On the relation of quark confinement and chiral symmetry breaking

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University of Heidelberg

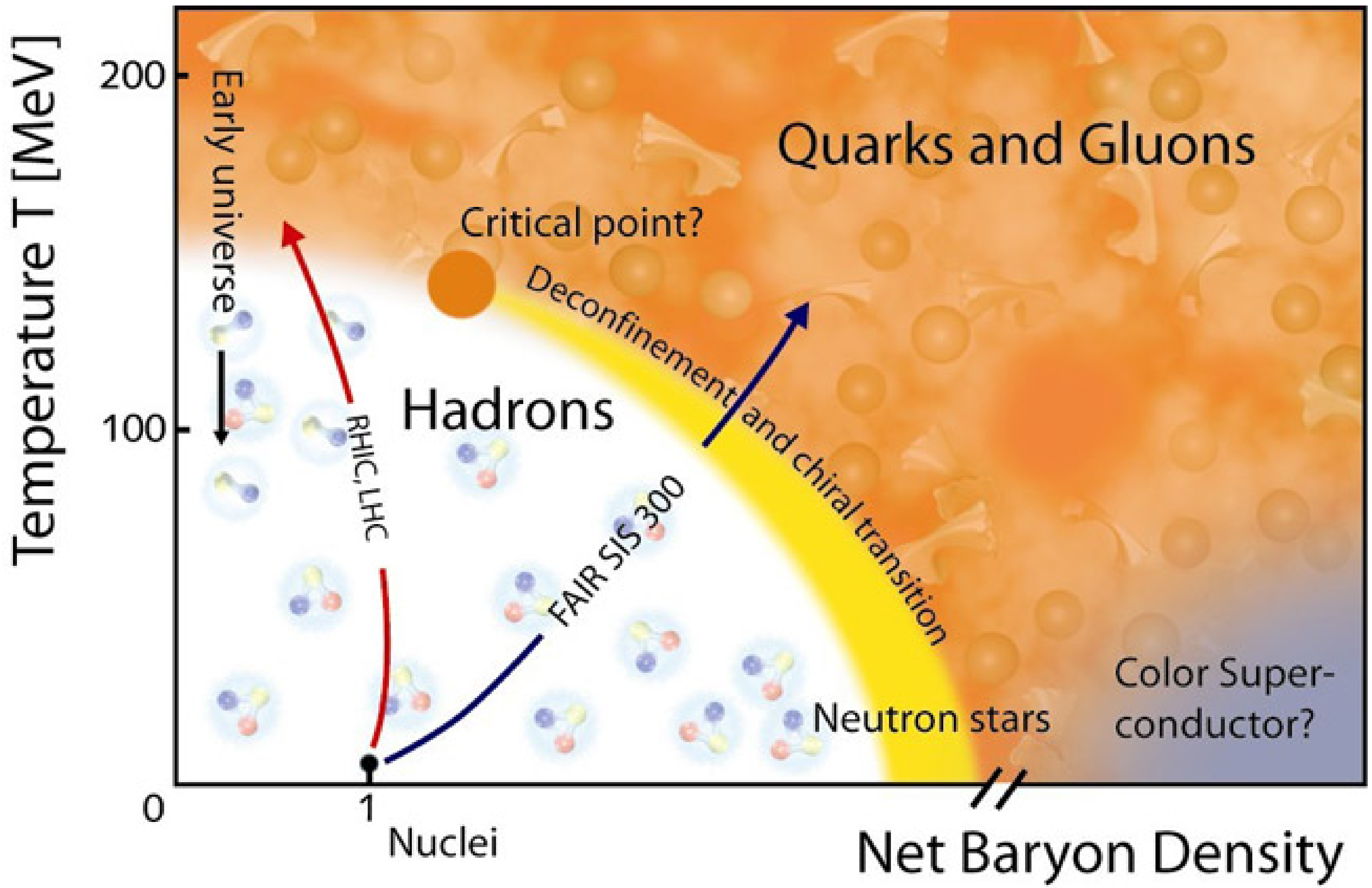
in collaboration with:
Jens Braun, Florian Marhauser, Jan M. Pawlowski

hep-ph/0908.0008 & diploma thesis (LMH) 2008

Norwegian Winter Workshop on QCD in Extreme Conditions
Trondheim 2010

Motivation & Results

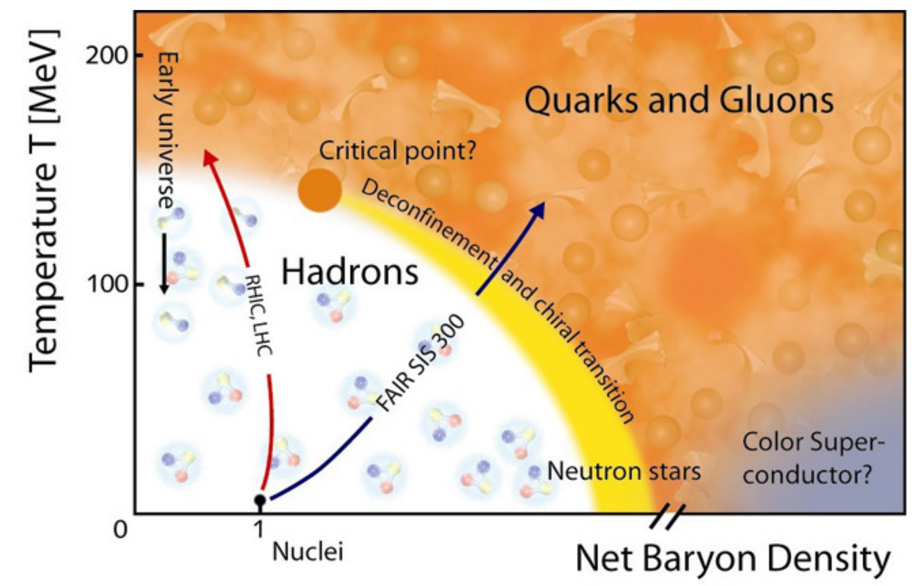
The QCD phase diagram-motivation



credits: GSI Darmstadt

The QCD phase diagram-motivation

Renormalisation Group flows in QCD



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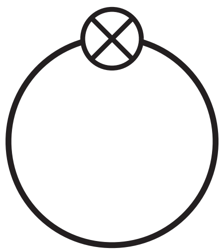
- continuum formulation
- incorporate full dynamics
- fermions straightforward
  - no sign problem
  - chiral fermions
  - bound states via dynamical hadronisation

