QCD thermodynamics at weak coupling

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Motivation

Why thermal QCD?

- study confinement and chiral symmetry breaking [cf. other talks]
- phenomenologically relevant for cosmology [$\rightarrow$ see below]
- phenomenologically relevant for RHIC, LHC [$\rightarrow$ see below]
- theoretical limit tractable with analytic methods [this talk]
  - ▷ goal: no models - stay within QCD!

Why weak-coupling methods?

- need to complement / understand other approaches (mainly LAT), e.g.:
  - ▷ $T \gg T_c$ -vs- $T \sim T_c$
  - ▷ m_q at low orders -vs- $m_q \gtrsim m_{phys}$
  - ▷ physical picture -vs- black box
- can be systematically improved (!)

Motivation

Focus on equilibrium thermodynamics of QCD

- typical questions to be addressed
  - ▷ equation of state (EoS)
  - ▷ structure of QCD phase diagram
    transition lines, order of transitions, critical points
  - ▷ medium properties: spectral functions, correlation lengths, ...

Interplay of methods

- QGP is strongly coupled system near $T_c \Rightarrow$ need e.g. LAT
- asymptotic freedom at high $T \Rightarrow$ weak-coupling approach
- cave: strict loop expansion not well-defined
  IR divergences at higher orders [Linde 1979; Gross/Pisarski/Yaffe 1981]

Discuss

- effective theories (here: $\mu = 0$; $\mu \lesssim T$ similar)
- spatial string tension
- basic thermodynamic observable: pressure $p(T)$ [⇐ main playground]
- quark mass effects on EoS

Motivation

$p(T)$ important for cosmology:

- cooling rate of the universe

$$\partial_t T = -\frac{\sqrt{24\pi}}{m_{\mathsf{Pl}}} \frac{\sqrt{e(T)}}{\partial_T \ln s(T)}$$

- with entropy $s = \partial_T \, p$ and energy density $e = Ts - p$
- $\Rightarrow$ cosmol. relics (dark matter, background radiation etc.) originate when an interaction rate $\tau(T)$ gets larger than the age of the universe $t(T)$.
  - ▷ Ex.: ''sterile'' ν_R with $m_\nu \sim$ keV can be warm dark matter, and decouple around $T \sim 150$ MeV [Abazajian, Fuller 02; Asaka, Shaposhnikov 05]

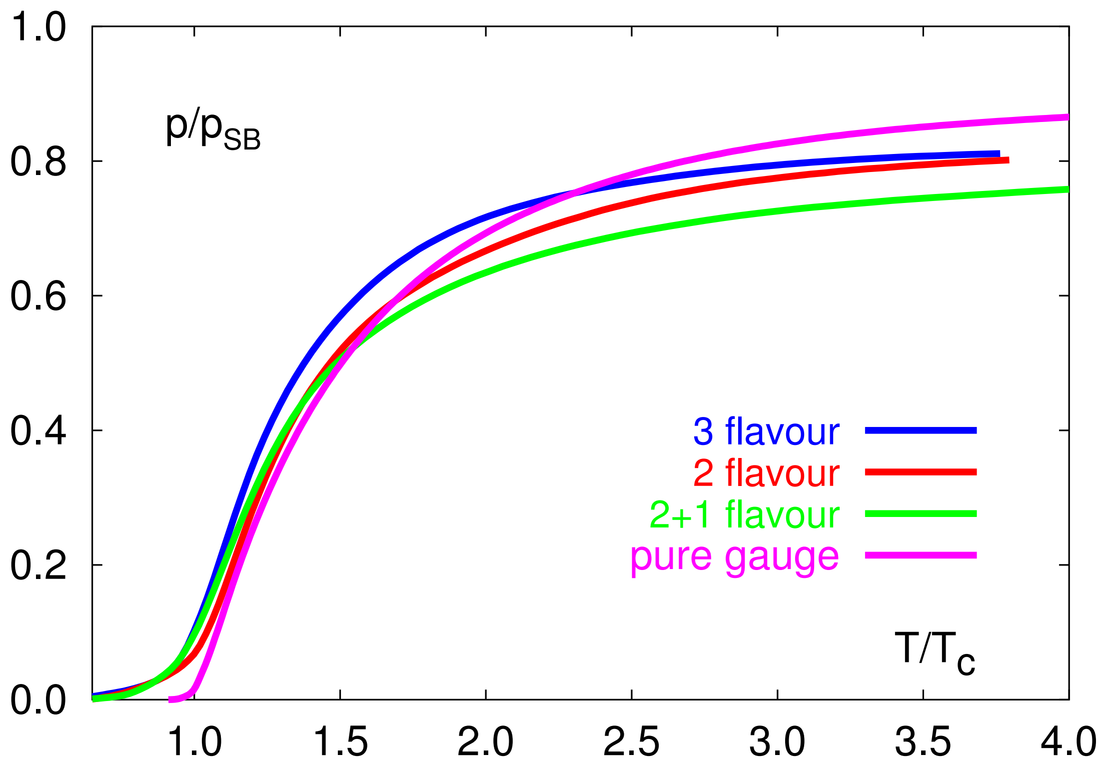
$p(T)$ in heavy ion collisions:

- expansion rate (after thermalization) given by

$$\partial_\mu \, T^{\mu\nu} = 0 \quad , \quad T^{\mu\nu} = [p(T) + e(T)] \, u^\mu u^\nu - p(T) g^{\mu\nu}$$

- with flow velocity $u^\mu(t, x)$
  - ▷ hydrodynamic expansion: hadronization at $T \sim 100 - 150$ MeV
    $\Rightarrow$ observed hadron spectrum depends (indirectly) on $p(T)$

$p(T)$ via (large) computer ($\mu_B = 0$)



[lattice data from Karsch et.al.]

at $T \to \infty$, expect ideal gas: $p_{\text{SB}} = \left(16 + \frac{21}{2} N_f\right) \frac{\pi^2 T^4}{90}$

confirms simplicity: 3 dofs (π) $\to$ 52 ($3 \times 3 \times 2 \times 2$ qu + 8×2 gl)

Energy scales in hot QCD

Interactions make QCD a multiscale system

At asymptotically high T, $g \ll 1 \Rightarrow$ clean separation of 3 scales

expansion parameter:

$$g^2\, n_b(|k|) = \frac{g^2}{e^{|k|/T}-1} \overset{|k| \lesssim T}{\approx} \frac{g^2 T}{|k|}$$

- $|k| \sim \pi T$ aka ''hard'': fully perturbative at high T
  thermal fluctuations; effective mass of non-static field modes
- $|k| \sim gT$ aka ''soft'': dynamically generated; barely perturbative at high T
  inverse screening length of static color-electric fluctuations; thermal/Debye mass
- $|k| \sim g^2 T$ aka ''ultrasoft'': dynamically generated; non-perturbative at high T
  inverse screening length of static color-magnetic fluctuations; ''magnetic mass''
- no smaller momentum scales / larger length scales due to confinement

treatment of a multiscale system: effective field theory !

