Influence of instantons and strong magnetic fields on quark matter

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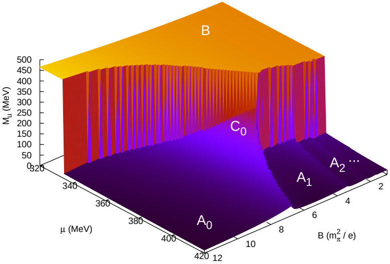
Outline

1. *B*-field effects on quark matter
2. Magnetic enhancement of χSB
3. Spontaneous isospin violation
4. Role of instantons
5. de Haas-van Alphen effect
6. Metastable phases
7. Nonzero temperature

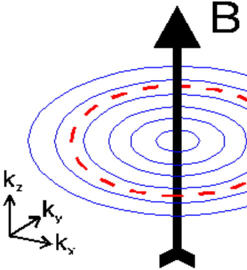


High magnetic fields

- Very strong magnetic fields occur in:
  - Heavy ion collisions: 10^{19} G
    Kharzeev, McLerran & Warringa (2009); Skokov, Illarionov & Toneev (2009)
  - Ordinary neutron stars up to 10^{13} G, magnetars up to 10^{15} G and possibly in core up to 10^{18} G
    Duncan & Thompson (1992); Thompson & Duncan (1993); Lattimer & Prakash (2007)
- In order to understand these systems we have to understand how magnetic fields affect quark matter

Charged particles in a strong magnetic field

- Effect: Landau quantization
  Landau & Lifshitz (1977)



- Discrete levels get subsequently filled → oscillation with B
- de Haas-van Alphen effect
- Similar effects on quark matter?
- Perform study using two-flavor NJL model

Instantons

- In heavy-ion collisions instantons can play an important role
  see for instance Kharzeev (2006); Kharzeev & Zhitnitsky (2007); Kharzeev, McLerran & Warringa (2008)
- Instantons affect the phase structure of quark matter
  Frank, Buballa & Oertel (2003); Boer & Boomsma (2008)
- Effects of instantons in an effective theory are mimicked by the 't Hooft determinant interaction
- This talk: combined study of the influence of instantons and magnetic fields

The NJL model

The Lagrangian

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_{\text{sym}} + \mathscr{L}_{\text{det}}$$

with

Free part $\mathscr{L}_0 = \bar{\psi}\left(i\partial\!\!\!/ - m + \gamma_0\mu\right)\psi$

Chiral symmetric interaction $\mathscr{L}_{\text{sym}} = G_1\left[\left(\bar{\psi}\tau_a\psi\right)^2 + \left(\bar{\psi}\tau_a i\gamma_5\psi\right)^2\right]$

Determinant interaction $\mathscr{L}_{\text{det}} = 2G_2\left[\det\left\{\bar{\psi}\left(1-\gamma_5\right)\psi\right\} + \text{h.c.}\right]$

and

$m = \text{diag}\left(m_u, m_d\right)$, We want to vary G_2

$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$, Usually $G_1 = G_2$

The τ_a are the generators of U(2)
Parameters are fitted to experimental values of m_π, f_π and $\langle\bar{\psi}\psi\rangle$

The effective potential in mean-field approximation

- The interaction terms are "linearized"

$$(\bar{\psi}\tau_a\psi)^2 \simeq 2\left\langle\bar{\psi}\tau_a\psi\right\rangle\bar{\psi}\tau_a\psi - \left\langle\bar{\psi}\tau_a\psi\right\rangle^2$$

- Consider only charge neutral condensates $\langle\bar{\psi}\tau_0\psi\rangle$ and $\langle\bar{\psi}\tau_3\psi\rangle$
- Lagrangian quadratic in the quark fields
- Perform the integration over the quark fields

Obtaining the effective potential II

The thermal effective potential is

$$\mathcal{V} = \frac{(M_0 - m)^2}{4(G_1 + G_2)} + \frac{M_3^2}{4(G_1 - G_2)}$$
$$- TN_c \sum_{f=u}^{d} \sum_{p_0=(2n+1)\pi T} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln \det \left[i\gamma_0 p_0 + \gamma_i p_i - M_f - \gamma_0 \mu \right]$$

where

$$\begin{aligned} M_0 &= m - 2(G_1 + G_2) \langle \bar{\psi} \tau_0 \psi \rangle \\ M_3 &= -2(G_1 - G_2) \langle \bar{\psi} \tau_3 \psi \rangle \end{aligned}$$

and $M_u = M_0 + M_3$, $M_d = M_0 - M_3$
We take $G_1 \neq G_2$

Including the magnetic field

- Choose magnetic field in z-direction
- Choose gauge such that $A^\mu = (0, -By, 0, 0)$
- Obtain new dispersion relation

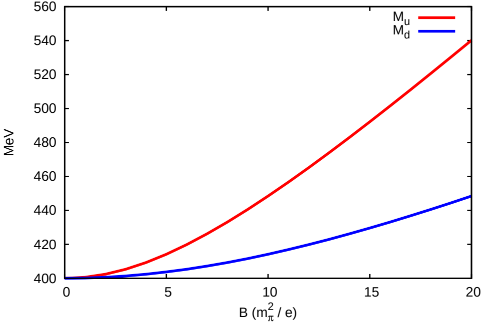
$$p_{0n}^2 = p_z^2 + M^2 + (2n + 1 - \sigma)|q|B \, .$$

- Thermal integral transforms according to Chakrabarty (1996); Fraga & Mizher (2008); Fukushima & Warringa (2008)

$$T \sum_{p_0} \int \frac{d^3p}{(2\pi)^3} \quad \rightarrow \quad \frac{|q|BT}{2\pi} \sum_{p_0} \sum_{n=0}^{\infty} \int \frac{dp_z}{2\pi} \, ,$$