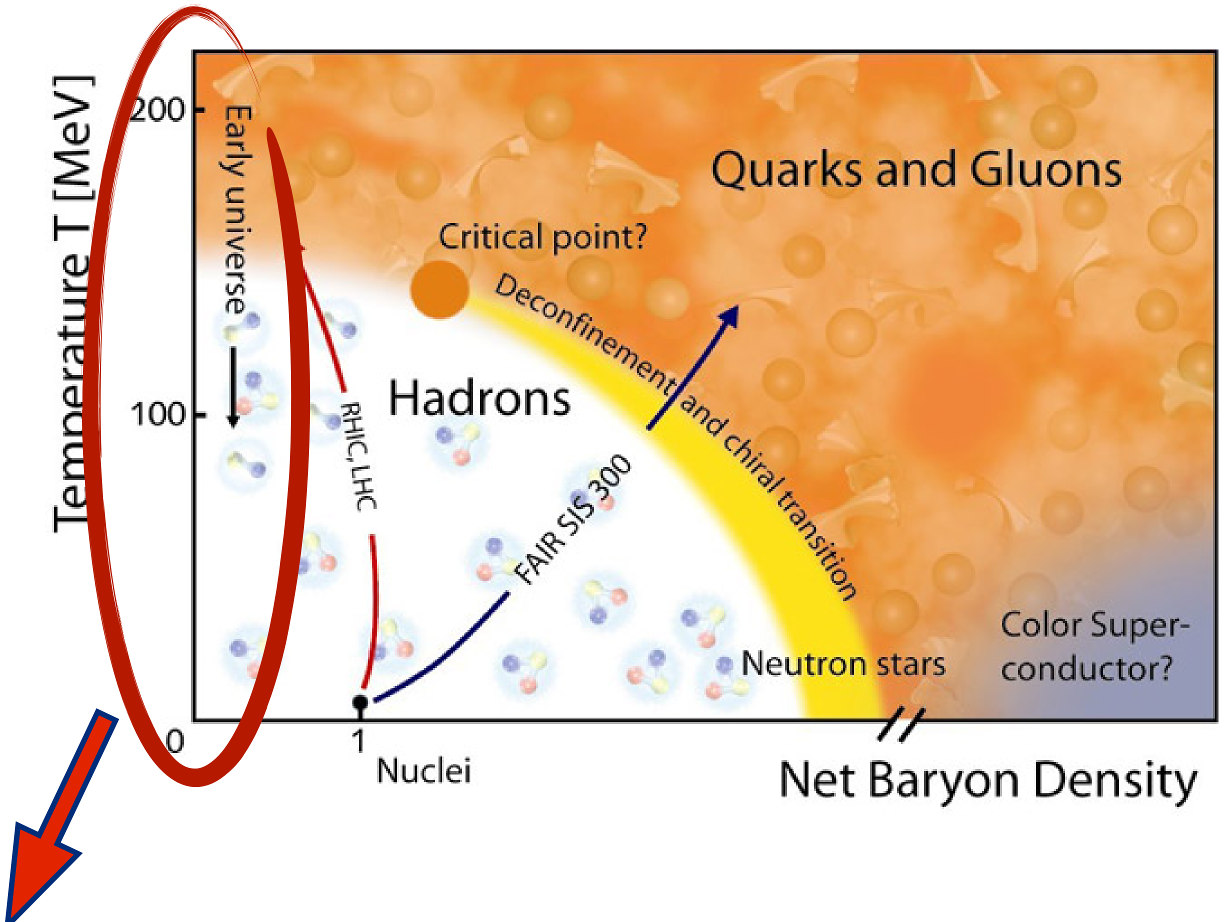
however: calculations more involved

functional RG flow: $\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}$ 

Wetterich '93

$t = k\frac{\mathrm{d}}{\mathrm{d}k}$

The QCD phase diagram-results $N_f=2$



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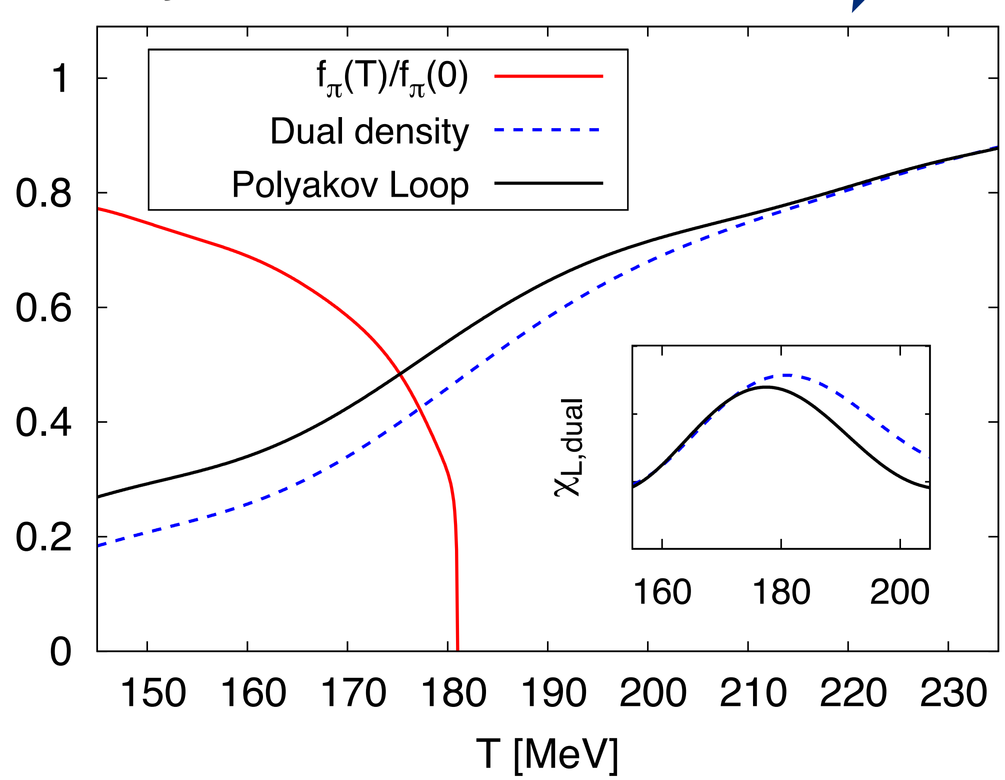

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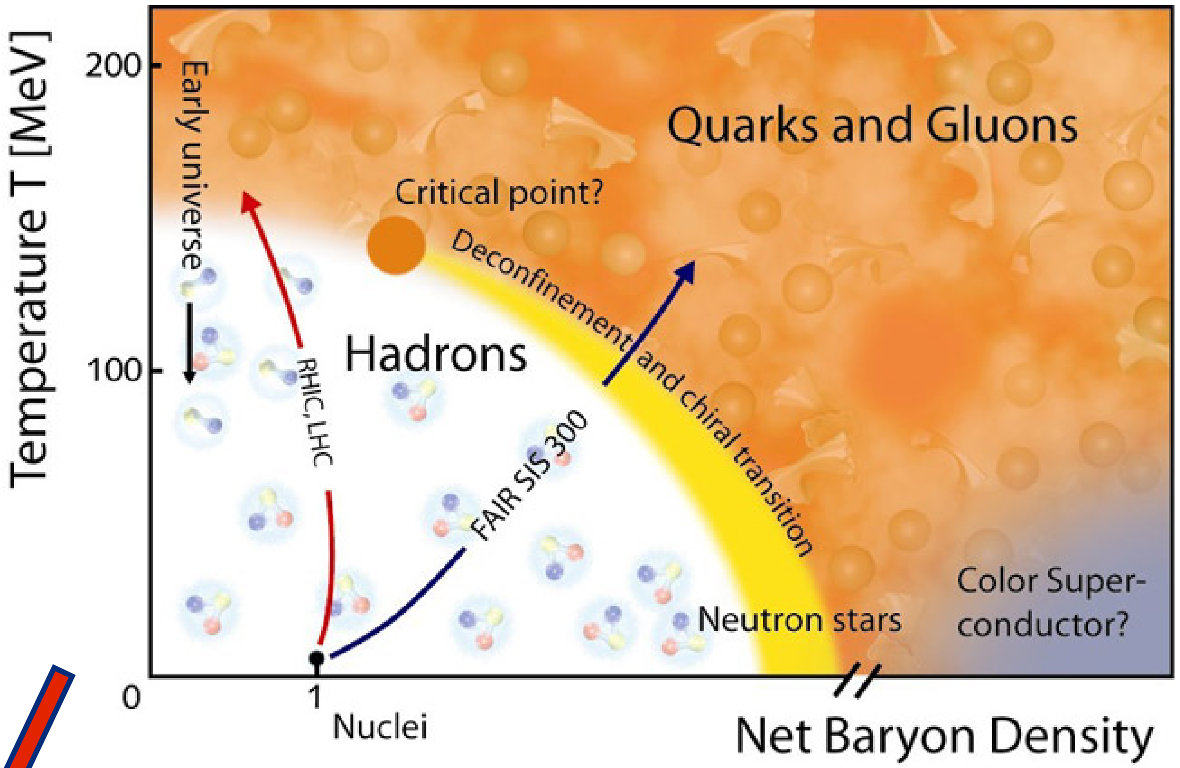

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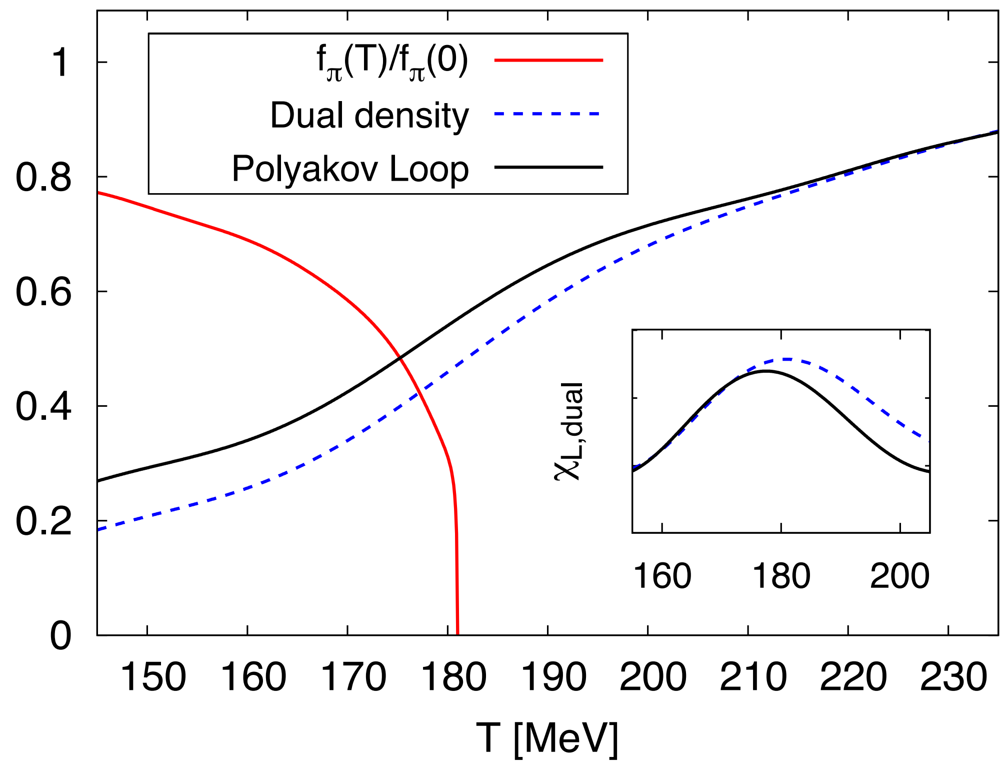

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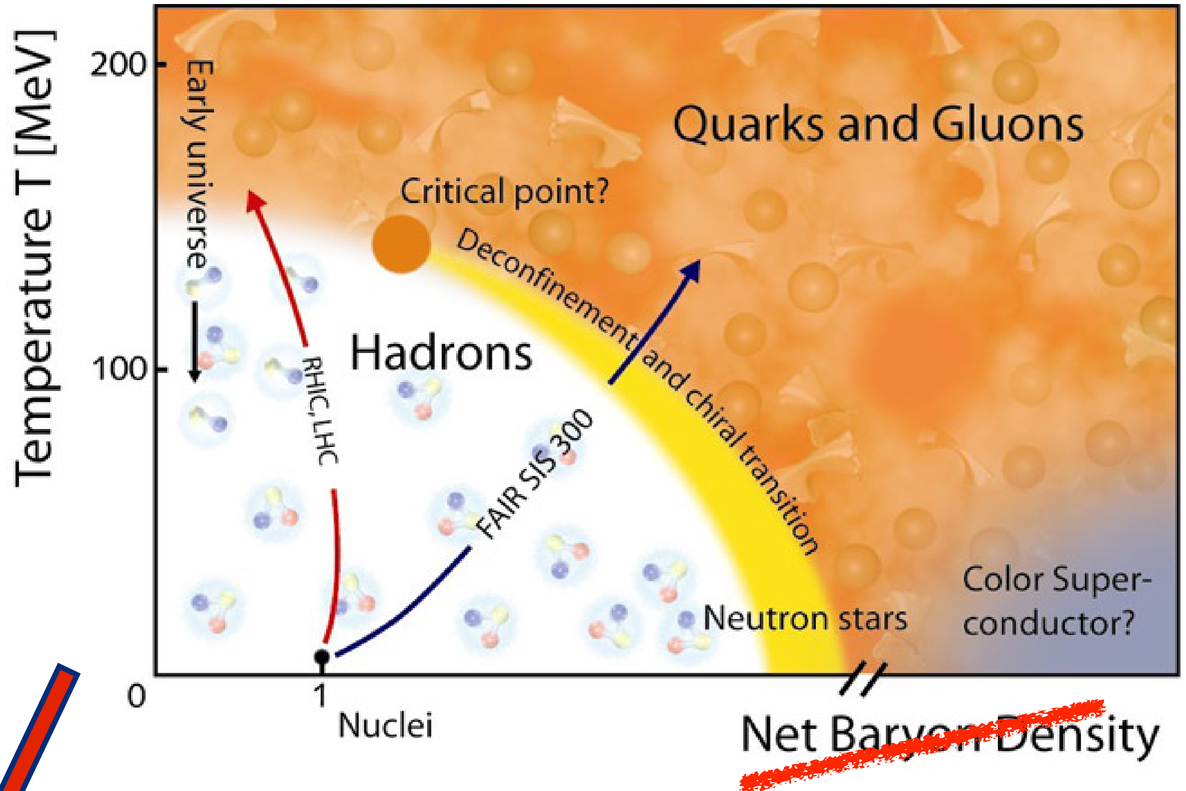