$p(T)$ via weak-coupling expansion

need to explain 20% deviation from ideal gas at $T \sim 4T_c$

- structure of pert series is non-trivial !
- $$p(T) \equiv \lim_{V\to\infty} \frac{T}{V} \ln \int \mathcal{D}[A^a_\mu, \psi, \bar\psi] \exp\left(-\frac{1}{\hbar}\int_0^{\hbar/T} d\tau \int d^{3-2\epsilon}x\, \mathcal{L}_{\text{QCD}}\right)$$
$$= c_0 + c_2 g^2 + c_3 g^3 + (c_4' \ln g + c_4) g^4 + c_5 g^5 + (c_6' \ln g + c_6) g^6 + \mathcal{O}(g^7)$$

[c_2 Shuryak 78, c_3 Kapusta 79, c_4' Toimela 83, c_4 Arnold/Zhai 94, c_5 Zhai/Kastening 95, Braaten/Nieto 96, c_6' KLRS 03]

- root cause of nonanalytic (in α_s) behavior well understood: above-mentioned dynamically generated scales
- clean separation best understood in effective field theory setup [$\mu = 0$: this talk]
  - ▷ generalizations, e.g. $\mu \neq 0$ [Vuorinen], standard model [Gynther/Vepsäläinen]
- other re-organizations possible, e.g. 2PI skeleton-expansion [eg Blaizot/Iancu/Rebhan]

Effective theory prediction for $p(T)$

$$\frac{p_{\mathsf{QCD}}(T)}{p_{\mathsf{SB}}} = \frac{p_{\mathsf{E}}(T)}{p_{\mathsf{SB}}} + \frac{p_{\mathsf{M}}(T)}{p_{\mathsf{SB}}} + \frac{p_{\mathsf{G}}(T)}{p_{\mathsf{SB}}} \quad , \quad p_{\mathsf{SB}} = \left(16 + \frac{21}{2}N_f\right)\frac{\pi^2 T^4}{90}$$

$$= 1 + g^2 \quad + g^4 \quad + g^6 \quad + \ldots \qquad \Leftarrow \text{4d QCD}$$

$$+ g^3 + g^4 + g^5 + g^6 + \ldots \qquad \Leftarrow \text{3d adj H}$$

$$+ \frac{1}{p_{\mathsf{SB}}}\frac{T}{V}\int \mathcal{D}[A_k^a]\exp\Big(-S_{\mathsf{M}}\Big) \qquad \Leftarrow \text{3d YM}$$

- this could be coined the *physical leading-order (!) approximation*
- collect contributions to $p(T)$ from all physical scales
  - ▷ weak coupling, effective field theory setup
  - ▷ faithfully adding up all Feynman diagrams
  - ▷ get long-distance input from clean lattice observable:

$$p_{\mathsf{G}}(T) \equiv \frac{T}{V}\ln\int \mathcal{D}[A_k^a]\exp\Big(-S_{\mathsf{M}}\Big) = T\#\, g_{\mathsf{M}}^6$$

only one non-perturbative (but computable!) coeff needed

Ingredients for c_6

$\ldots + g^6$

- 4-loop sum-integrals needed, const term
- DOABLE?! manpower OR brainpower? [YS/AV ??]

matching coeffs

- 2-loop ϵ-terms for m_{E}^2, g_{E}^2 DONE. ML/YS 05: IBP, reduction, master sum-ints

$\ldots + g^6$

- 4-loop integrals needed DONE. KLRS 03: reduction, master ints, HPL

match $\overline{\text{MS}}$/LAT

- 4-loop const in LAT reg via NSPT DONE. LMRST 06: LAT pert

$\ldots + g^6$

- measure $<$ Plaquette $>$ in 3d SU(N) DONE. HKLRS 05: LAT Monte Carlo

Ingredients for c_6

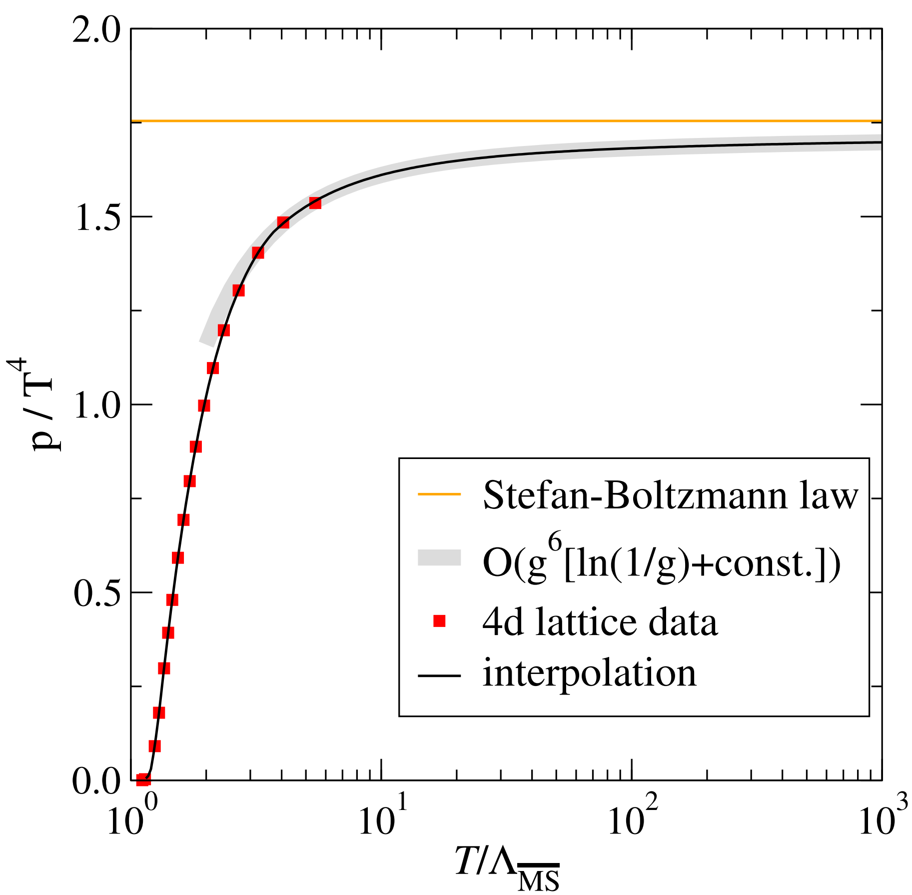
4-loop sum-integrals?

- a single one has already been computed
  - ▷ painfully disentangled (sub-)divergences by hand
  - ▷ constant term only numerically
  - ▷ gave the g^6 term in scalar ϕ^4 [GLSTV 2008]
  - ▷ fermionic generalization for $g^6 N_f^3$ in QCD [→ see talk by A.Gynther]
- in QCD, need $\mathcal{O}(10^8)$ of them
- ideas to profit from algorithmic $T = 0$ methods not fruitful (yet?)
  - ▷ as used - and tested extensively - for the 3d part
- find a smart duality to map the problem to sth simpler?

