)\ \phi(T,m)\ \exp(-s^* v)} \cong \frac{1}{\Delta s(T)} \frac{\Gamma(a+1, V_0\,\Delta s(T))}{\Gamma(a, V_0\,\Delta s(T))}$$



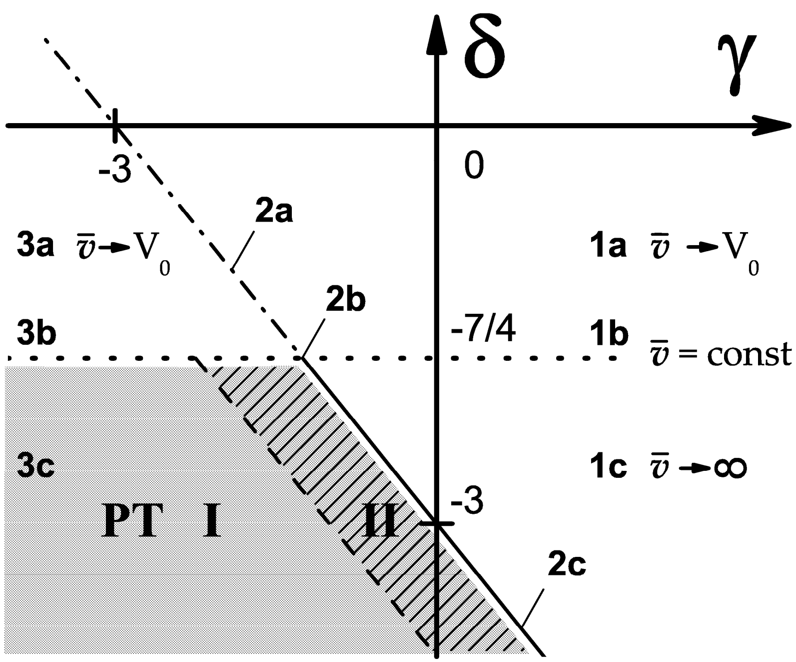
$1a,\ 2a,\ 3a\ :\quad \Delta s(T) \sim \ln T \to \infty\,, \qquad \overline{v}(T) \to V_0\,,$

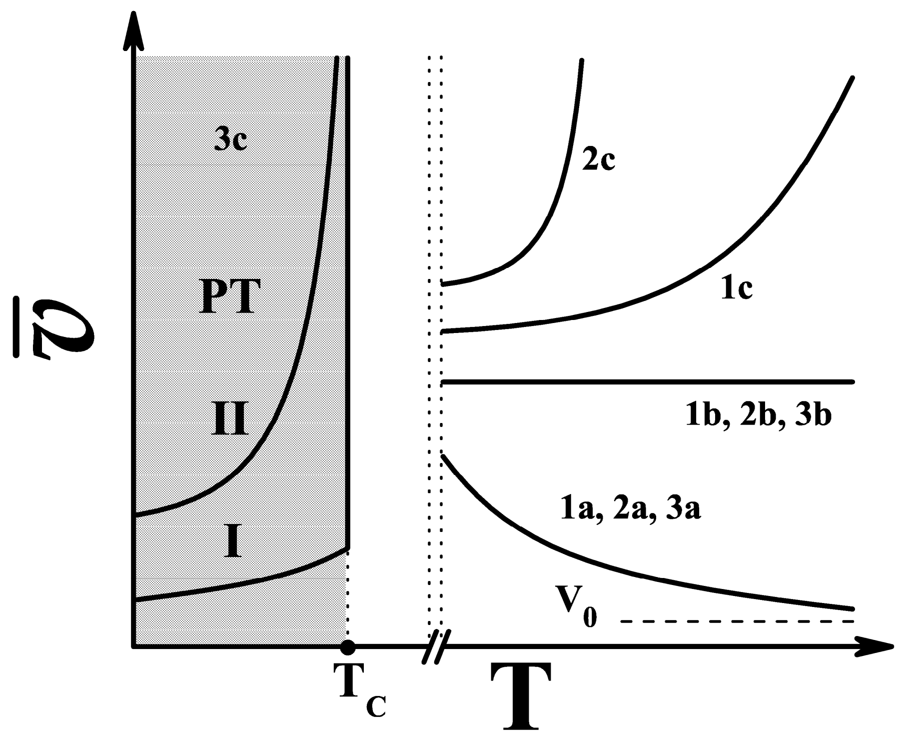
$1b,\ 2b,\ 3b\ :\quad \Delta s(T) \cong \mathrm{const} > 0 \qquad \overline{v}(T) \to \mathrm{const}\,,$