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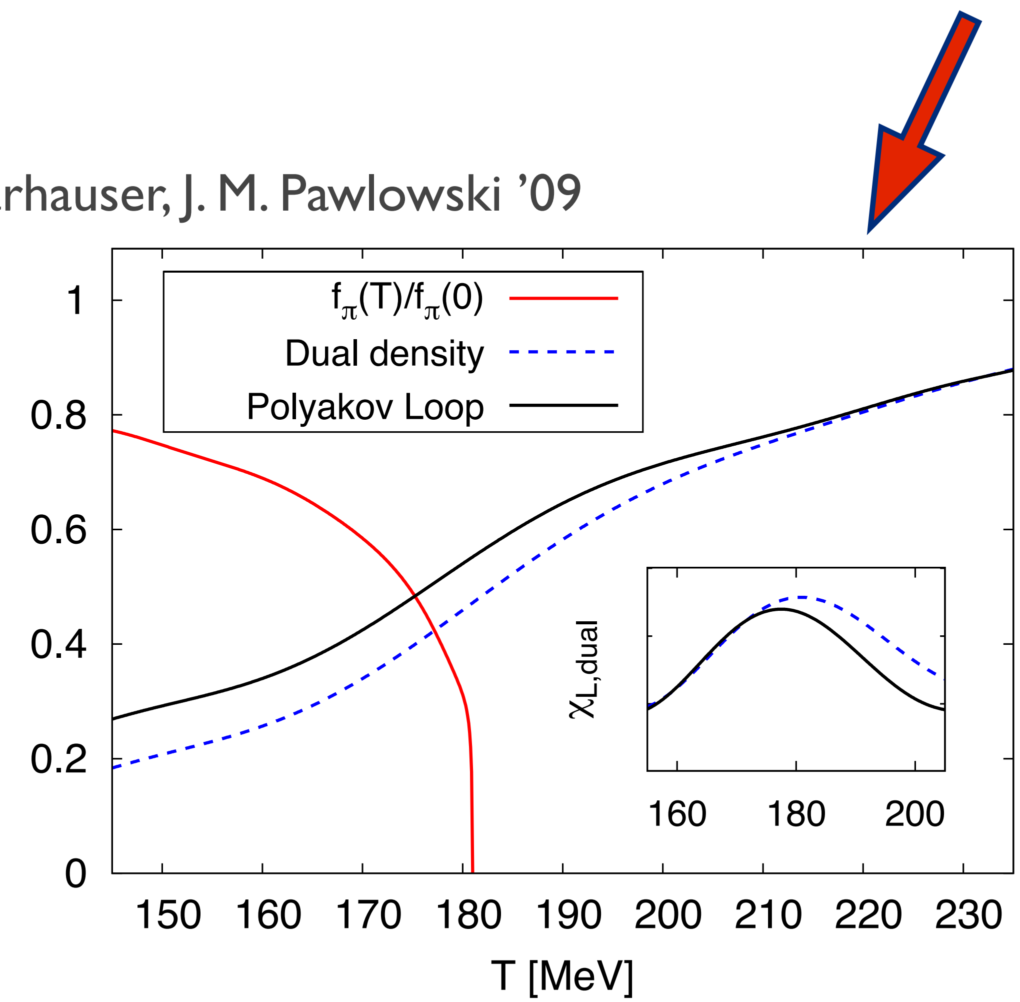

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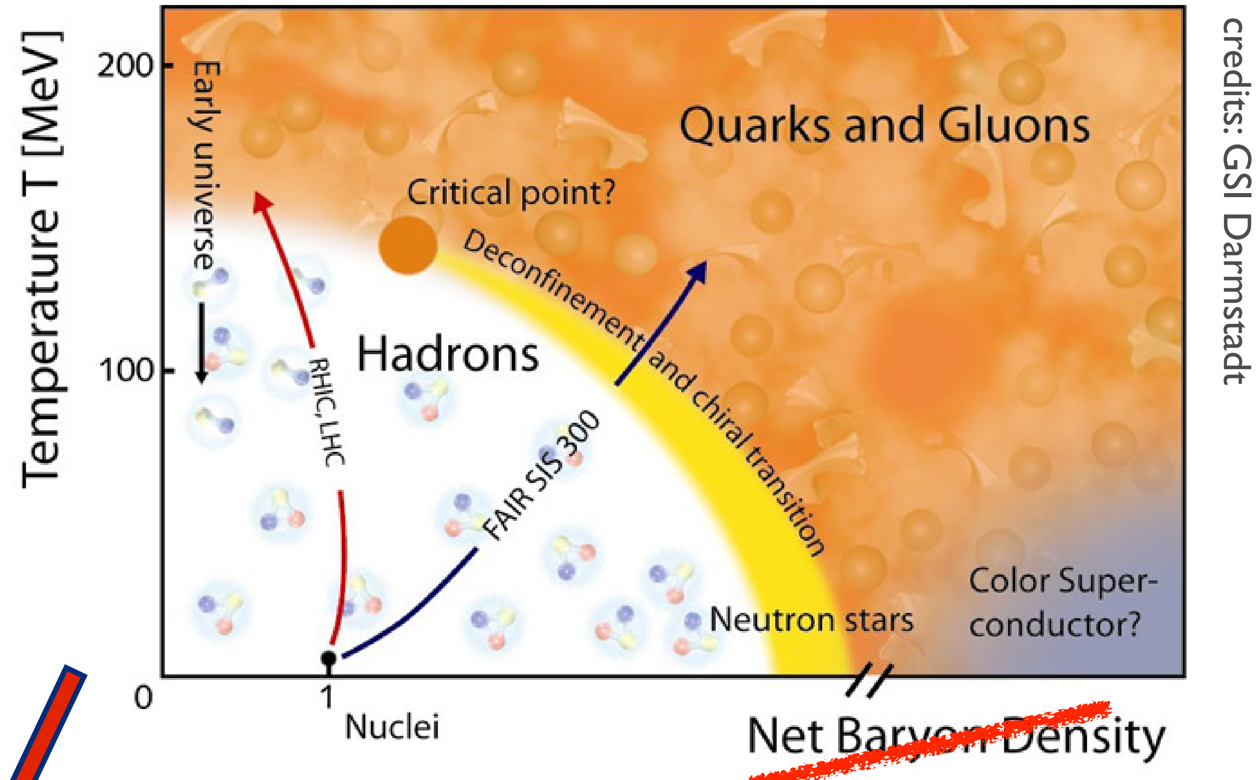

(quark chemical potential)^2

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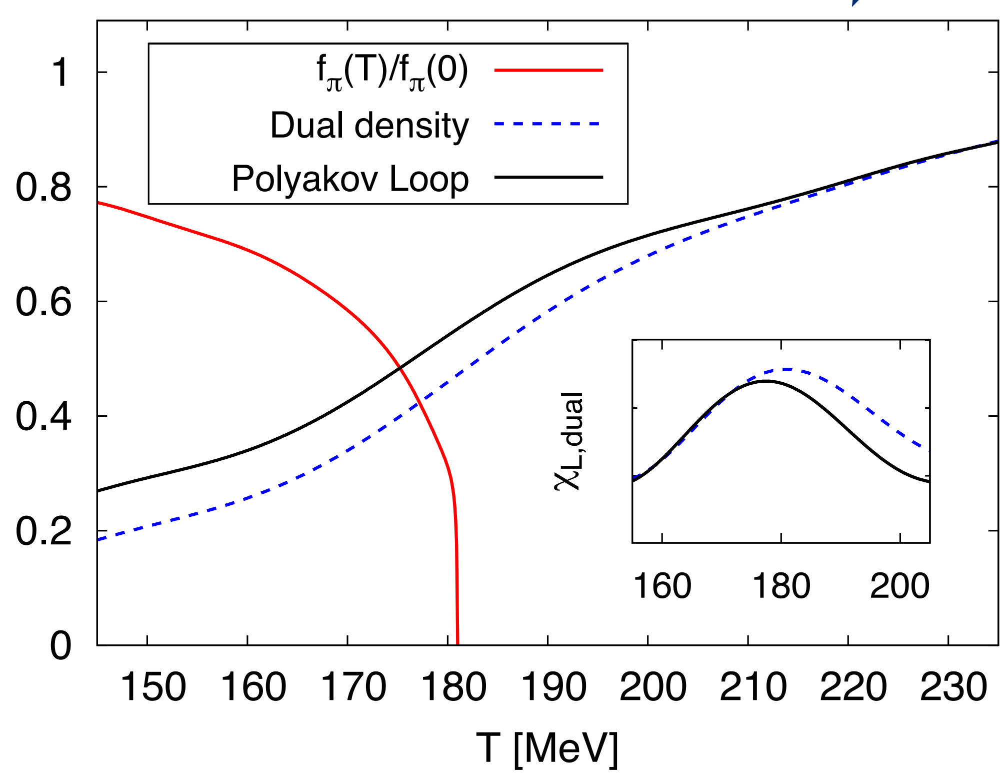