Matching $p(T)$ at $N_f = 0$

in the meantime ...

- want to show results / tackle simpler problems / phenomenology
- strive for best possible description of pure-glue sector

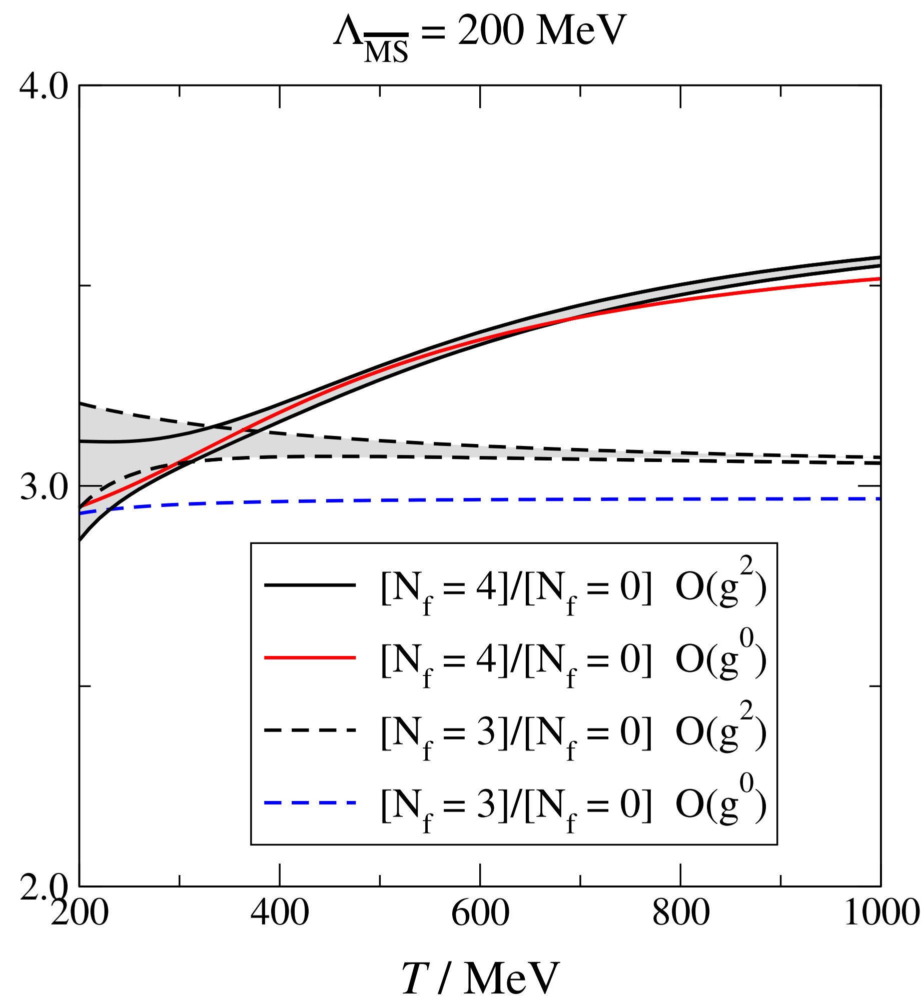


- fix unknown perturbative $\mathcal{O}(g^6)$ coeff
- use available lattice data
  here: at 3-5T_c [Boyd et al 1996]
- translate via $T_c/\Lambda_{\overline{\text{MS}}} \approx 1.20$
- match at intermediate $T \sim 3 - 5T_c$
- (at high T? [Endrödi et al LAT07; Borsanyi at QHPD09])

Quark mass dependence

analyze quark mass dependence to NLO

- strategy: ''unquenching''
  start from $N_f = 0$, i.e. $m_q = \infty$
  lower N_f quark masses to $m_{q,phys}$
  p at any T increases
- estimate this ''correction factor''
- aproach is systematic
  LO: $c_0(N_f)/c_0(0)$
  NLO: $[c_0 + g^2 c_2](N_f) \,/\, [c_0 + g^2 c_2](0)$
- computed $c_{0,2}(T, N_c, N_f, m_i, \mu_i)$
- good convergence LO→NLO
  - ▷ $N_f = 3$: 5% effect
  - ▷ $N_f = 4$: even better