$1c\ :\quad \Delta s(T) \sim T^{(7+4\delta)/a} \to 0\,, \qquad \overline{v}(T) \sim T^{-(7+4\delta)/a} \to \infty\,,$

$2c\ :\quad \Delta s(T) \sim \exp\left(-\,T^{-7-4\delta}\right) \to 0\,, \qquad \overline{v}(T) \sim \exp\left(T^{-7-4\delta}\right) \to \infty$

The $\gamma - \delta$ phase diagram, average volume & mass of bags





$$\overline{v}(T) = \frac{\int dvdm\ v\ \rho(m,v)\ \phi(T,m)\ \exp(-s^* v)}{\int dvdm\ \rho(m,v)\ \phi(T,m)\ \exp(-s^* v)} \cong \frac{1}{\Delta s(T)} \frac{\Gamma(a+1, V_0\,\Delta s(T))}{\Gamma(a, V_0\,\Delta s(T))}$$

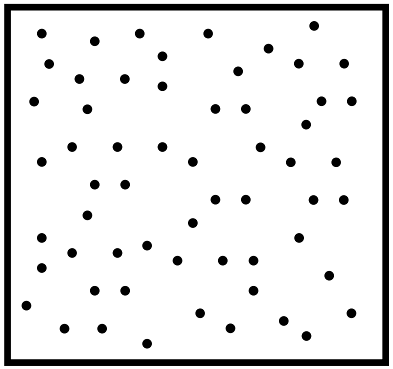
$$\overline{m}(T) \cong \overline{v}(\sigma_Q T^4 + B)$$

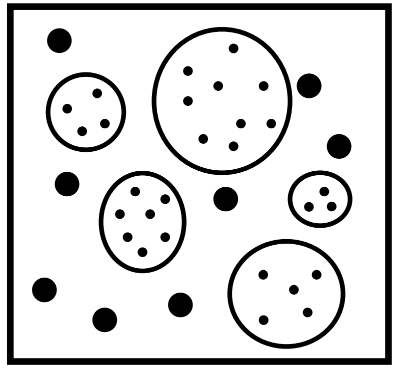
Summary:

1. The system depends crucially on γ and δ .
2. The pressure and energy density for different γ and δ have the same ideal QGP behavior at $T \rightarrow \infty$
3. The average volume and mass of the bag have different behavior in different regions of the $\gamma - \delta$ phase diagram.
4. Possible cluster QGP can be rather different from the ideal QGP despite of the similar to that equation of state.
5. The kinetic properties of the cluster QGP, e.g. the shear and bulk viscosity, may deviate strongly from quark-gluon gas.