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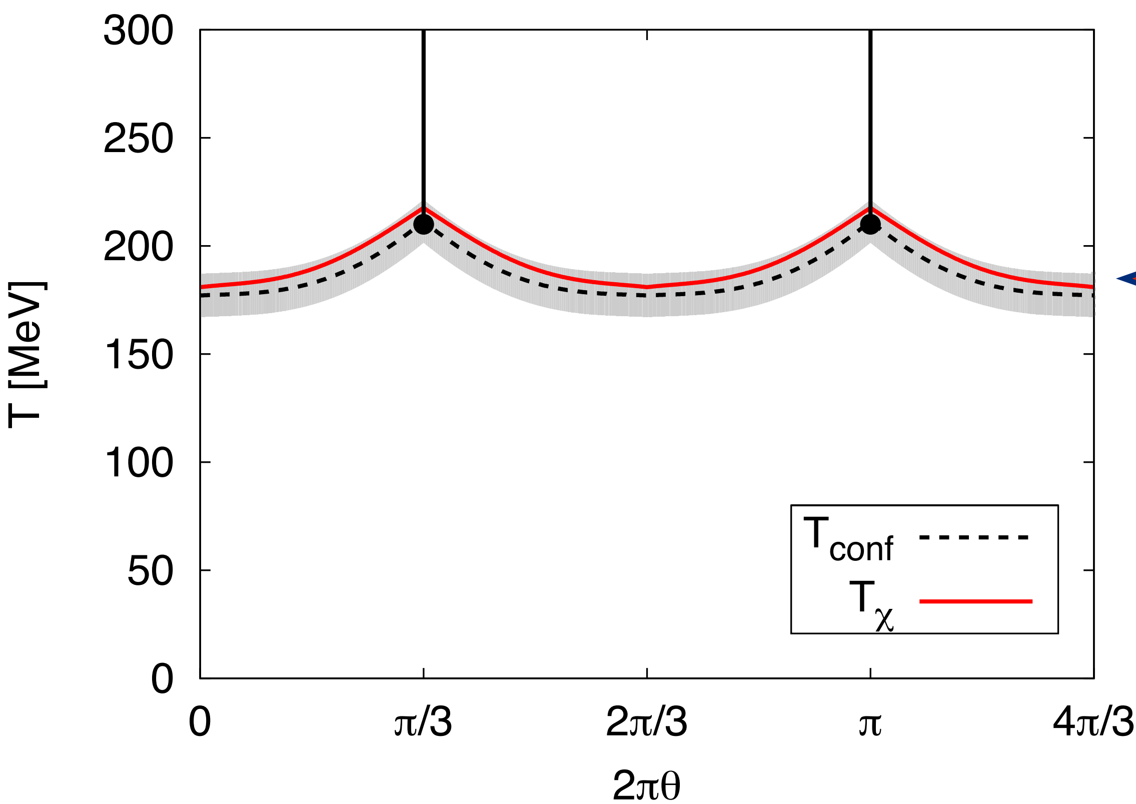

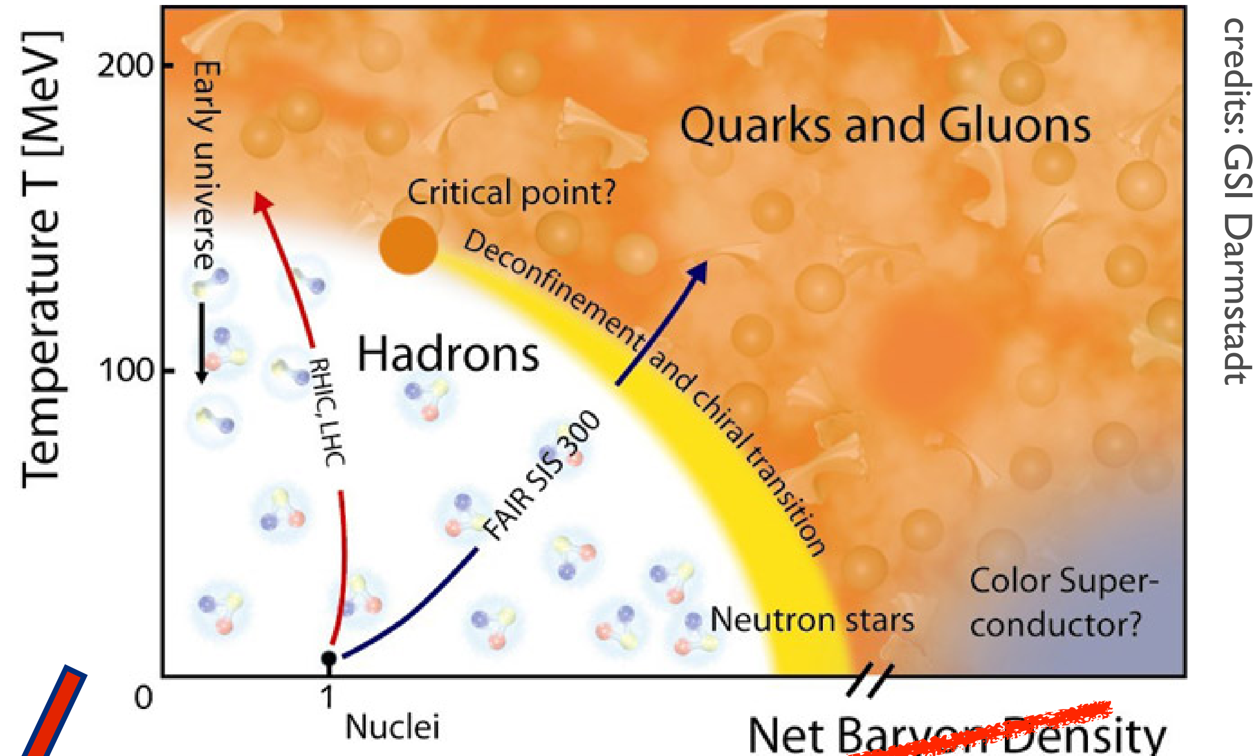

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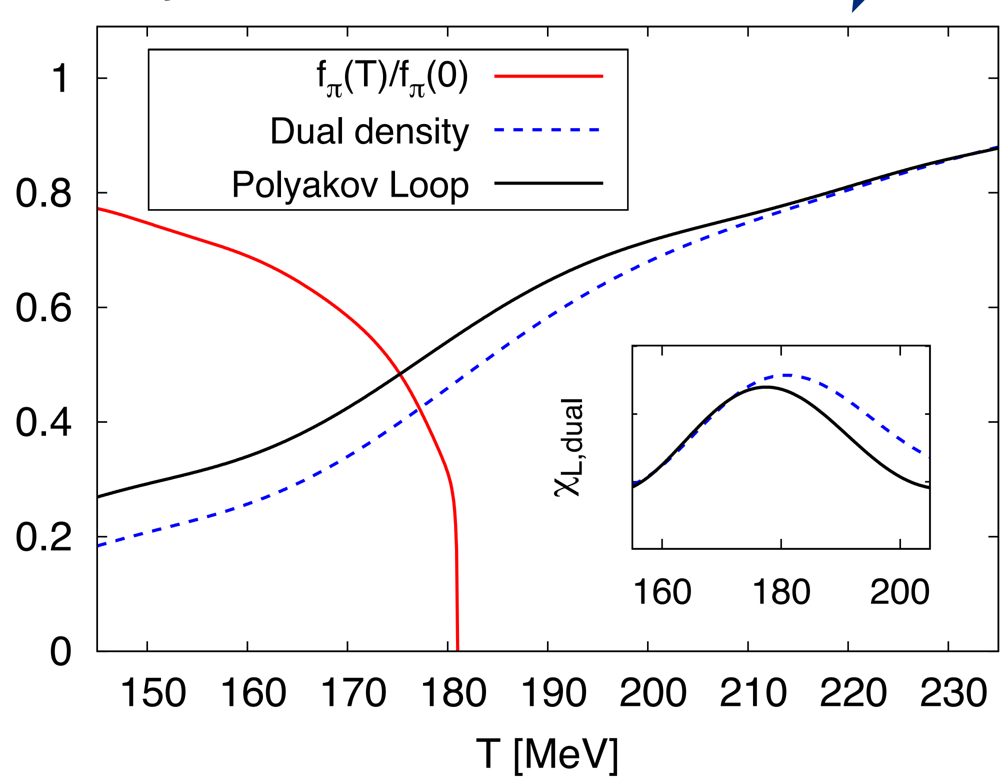

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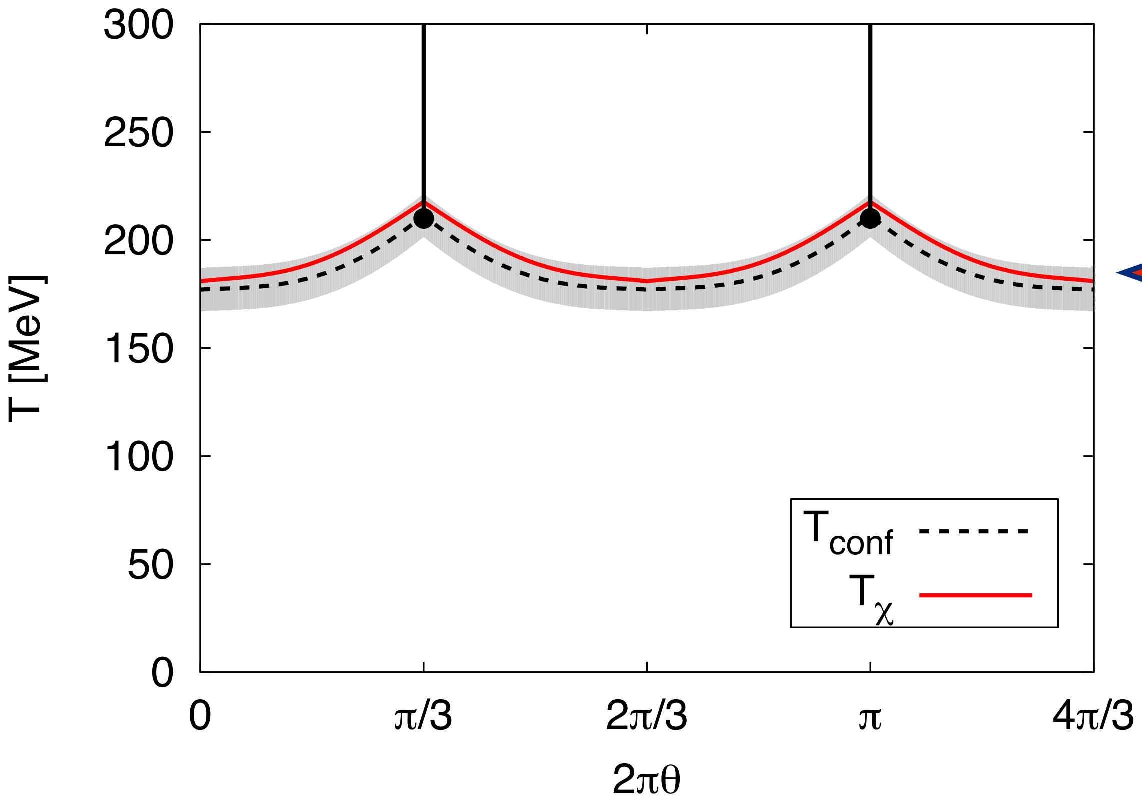