charm quark contributes already at low $T \sim 350 MeV$

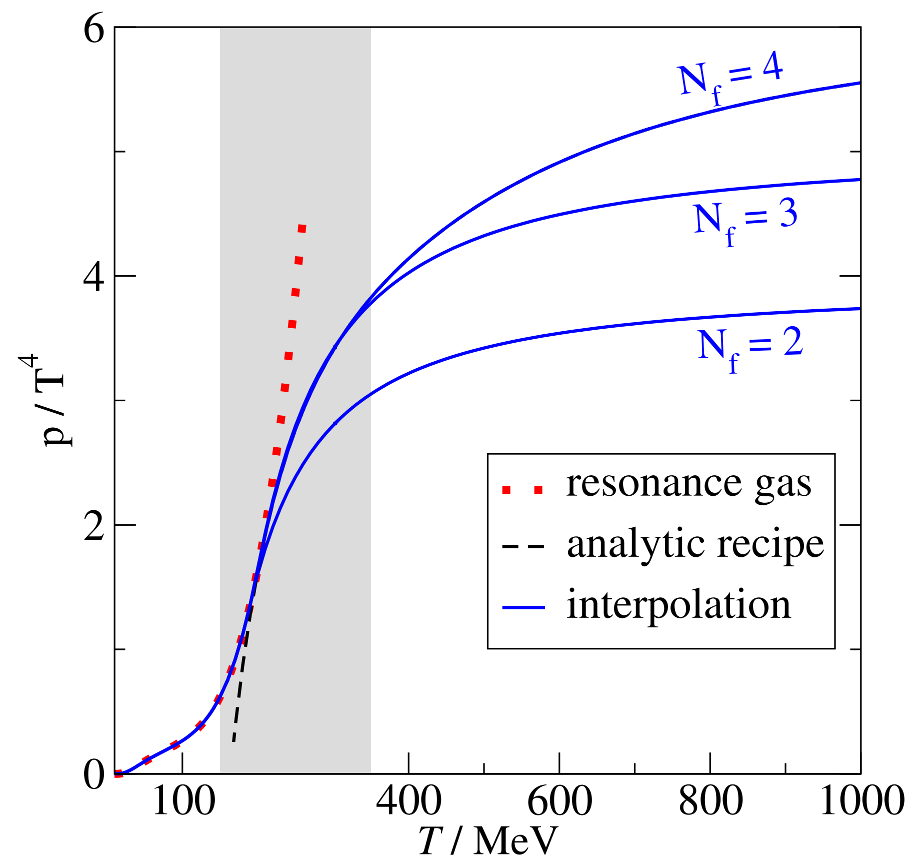
Setting the scale

now ready to estimate thermodynamic quantities

multiply best $N_f = 0$ result with correction factor

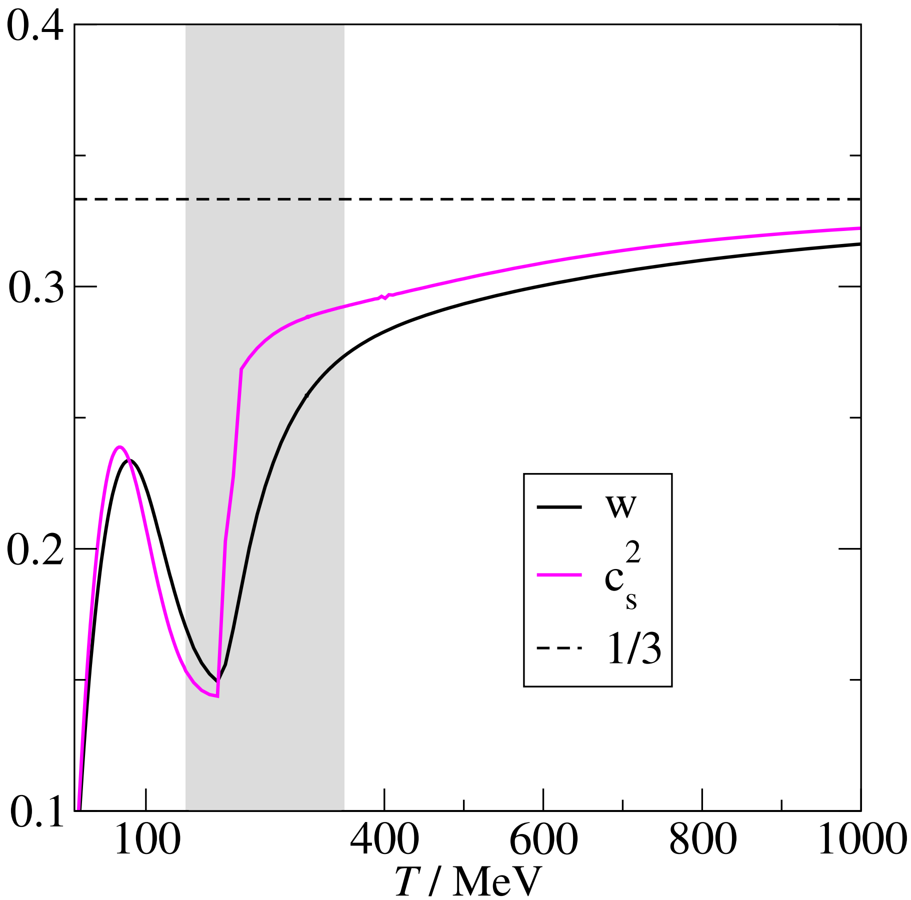
$$g^2(\bar{\mu}) = \frac{24\pi^2}{(11C_A - 2N_f)\ln(\bar{\mu}/\Lambda_{\overline{\text{MS}}})}\,,\quad m_i(\bar{\mu}) = m_i(\bar{\mu}_{\text{ref}})\left[\frac{\ln(\bar{\mu}_{\text{ref}}/\Lambda_{\overline{\text{MS}}})}{\ln(\bar{\mu}/\Lambda_{\overline{\text{MS}}})}\right]^{\frac{9C_F}{11C_A-2N_f}}$$

- need to fix $\Lambda_{\overline{\text{MS}}}$ in physical units!
- strategy: matching
  take p of hadronic resonances
  match p and p' to our recipe
- obtain $\Lambda_{\overline{\text{MS}}}^{(eff)} \approx 175...180 MeV$
- shaded: lattice simulations needed!



Thermodynamic quantities

now use the recipe $\boxed{p(N_f{=}0)\times \text{ corr.fct}}$ and plot dimensionless ratios



- equation of state

$$w(T) \equiv \frac{p(T)}{e(T)} = \frac{p(T)}{Tp'(T) - p(T)}$$

- sound speed (squared)

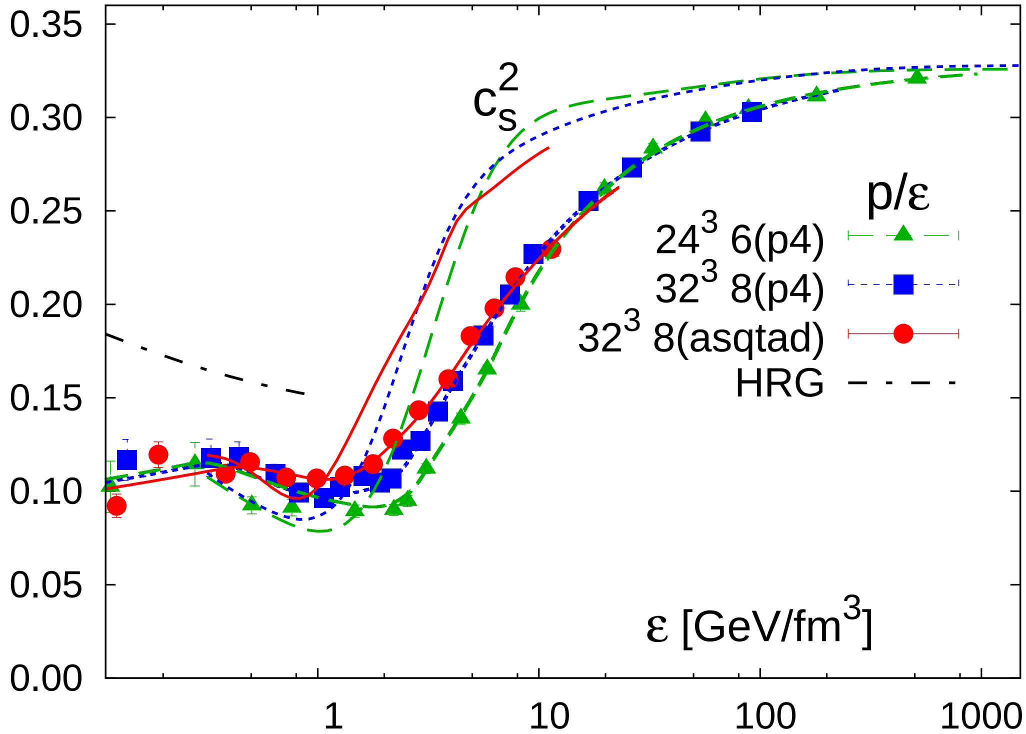
$$c_s^2(T) \equiv \frac{p'(T)}{e'(T)} = \frac{p'(T)}{Tp''(T)} = \frac{s(T)}{c(T)}$$