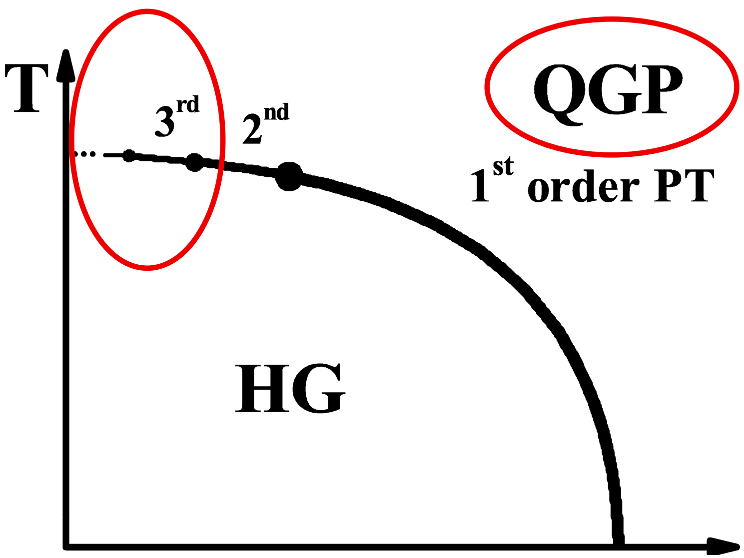
Extra slides

Free particles, bags, clusters or polymer chains?

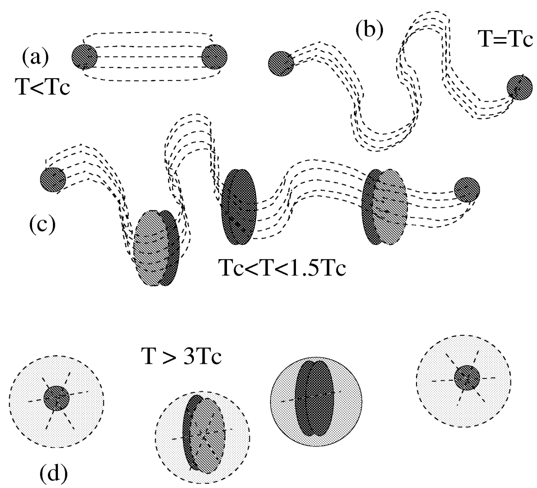




Ferroni, Koch,
Phys. Rev. C (2009)
Crossover ?