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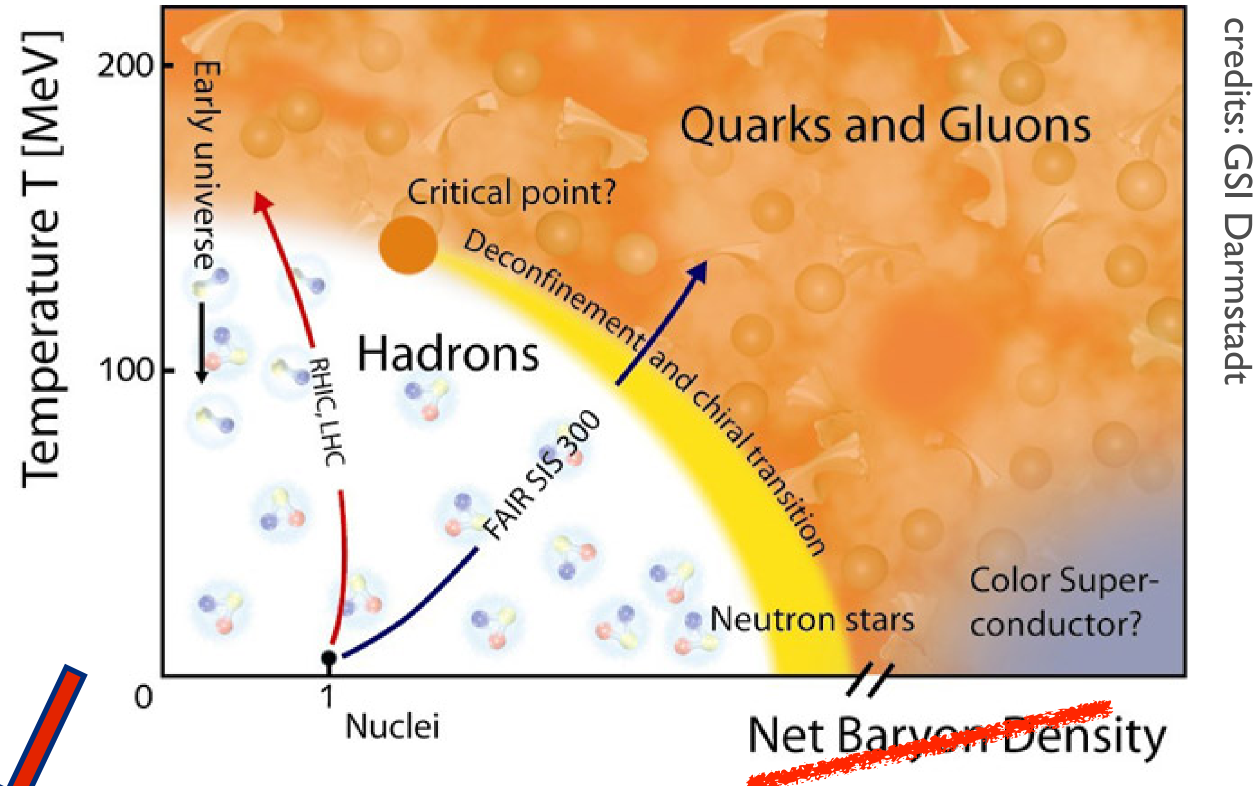

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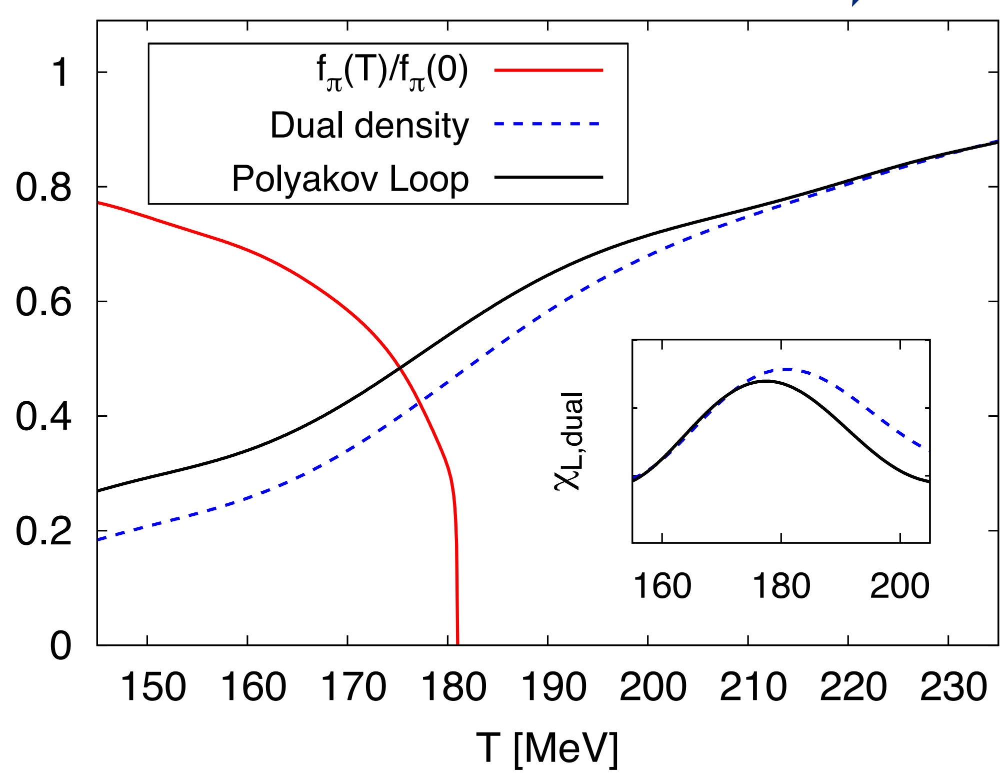
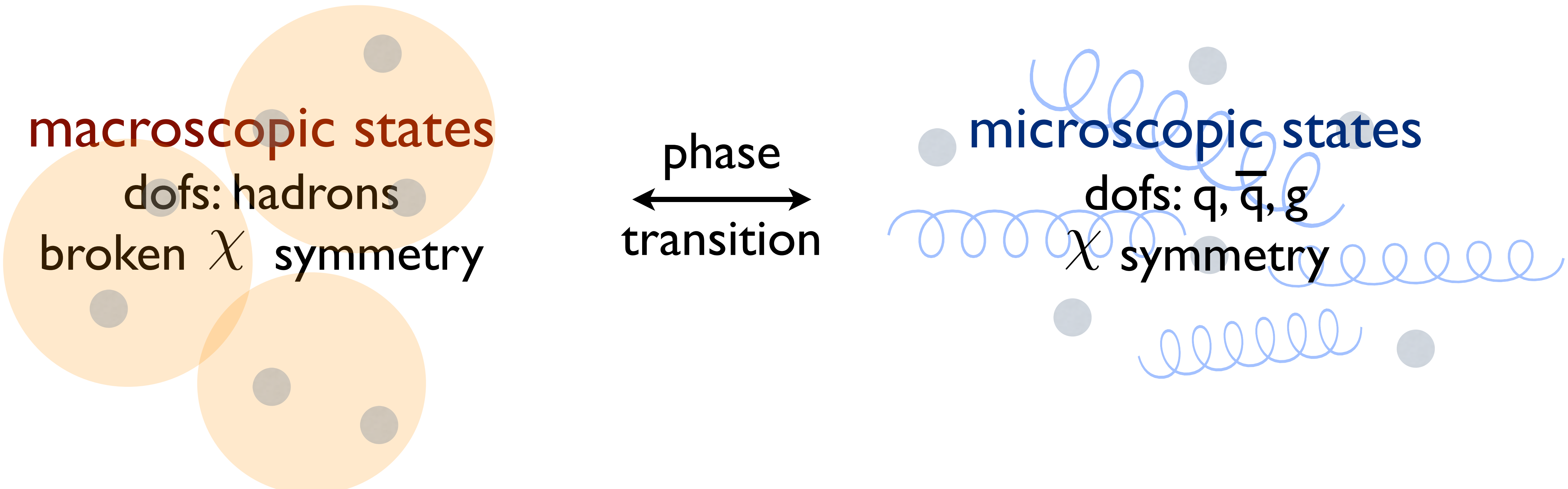


$$T_{\mathrm{conf},cr} \simeq T_{\chi,cr} \approx 180\ \mathrm{MeV}$$

Quark confinement & chiral symmetry breaking

Chiral phase transition



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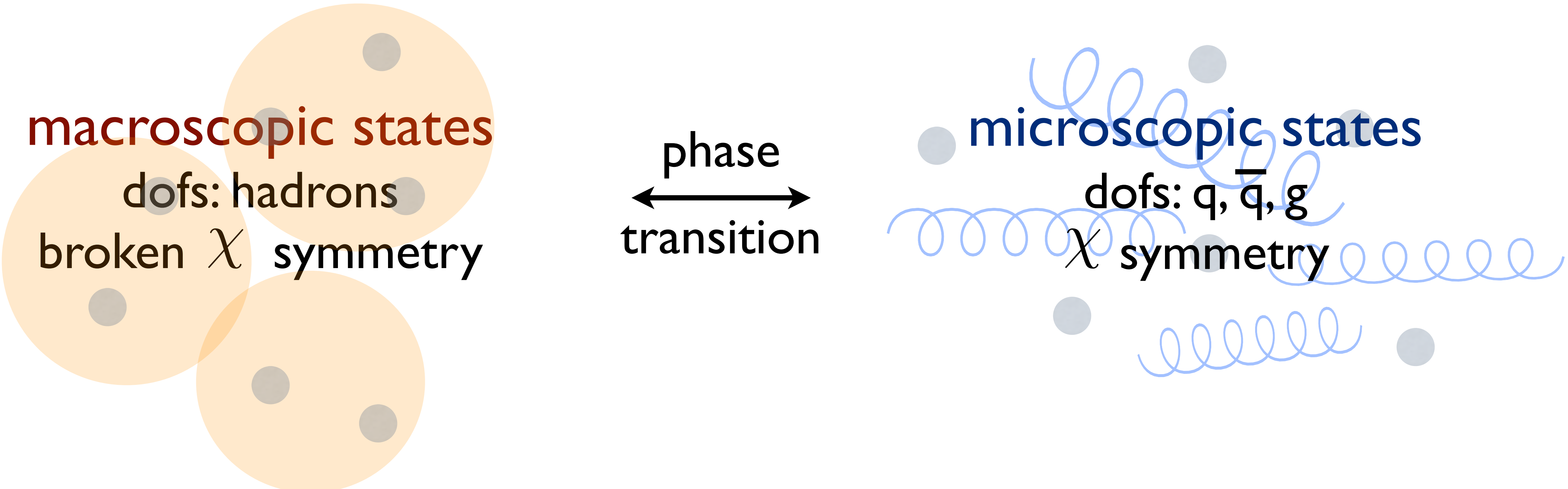


symmetry of matter sector of QCD for $m_q = 0$

order parameter: chiral condensate $\langle\bar{\psi}\psi\rangle$

$$\langle\bar{\psi}\psi\rangle = \begin{cases} 0 & T > T_{c,\chi} \\ > 0 & T < T_{c,\chi}. \end{cases}$$