- $\left(\frac{1}{3} - w(T)\right) \propto$ "trace anomaly"
- observe significant structure
- at 2nd order phase transition $c(T) \sim (T - T_c)^{-\gamma}$

peak around 70MeV not (yet) visible in lattice simulations

Thermodynamic quantities

most recent lattice data [Bazavov et al, 2009]



- HotQCD 2009
- $N_f = 2 + 1$
  m_s physical
  light quarks not
- $N_\tau = 8$
  two (staggered) actions

Outlook for $p(T)$: $g^6 \to g^7 \to g^8$

$$\frac{p_\mathsf{G}}{p_\mathsf{SB}} = \#_{(6)}\left(\frac{g_\mathsf{M}^2}{T}\right)^3 + [\delta\mathcal{L}_\mathsf{M}]_{(9)}$$

$$g_\mathsf{M}^2 = g_\mathsf{E}^2\left[1 + \#_{(7)}\frac{g_\mathsf{E}^2}{m_\mathsf{E}} + \left(\frac{g_\mathsf{E}^2}{m_\mathsf{E}}\right)^2\left(\#_{(8)} + \#_{(10)}\frac{\lambda_\mathsf{E}}{g_\mathsf{E}^2}\right) + \dots_{(9)}\right]$$

$$\frac{p_\mathsf{M}}{p_\mathsf{SB}} = \frac{m_\mathsf{E}^3}{T^3}\left[\#_{(3)} + \frac{g_\mathsf{E}^2}{m_\mathsf{E}}\left(\#_{(4)} + \#_{(6)}\frac{\lambda_\mathsf{E}}{g_\mathsf{E}^2}\right) + \left(\frac{g_\mathsf{E}^2}{m_\mathsf{E}}\right)^2\left(\#_{(5)} + \#_{(7)}\frac{\lambda_\mathsf{E}}{g_\mathsf{E}^2} + \#_{(9)}\left(\frac{\lambda_\mathsf{E}}{g_\mathsf{E}^2}\right)^2\right)\right.$$
$$+ \left(\frac{g_\mathsf{E}^2}{m_\mathsf{E}}\right)^3\left(\#_{(6)} + \#_{(8)}\frac{\lambda_\mathsf{E}}{g_\mathsf{E}^2} + \#_{(10)}\left(\frac{\lambda_\mathsf{E}}{g_\mathsf{E}^2}\right)^2 + \#_{(12)}\left(\frac{\lambda_\mathsf{E}}{g_\mathsf{E}^2}\right)^3\right)$$
$$\left. + [\text{3d 5loop 0pt}]_{(7)} + [\delta\mathcal{L}_\mathsf{E}]_{(7)} + [\text{3d 6loop 0pt}]_{(8)} + \dots_{(9)}\right]$$

$$m_\mathsf{E}^2 = T^2\left[\#_{(3)}g^2 + \#_{(5)}g^4 + [\text{4d 3loop 2pt}]_{(7)} + \dots_{(9)}\right]$$

$$\lambda_\mathsf{E} = T\left[\#_{(6)}g^4 + \#_{(8)}g^6 + \dots_{(10)}\right]$$

$$g_\mathsf{E}^2 = T\left[g^2 + \#_{(6)}g^4 + \#_{(8)}g^6 + \#_{(10)}g^8 + \dots_{(12)}\right]$$

$$\frac{p_\mathsf{E}}{p_\mathsf{SB}} = \#_{(0)} + \#_{(2)}g^2 + \#_{(4)}g^4 + \#_{(6)}g^6 + [\text{4d 5loop 0pt}]_{(8)} + \dots_{(10)}$$

notation: $\#_{(n)}$ enters p_QCD at g^n

[cave: no $\frac{1}{\epsilon} + 1 + \epsilon$, no IR/UV, and no logs shown above]

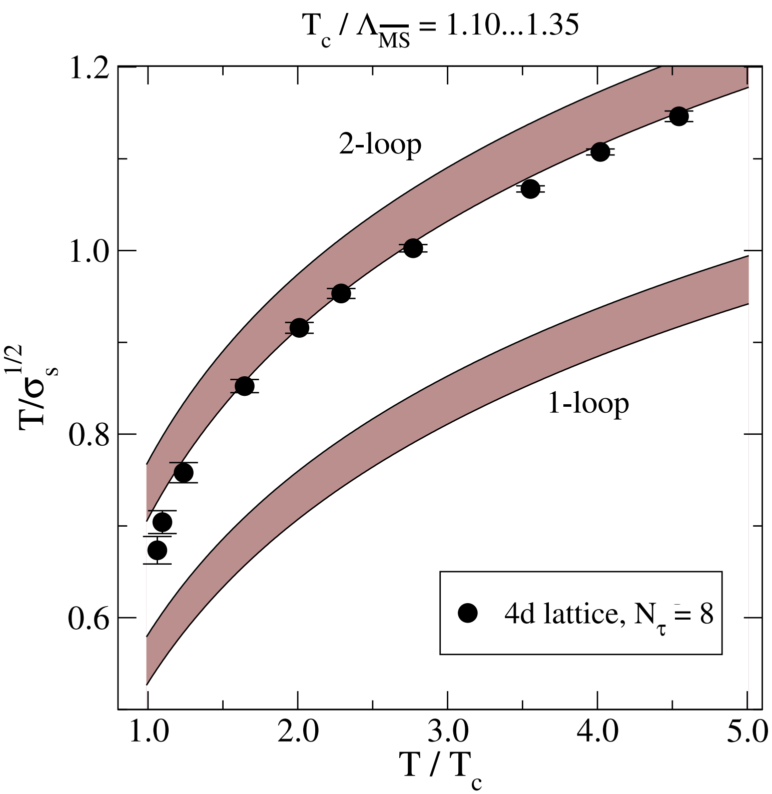
Higher precision on QCD $\to$ EQCD matching coeffs

- seems doable: need 3-loop sum-integrals
- take as arena for testing methods [Jan Möller, YS]
- 3-loop correction to Debye mass m_E^2
  - ▷ status: reduction done, result gauge independent
  - ▷ expressed in terms of $\mathcal{O}(5_b + 20_f)$ master sum-integrals
  - ▷ dim[ints] $= T^2 \;\Rightarrow$ only three are known [GLSTV 08; Andersen/Kyllingstad 08; Möller 09]
  - ▷ application: contributes to $p(T)$ at ''NLO'' i.e. g^7
    (g^7 done for ϕ^4 by [Andersen/Kyllingstad/Leganger 09])
- 3-loop correction to 3d gauge coupling g_E^2
  - ▷ status: reduction done, result gauge independent
  - ▷ expressed in terms of $\mathcal{O}(6_b + 30_f)$ master sum-integrals
  - ▷ dim[ints] $= T^0 \;\Rightarrow$ only two are known [Möller 10]
  - ▷ application: σ_s, precision test of eff. theory setup [$\to$ see below]

Spatial string tension: $W_s(R_1, R_2) = \exp(-\sigma_s R_1 R_2)$ at large R_1, R_2