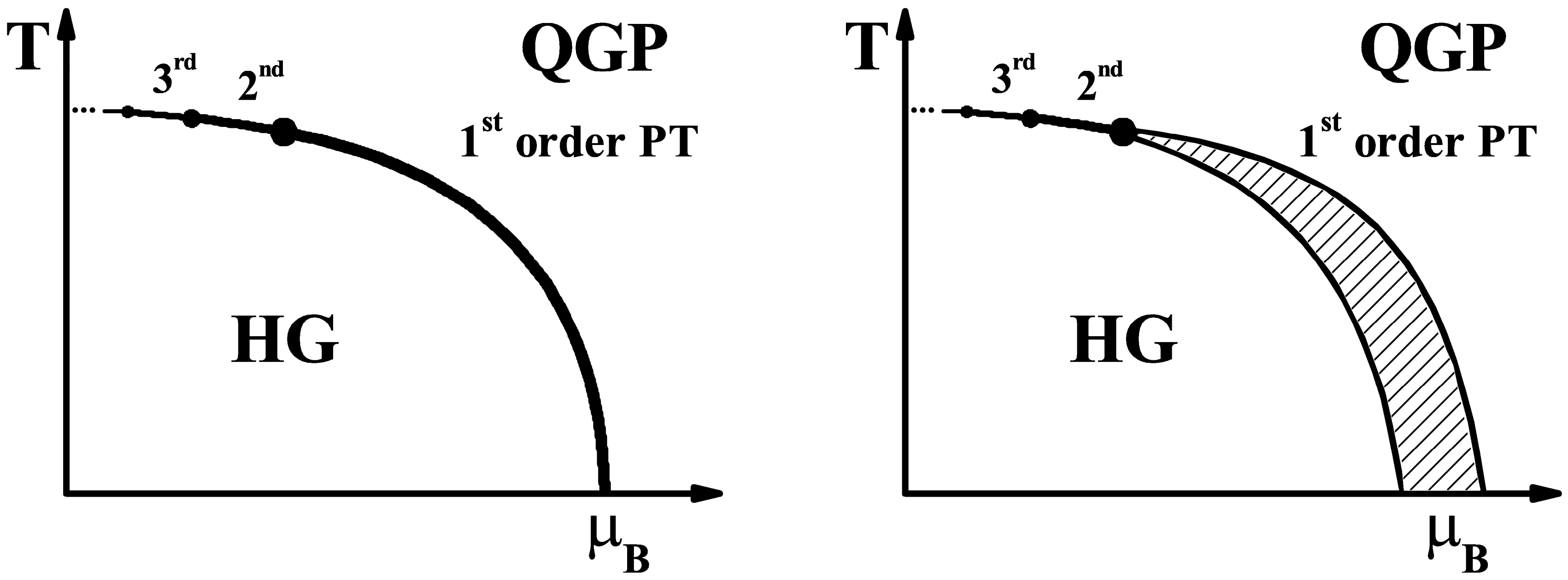


Gorenstein, Gazdzicki, Greiner,
Phys. Rev. C (2005)



Liao, Shuryak,
Nucl.Phys.A (2006)

Critical line of the deconfinement phase transition



Phys. Rev. C72 (2005), nucl-th/0505050

Laplace transform

$$\hat{Z}(T,s) \equiv \int_0^\infty dV \exp(-sV)\ Z(V,T) = [\,s - f(T,s)]^{-1}$$

$$f(T,s) = \int_0^\infty dm_i dv_i\ \exp(-v_i s)\ \rho(m_i,v_i)\,\phi(T,m_i)$$

$$\frac{1}{N!}\Big(V-\sum\Big)^N \theta\Big(V-\sum\Big) \longrightarrow \exp\Big(-s\cdot\sum\Big)\Big/ s^{N+1}$$

Pressure

$$\hat{Z}(T,s) \equiv \int_0^\infty dV \exp(-sV)\; Z(V,T) = [\, s - f(T,s)]^{-1}$$

$$f(T,s) = \int_0^\infty dm_i dv_i \exp(-v_i s)\; \rho(m_i, v_i)\, \phi(T, m_i)$$

$$V \to \infty \quad \Rightarrow \quad Z(V,T) \cong \exp\left[p\, V/T\right]$$

$$s_H : \quad s_H(T) = f(T, s_H(T))$$

$$s_Q : \quad f(T, s_Q(T)) \to \infty$$

Pressure

$$\hat{Z}(T,s) \equiv \int_0^\infty dV \exp(-sV)\; Z(V,T) = [\, s - f(T,s)]^{-1}$$

$$f(T,s) = \int_0^\infty dm_i dv_i \exp(-v_i s)\; \rho(m_i, v_i)\, \phi(T, m_i)$$

$$V \to \infty \quad \Rightarrow \quad Z(V,T) \cong \exp\left[p\,V/T\right]$$

$$p(T) = T \lim_{V\to\infty} \frac{\ln Z(V,T)}{V} = T\; s^*(T)$$

$$= T \cdot \max\{s_H(T), s_Q(T)\}$$

QGP pressure & energy density

$$f(T,s) \simeq u(T)\int_{V_0}^{\infty} dv\; v^{2+\gamma+\delta}\;\; \exp\left[-v\left(s - s_Q(T)\right)\right]$$

$$u(T) = C\pi^{-1}\sigma_Q^{\delta+1/2}\; T^{4+4\delta}\;\left(\sigma_Q T^4 + B\right)^{3/2}$$

$$s_Q(T) \equiv \frac{1}{3}\,\sigma_Q\; T^3 - \frac{B}{T}$$

$$p(T) = T\,s_Q = \frac{\sigma_Q}{3}\;T^4 - B\;, \qquad \varepsilon(T) = T^2\,\frac{ds_Q}{dT} = \sigma_Q\,T^4 + B$$