Confinement phase transition

Symmetry of gauge sector of QCD:

center symmetry Z_3 for SU(3)

Confinement phase transition

Symmetry of gauge sector of QCD:

center symmetry Z_3 for SU(3)

symmetry present in the limit of static quarks ($m_q \to \infty$)

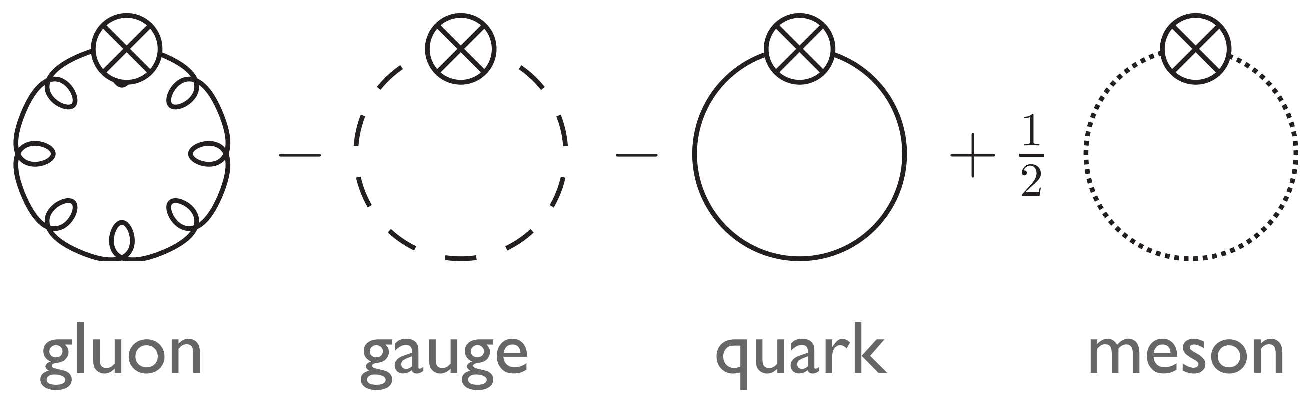
order parameter: Polyakov loop ϕ

$$\phi = \frac{1}{N_c} \mathrm{Tr}\, \mathcal{P}\, e^{i \int_0^{1/T} dt \, \langle A_0 \rangle} = \begin{cases} > 0 & T > T_{c,\mathrm{conf}} \\ 0 & T < T_{c,\mathrm{conf}}. \end{cases}$$

Approximation

imaginary $\mu = 2\pi \mathrm{i}\theta T$ and finite T, $N_f = 2$, chiral limit

$\partial_t \Gamma_k[\phi] = \frac{1}{2}$ (gluon) $-$ (gauge) $-$ (quark) $+\frac{1}{2}$ (meson)



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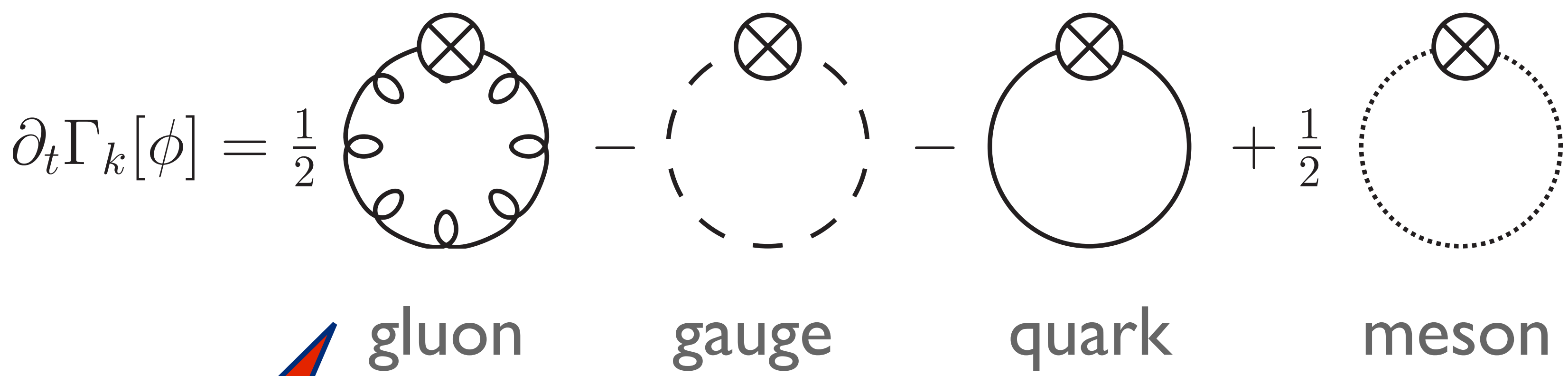

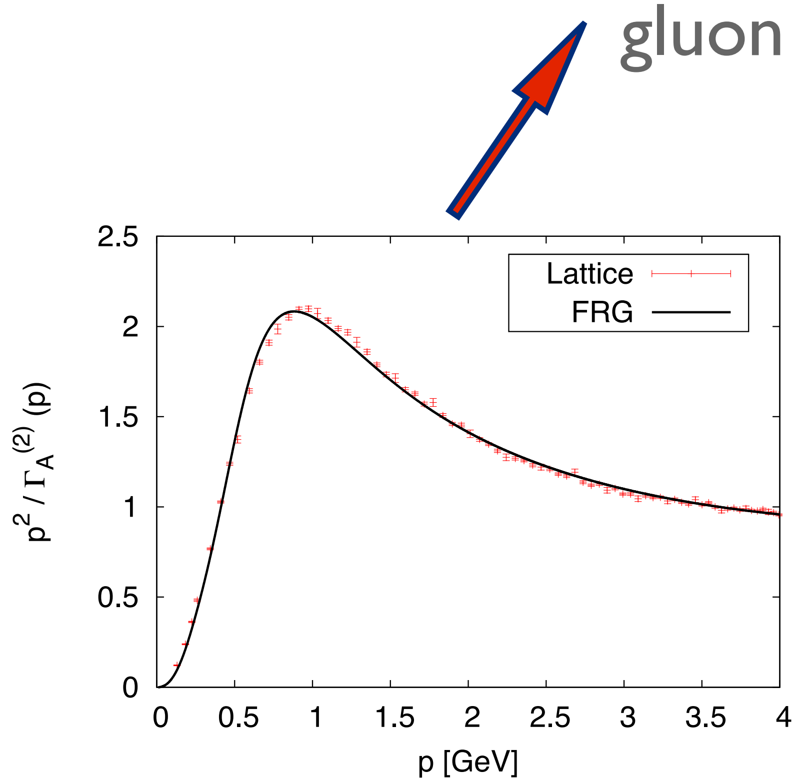

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J. M. Pawlowski, in preparation

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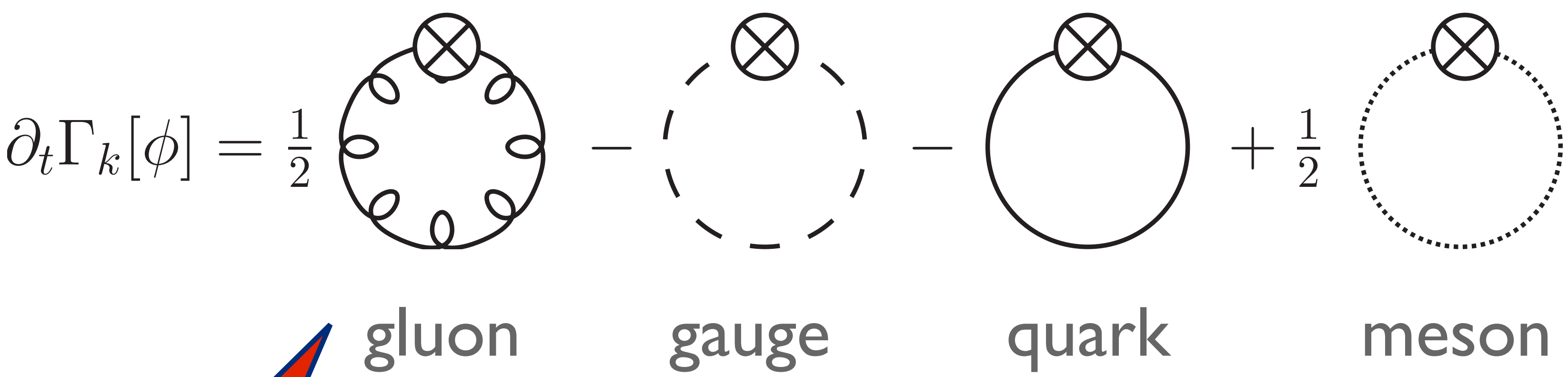