SU(3), 4d lat: $\frac{\sqrt{\sigma_s}}{T} = \text{fct}\left(\frac{T}{T_c}\right)$; $T_c \approx 1.2\Lambda_{\overline{\text{MS}}}$

SU(3), 3d MQCD: $\frac{\sqrt{\sigma_s}}{T} = \# \frac{g_{\text{M}}^2}{g_{\text{E}}^2} \frac{g_{\text{E}}^2}{T} = \text{fct}\left(\frac{T}{\Lambda_{\overline{\text{MS}}}}\right)$; $\# = 0.553(1)$ [Teper, Lucini 02]



[4d lattice data from Boyd et al, 96] (cave: no cont. extrapolation)

parameter-free comparison; support for hard/soft+ultrasoft picture

Summary

- thermodynamic quantities of QCD are relevant for cosmology and heavy ion collisons
- these quantities can be determined numerically at $T \sim 200$ MeV, and analytically at $T \gg 200$ MeV; multi-loop sports, eff. theories convenient
- effective field theory opens up tremendous opportunities: analytic treatment of fermions, universality, superrenormalizabilty
- QCD pressure
  - ▷ not even known at ''physical leading order''
  - ▷ problem reduced to one (hard) perturbative computation
  - ▷ shows friendly functional behavior with fitted unknown coefficient
- quark mass dependence in EoS
  - ▷ shows good convergence
  - ▷ charm quark contributes already at fairly low T
  - ▷ need reliable lattice simulations in transition region
- spatial string tension
  - ▷ successful test of effective theory setup
  - ▷ even higher precision under investigation

Effective theory setup: QCD $\to$ EQCD

high T: QCD dynamics contained in 3d EQCD

integrate out $|p| \gtrsim 2\pi T$: ψ, $A_\mu(n \neq 0)$

$$p_{\mathsf{QCD}}(T) \equiv p_{\mathsf{E}}(T) + \frac{T}{V}\ln\int\mathcal{D}[A_k^a, A_0^a]\exp\Big(-\int d^{3-2\epsilon}x\,\mathcal{L}_{\mathsf{E}}\Big)$$

$$\mathcal{L}_{\mathsf{E}} = \frac{1}{2}Tr\,F_{kl}^2 + Tr\,[D_k, A_0]^2 + m_{\mathsf{E}}^2 Tr\,A_0^2 + \lambda_{\mathsf{E}}^{(1)}(Tr\,A_0^2)^2 + \lambda_{\mathsf{E}}^{(2)}Tr\,A_0^4 + \ldots$$

five matching coefficients [E. Braaten, A. Nieto, 95; KLRS 02; M. Laine, YS, 05]

$$p_{\mathsf{E}} = T^4\left[\# + \#g^2 + \#g^4 + \#g^6 + \ldots\right],\; m_{\mathsf{E}}^2 = T^2\left[\#g^2 + \#g^4 + \ldots\right],$$

$$g_{\mathsf{E}}^2 = T\left[g^2 + \#g^4 + \#g^6 + \ldots\right],\; \lambda_{\mathsf{E}}^{(1),(2)} = T\left[\#g^4 + \ldots\right].$$

higher order operators do not (yet) contribute [S. Chapman, 94; Kajantie et al, 97, 02]

$$\frac{\delta p_{\mathsf{QCD}}(T)}{T} \sim \delta\mathcal{L}_{\mathsf{E}} \sim g^2\frac{D_k D_l}{(2\pi T)^2}\mathcal{L}_{\mathsf{E}} \sim g^2\frac{(gT)^2}{(2\pi T)^2}(gT)^3 \sim g^7 T^3$$

Effective theory setup: QCD $\rightarrow$ EQCD $\rightarrow$ MQCD

the IR of 3d EQCD is contained in 3d MQCD

integrate out $|p| \gtrsim gT$: A_0

$$
\begin{aligned}
p_{\mathsf{QCD}}(T) &\equiv p_{\mathsf{E}}(T) + p_{\mathsf{M}}(T) + \frac{T}{V}\ln\int\mathcal{D}[A_k^a]\exp\Big(-\int d^{3-2\epsilon}x\,\mathcal{L}_{\mathsf{M}}\Big) \\
\mathcal{L}_{\mathsf{M}} &= \frac{1}{2}Tr\,F_{kl}^2 + \ldots
\end{aligned}
$$

two matching coefficients [KLRS 03; P. Giovannangeli 04, M. Laine/YS 05]

$$
p_{\mathsf{M}} = Tm_{\mathsf{E}}^3\left[\# + \#\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}} + \#\frac{g_{\mathsf{E}}^4}{m_{\mathsf{E}}^2} + \#\frac{g_{\mathsf{E}}^6}{m_{\mathsf{E}}^3} + \ldots\right],\; g_{\mathsf{M}}^2 = g_{\mathsf{E}}^2\left[1 + \#\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}} + \#\frac{g_{\mathsf{E}}^4}{m_{\mathsf{E}}^2} + \ldots\right].
$$

higher order operators do not (yet) contribute

$$
\frac{\delta p_{\mathsf{QCD}}(T)}{T} \sim \delta\mathcal{L}_{\mathsf{M}} \sim g_{\mathsf{E}}^2\frac{D_kD_l}{m_{\mathsf{E}}^3}\mathcal{L}_{\mathsf{M}} \sim g_{\mathsf{E}}^2\frac{(g^2T)^2}{m_{\mathsf{E}}^3}(g^2T)^3 \sim g^9T^3
$$

Summing up IR contributions beyond 4-loop

Can one do without the EQCD $\rightarrow$ MQCD matching?

- treat 3d EQCD on the lattice
- still much simpler than full QCD
- measure condensates on physical line
- 3d EQCD is superrenormalizable
  - ▷ can perform LAT $\leftrightarrow$ $\overline{\text{MS}}$ matching exactly in perturbation theory
  - ▷ before cont. limit, all numbers are $f(am)$
  - ▷ action not (yet) completely $\mathcal{O}(a)$ improved
  - ▷ large discretization effects such as $a\ln(a)$ present

Compute parameters needed for reliable continuum extrapolation

- need to compute (4-loop) diagrams in lattice regularization
  - ▷ via Numerical Stochastic Perturbation Theory [Di Renzo et al 2008]
- invested $\sim 4 \cdot 10^{18}$ flops on APE (Parma), Ben (Trento)

