The $N_f^3 g^6$ term in the pressure of hot QCD

Antti Gynther
(together with Aleksi Jr. & Aleksi Sr.)
arXiv:0909.3521, PRD 80:096002,2009

Vienna University of Technology

EQCD at Trondheim, February 25, 2010

Motivation

- Theoretical: pressure is a fundamental quantity.
- Phenomenological: hydrodynamical simulations of heavy-ion experiments require equation of state as input.
- Cosmological: expansion rate of the universe depends on the equation of state.
  - predictions for WIMP abundances Hindmarsh & Philipsen

Pressure

$$p(T) = \frac{\pi^2 T^4}{45}\Big(8 + \frac{21}{4}N_f + c_2 g^2 + c_3 g^3 + (c_4 + c_4' \ln g)g^4 + c_5 g^5 \\ + (c_6 + c_6' \ln g)g^6 + \dots\Big)$$

- Expansion of the pressure in terms of the gauge coupling known to order $g^6 \ln g$. Kajantie *et al.*
- Order g^6 contains relevant physics not included yet at lower order.
- A systematic framework in which contributions from different energy scales are taken into account is needed.

Dimensional reduction

- At weak coupling (high temperatures) there exists a scale hierarchy:
  - typical momenta of particles $\sim 2\pi T$
  - electric fields screened at length scales larger than $\sim 1/(gT)$
  - magnetic fields screened at length scales larger than $\sim 1/(g^2 T)$

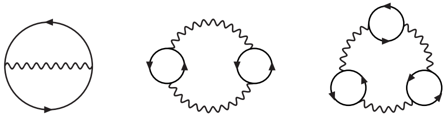
  $\Rightarrow$ Effective field theories
- EQCD: $\mathcal{L}_E = (D_i A_0)(D_i A_0) + m_E^2 A_0^2 + \frac{1}{4} F_{ij}^2 + \ldots$
- MQCD: $\mathcal{L}_M = \frac{1}{4} F_{ij}^2 + \ldots$
- Relate the parameters of the effective theories to physical observables by requiring that the effective theories reproduce the relevant correlators to wanted accuracy.

The g^6 coefficient of the pressure

- In this framework, the coefficient c_6 receives contributions from all the presented theories.
  - pressure of MQCD: $p_G = \# \times g_M^6$ (non-perturbative)
  - four loop pressure of EQCD: $p_M = p_M(m_E, g_E, \lambda_E)$
  - determination of the EQCD screening mass and coupling constants to order $\sim g^4$
  - four loop pressure of QCD: $p_E/T^4 = \cdots + p_6 g^6/(4\pi)^6$
- p_G from lattice computations of the plaquette expectation value of 3d YM theory. Di Renzo *et al*., Hietanen *et al*.
- presently only p_E up to order g^6 unknown.
  - solved in scalar φ^4 theory AG, Laine, Schröder, Torrero & Vuorinen

The $N_f \to \infty$ limit

- QCD simplifies at the limit in which $N_f \to \infty$ and $N_f g^2 \to$ const.



$$\frac{p_{\mathrm{E}}}{d_A} = \frac{1}{2} Z_g g^2 I_1 + \frac{1}{4} Z_g^2 g^4 I_2 + \frac{1}{6} Z_g^3 g^6 I_3 + \dots$$

where

$$I_n = (-1)^n \sumint_P \frac{\Pi_L(P)^n + (d-2)\Pi_T(P)^n}{P^{2n}}$$

- Explicitly extracting the UV divergences of the self-energies cancels against renormalization of the coupling.

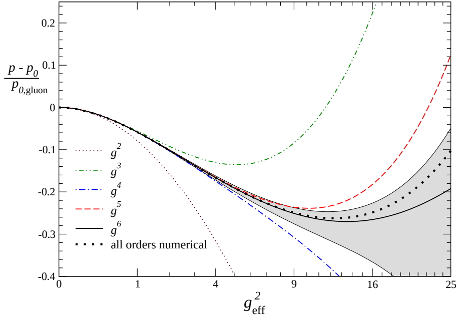
$$\Pi_{L/T}(P) = \overline{\Pi}_{L/T}(P) + \frac{\beta_1}{(4\pi)^2}\frac{P^2}{\epsilon}$$
$$\Rightarrow \frac{p_{\mathrm{E}}}{d_A} = \frac{1}{2}g^2\bar{I}_1 + \frac{1}{4}g^4\bar{I}_2 + \frac{1}{6}g^6\bar{I}_3 + \ldots$$

- The self-energy functions have compact representations at various limits
- Obtain a reasonably finite set of integrals, UV and IR divergent parts can be dealt with analytically and finite parts numerically.

$$p_6 = \frac{\pi^2 N_f^3}{360}\Bigg[-\frac{800}{9}\ln^2\frac{\Lambda}{4\pi T} - \frac{16}{3}\left(\frac{5}{3} + 20\gamma_{\mathrm{E}} - 88\ln 2 + \frac{80}{3}\frac{\zeta'(-1)}{\zeta(-1)} - \frac{40}{3}\frac{\zeta'(-3)}{\zeta(-3)}\right)\ln\frac{\Lambda}{4\pi T} + 296.055373\Bigg] + \mathcal{O}(N_f^2).$$

- Estimate from numerical all-orders computation: Ipp, Moore, Rebhan
  $C_6 \approx 20(2) \Rightarrow C_6 = 21.8597689$

Convergence



- The expansion converges very well!
- Contributions from all the theories are given as expansions in $g^2/(4\pi)^2$ at the large N_f limit (at least up to g^6)

Future

- Wanted: pressure of QCD up to order g^6
- # of diagrams contributing to $g^6 N_f^2$: ~ 10
  - maybe possible (?)
- Far more challenging to go beyond that, must find and apply new methods.