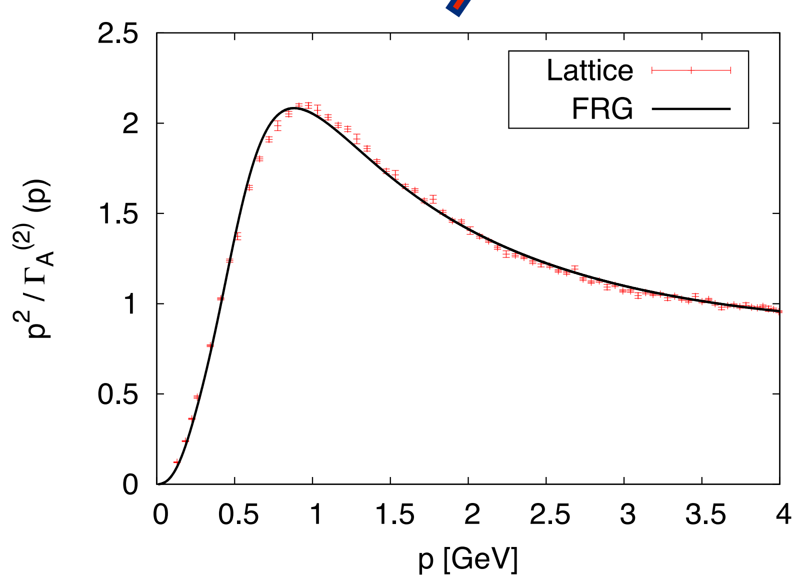

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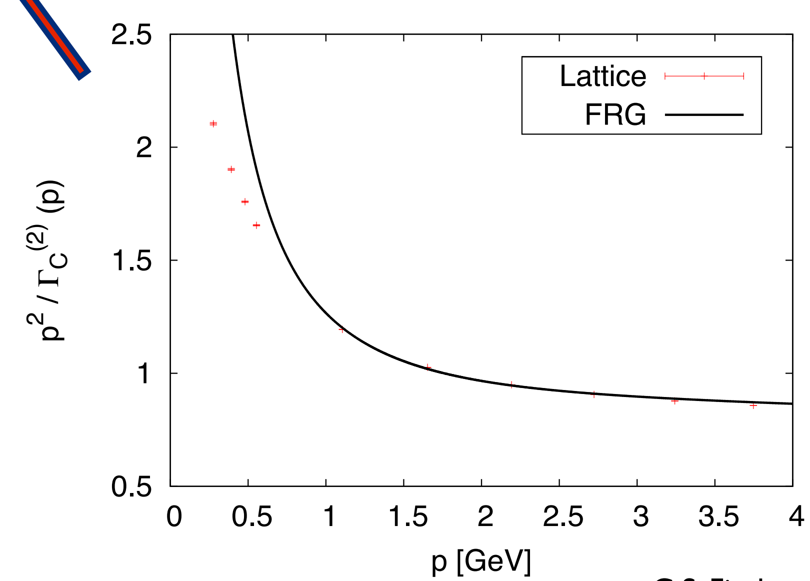

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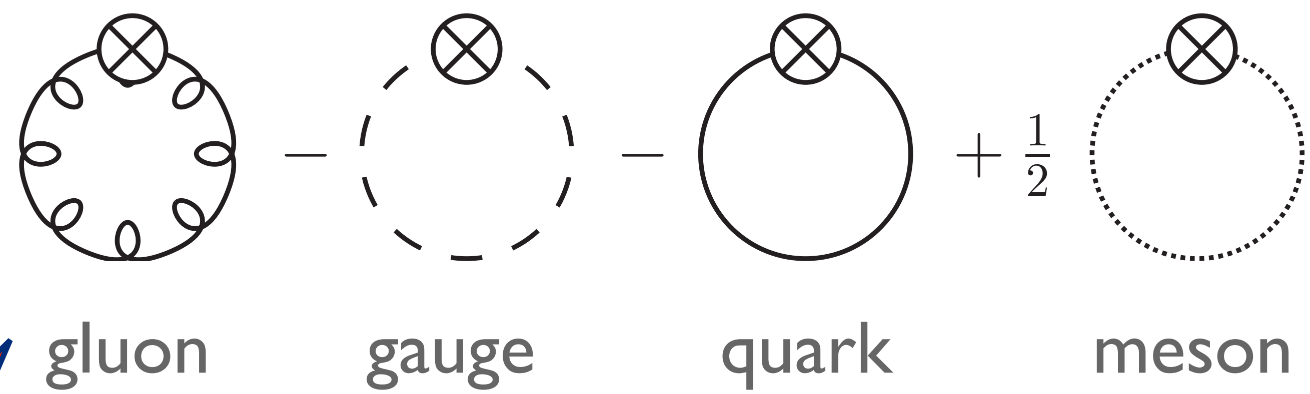
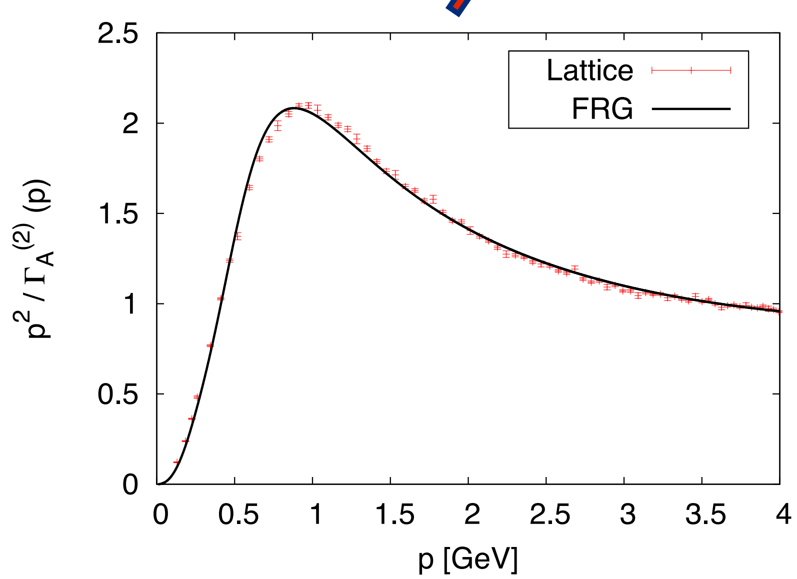
$\partial_t \Gamma_k[\phi] = \frac{1}{2}$ 

mesonic quantum fluctuations



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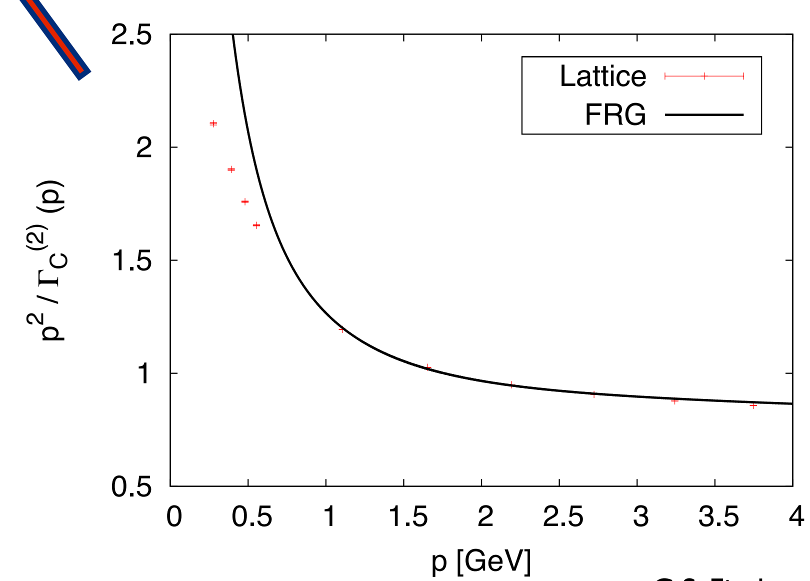

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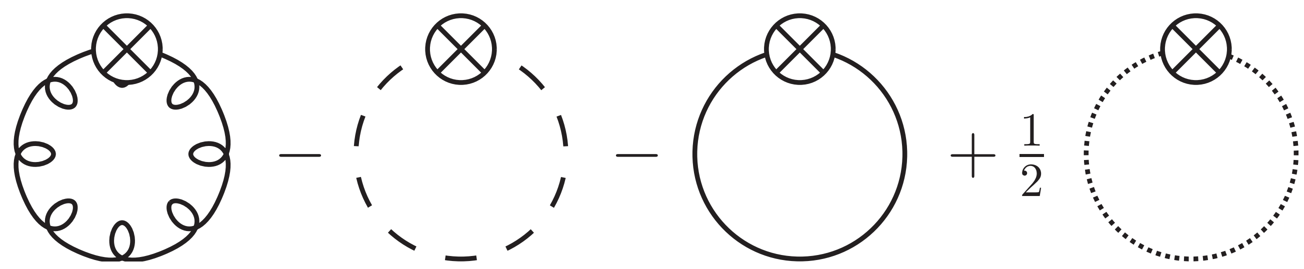

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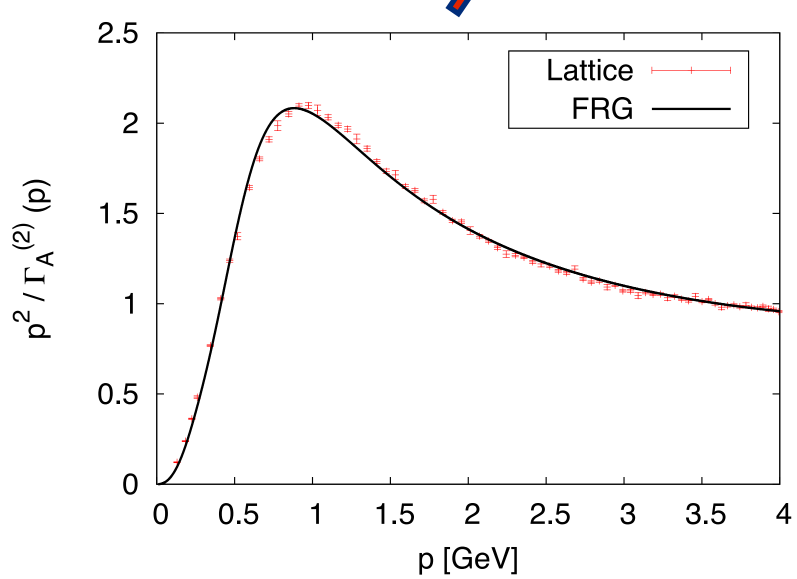