Summing up IR contributions beyond 4-loop

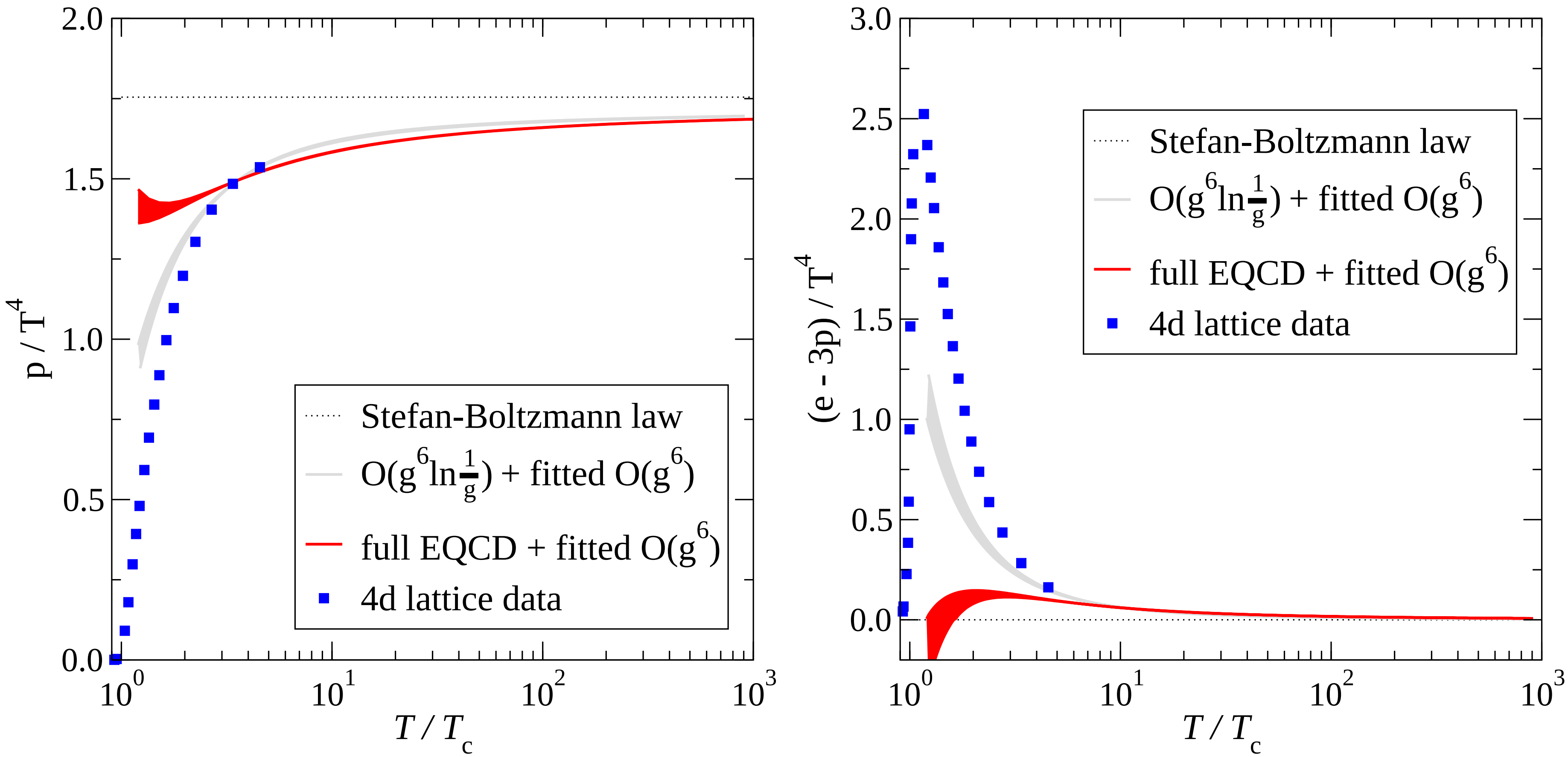
Measure condensates

[HKLRS 2008]

- main idea: measure $\langle Tr A_0^2 \rangle \sim \partial_{m^2} \mathcal{F}$ and integrate
- know integration constant from perturbative analysis
  - ▷ since dimensionless expansion parameter is $g_{\mathsf{E}}^2/m_{\mathsf{E}}$
- sampled the function at 13 points
  used 186 lattices, with $(\beta = \frac{6}{a g_{\mathsf{E}}^2}, N_s)$ from (24,48) up to (240,512)
- get a good perturbation-theory inspired fit and integrate
- invested $\sim 1.4 \cdot 10^{18}$ flops at CSC (Fin)

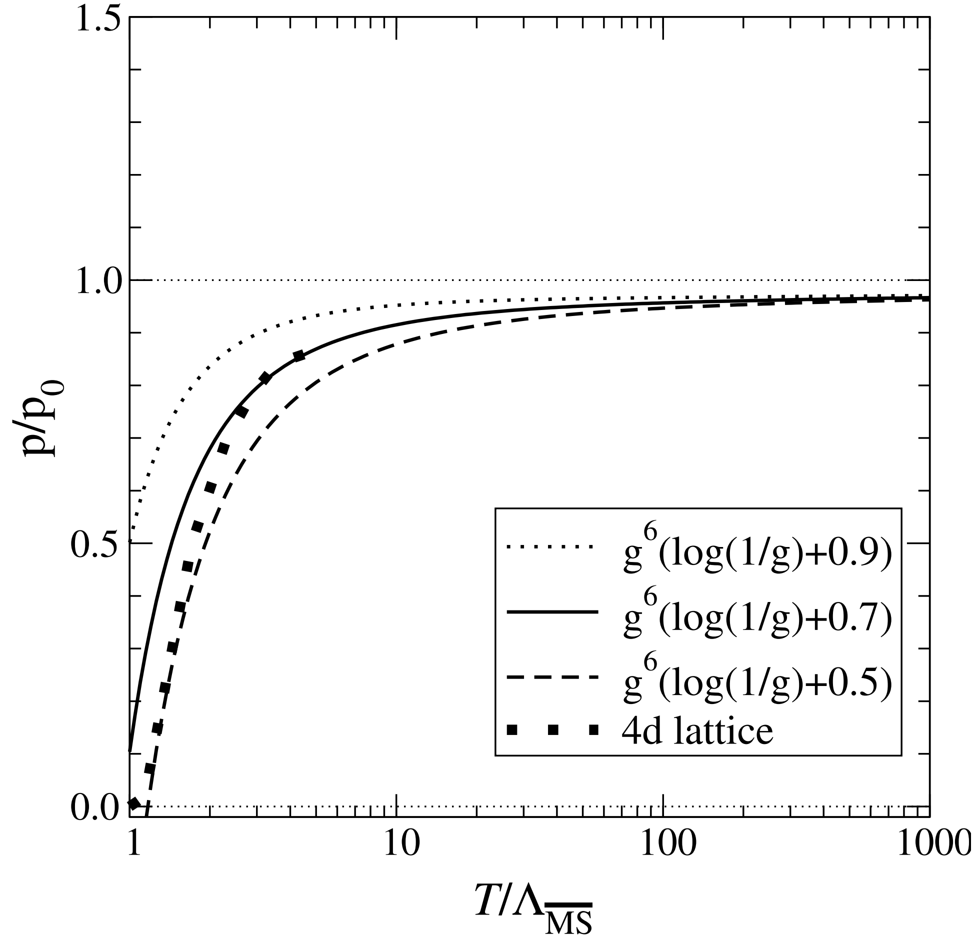
... do not yet fully understand the result ...