B-dependence of masses without instanton interaction

- B field anti-aligns helicities of quarks and antiquarks $\longrightarrow$ more strongly bound by interaction
  Klevansky (1992)
- Constituent quark masses increase with magnetic field ($m_\pi^2/e = 0.33 \times 10^{19}$ G)

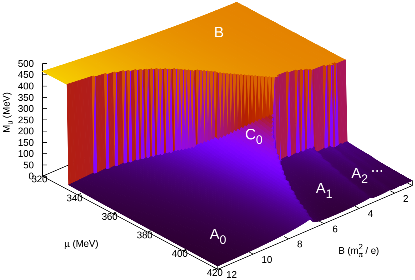


Magnetic enhancement of χSB

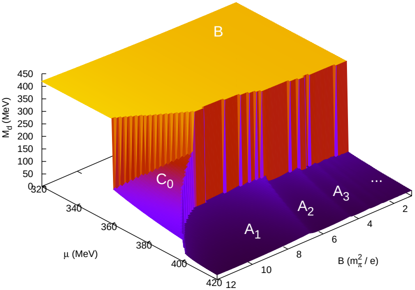
small isospin violation

Nonzero chemical potential at $G_2 = 0$

Up quark



Down quark



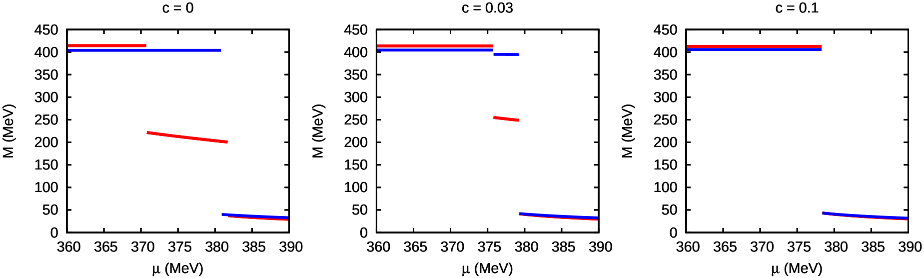
- Similar phase structure found by Ebert et al. 2000; Ebert and Klimenko (2003) with $G_2 = G_1$
- Discontinuous de Haas-van Alphen effect
- A region exists with a considerable mass difference $\rightarrow$ large spontaneous breaking of isospin ($\langle\bar{\psi}\tau_3\psi\rangle \neq 0$)

Effect of the instanton interaction I

- Interaction is flavor mixing $\rightarrow$ counters effect of magnetic field
- Investigate this competition as function of G_2
- Keep $G_1 + G_2$ fixed, determines $B = 0$-physics
- Vary $c = G_2/(G_1 + G_2)$ between 0 and $1/2$
  Frank, Buballa & Oertel (2003)
- Exact value of c is unknown in Nature, from $N_f = 3$ and m_η considerations probably $c \approx 0.2$
  Frank, Buballa & Oertel (2003)

Effect of the instanton interaction II

$$B = 5m_\pi^2/e$$

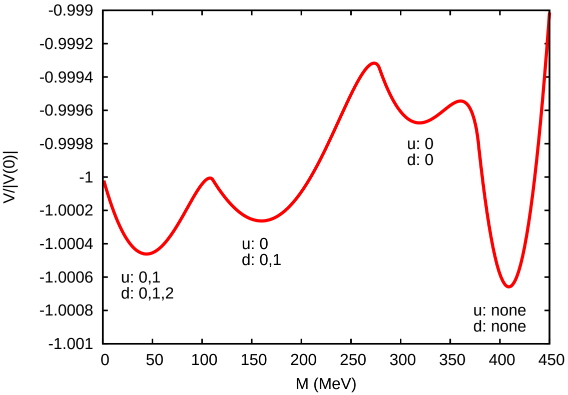


- C_0 phase disappears with increasing c $(= G_2/(G_1 + G2))$
- Regions with large mass differences disappear
- Behavior similar to Frank, Buballa & Oertel (2003) $(\mu_I \neq 0, B = 0)$

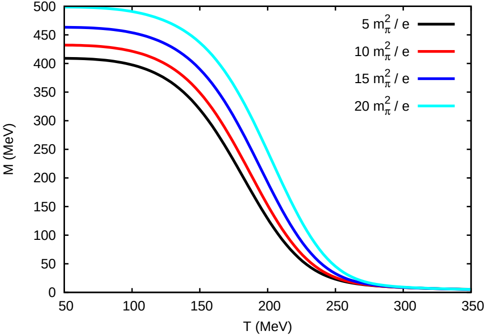
Nearly degenerate metastable states

Normalized effective potential as function of $M = M_u = M_d$ at

$B = 5m_\pi^2/e$, $c = 1/2$ $(G_1 = G_2)$, $\mu = 378\,\text{MeV}$



T-dependence of masses at $c = 1/2$



- High T χ symmetry restoring transition remains crossover ($m_q \neq 0$) at nonzero B
- First order transition in linear sigma model coupled to quarks
  Stronger at larger B
  Mizher & Fraga (2009)

Conclusions

- A magnetic field enhances chiral symmetry breaking and allows for the possibility of spontaneous isospin breaking
- The instanton interaction counters the effect of magnetic field
- de Haas-van Alphen effect is discontinuous in NJL model
- Also metastable states develop that differ considerably in the amount of chiral symmetry breaking
- High-temperature phase transition remains crossover at nonzero B
- NJL: $G_1 \neq G_2$ gives important qualitative differences

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