Summing up IR contributions beyond 4-loop



- need to include $\langle (TrA_0^2)^2 \rangle$?
- $\langle TrA_0^2 \rangle$ fluctuates too much at small m_E (or T) $\longrightarrow$ systematically overestimated?
  - ▷ physical phase of EQCD is metastable [Kajantie et al 1997]
  - ▷ cure this by improved eff. theories? [Vuorinen/Yaffe; Pisarski; ...]

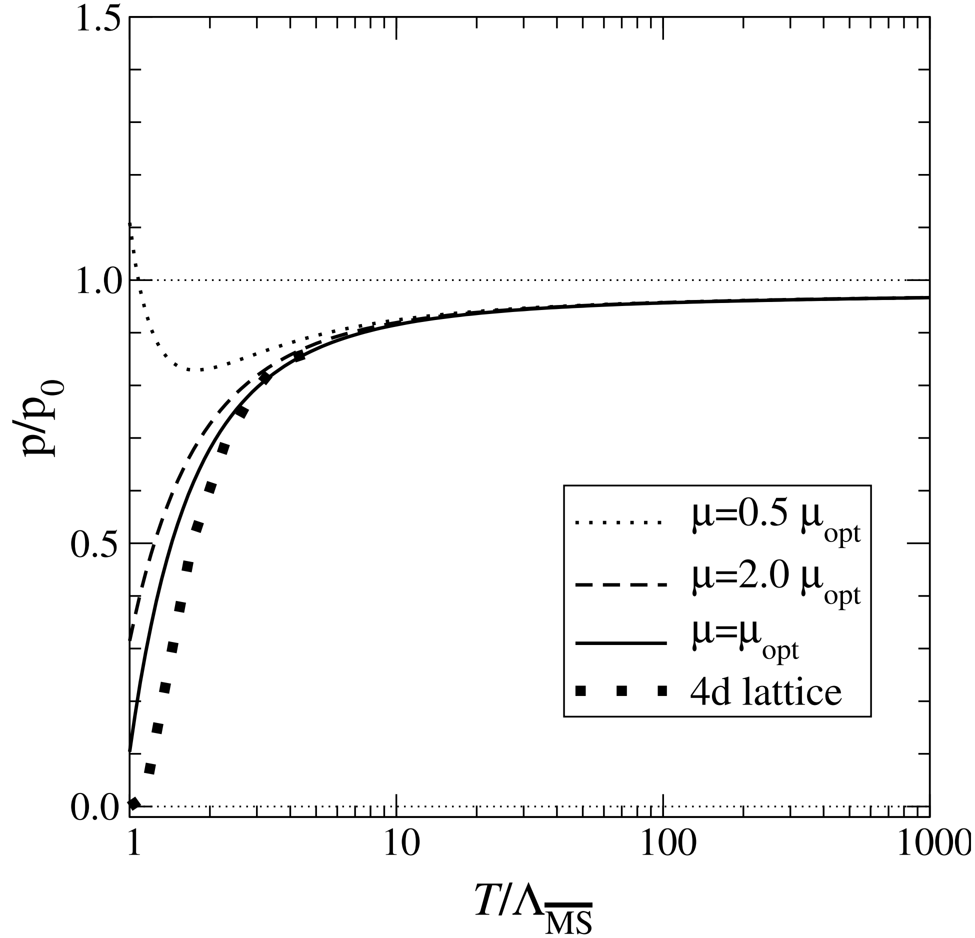
Thermal pressure $p(T)$: 4d vs 3d $(N_f = 0)$



depecence on g^6 constant

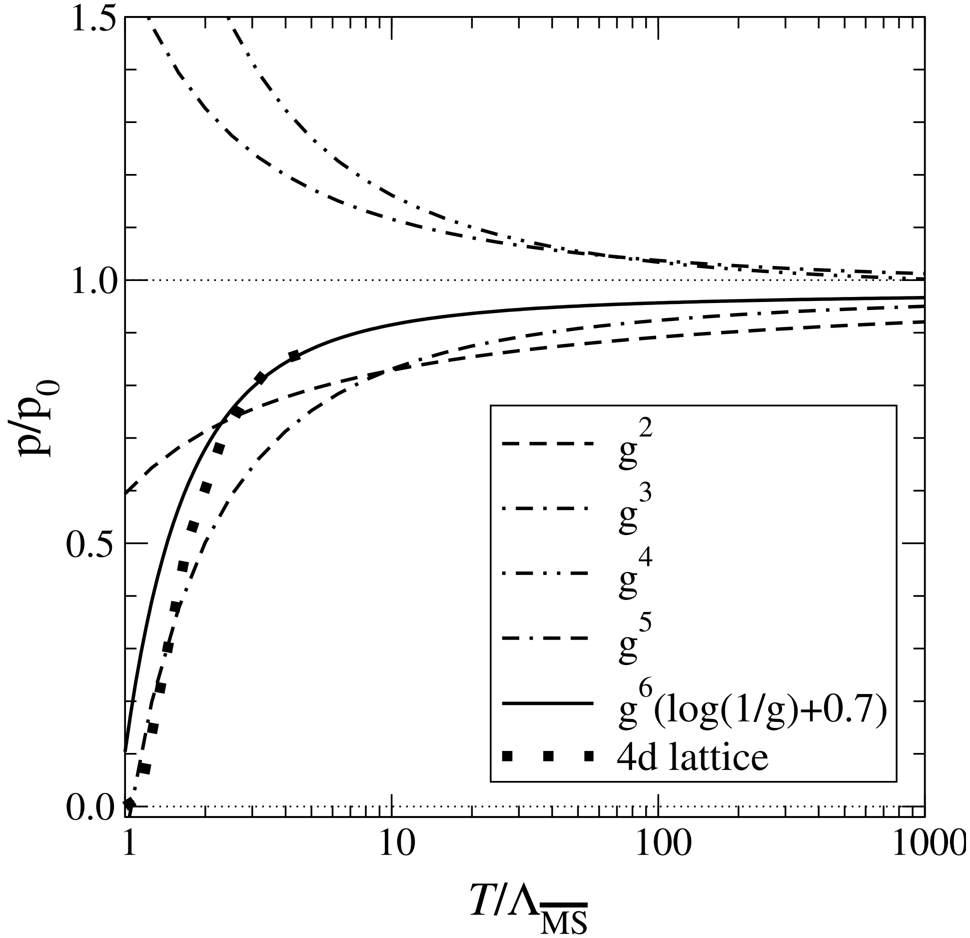
this non-perturbative contribution is unknown, but computable!

Thermal pressure $p(T)$: 4d vs 3d $(N_f = 0)$



scale dependence

Thermal pressure $p(T)$: 4d vs 3d $(N_f = 0)$



g^6 constant is a guess.

non-perturbative contrib not known, but computable!