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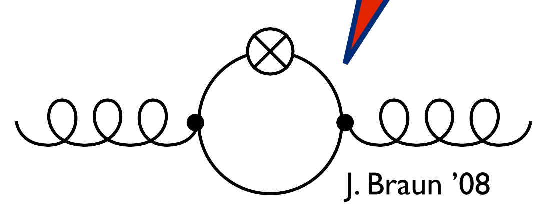
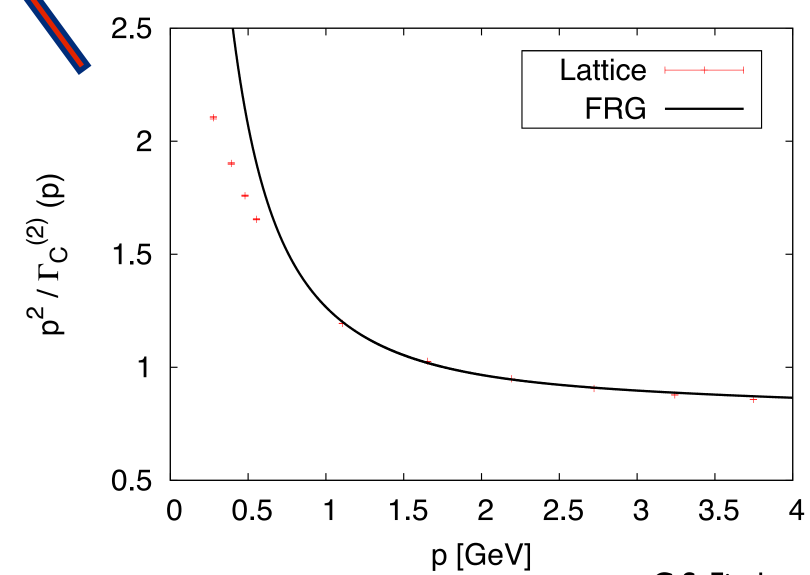


J. Braun '08
LMH diploma thesis '08



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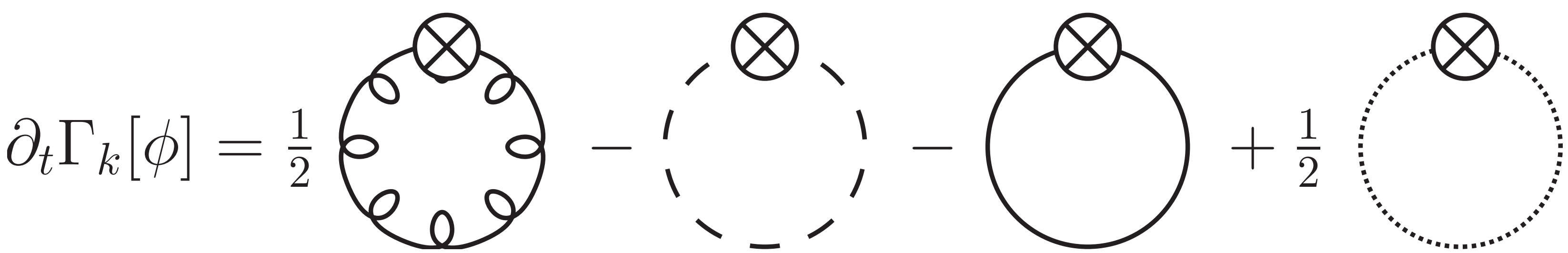

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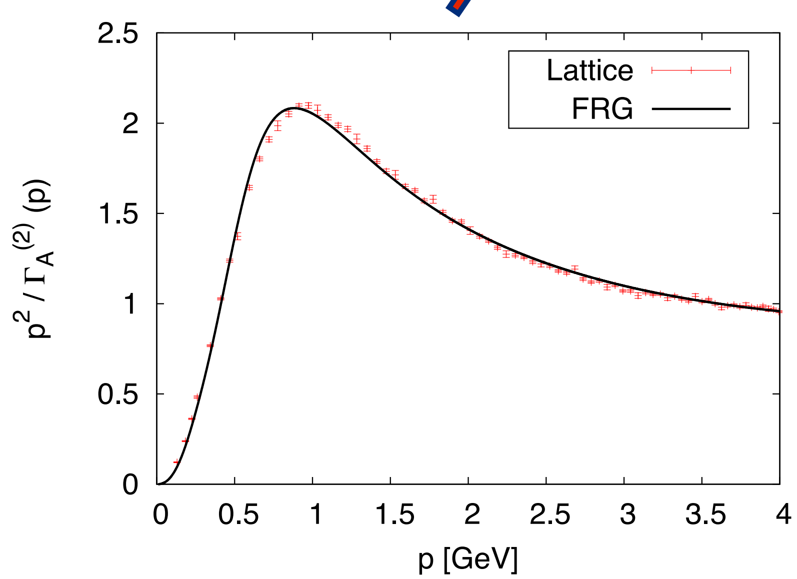

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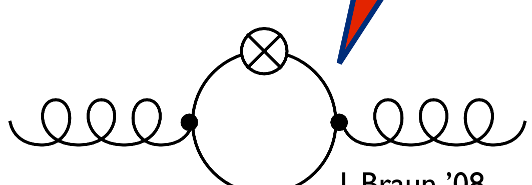

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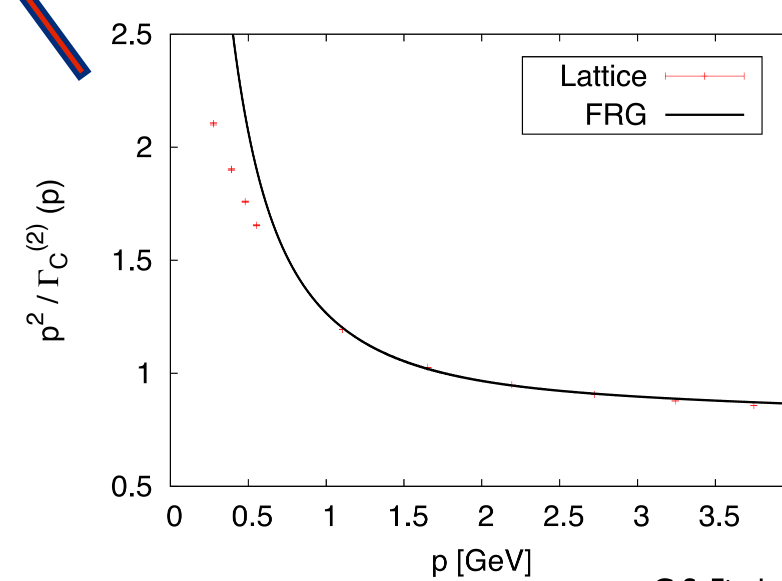

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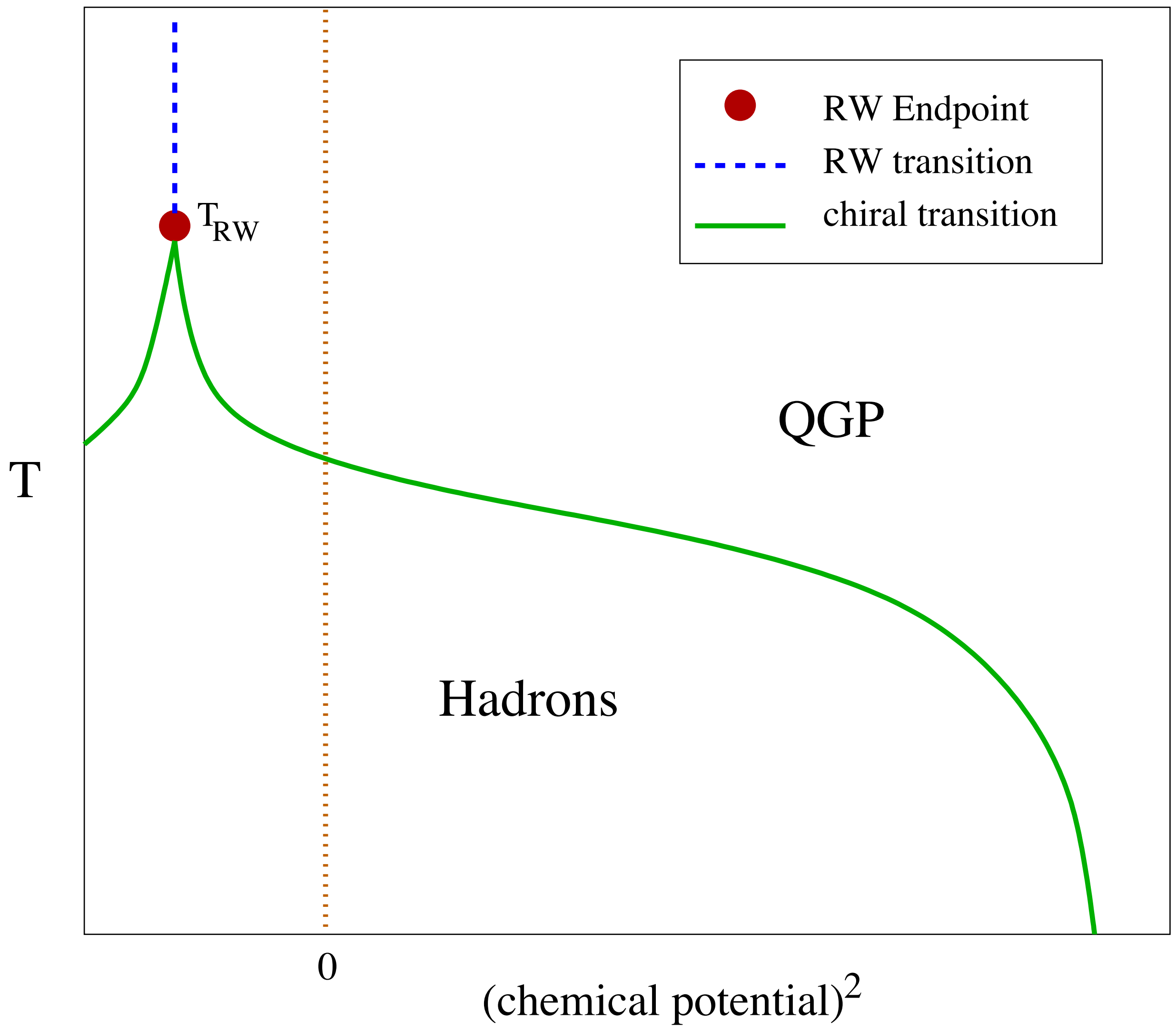
confining properties: full momentum dependence of ghost & gluon propagator

Roberge-Weiss periodicity & p.t.



$\theta = -\mathrm{i}\mu/2\pi T$:

$$\mathrm{QCD}_\theta = \mathrm{QCD}_{\theta+\theta_z}$$ periodic

where $\theta_z = 0, 1/3, 2/3$ for SU(3)

quantities related to effective action: show same periodicity (RW periodicity) e.g. $\langle\bar{\psi}\psi\rangle$

QCD_θ : smooth until $\theta = \pi/3$, then shows discontinuity: Polyakov loop RW phase transition

Order parameters

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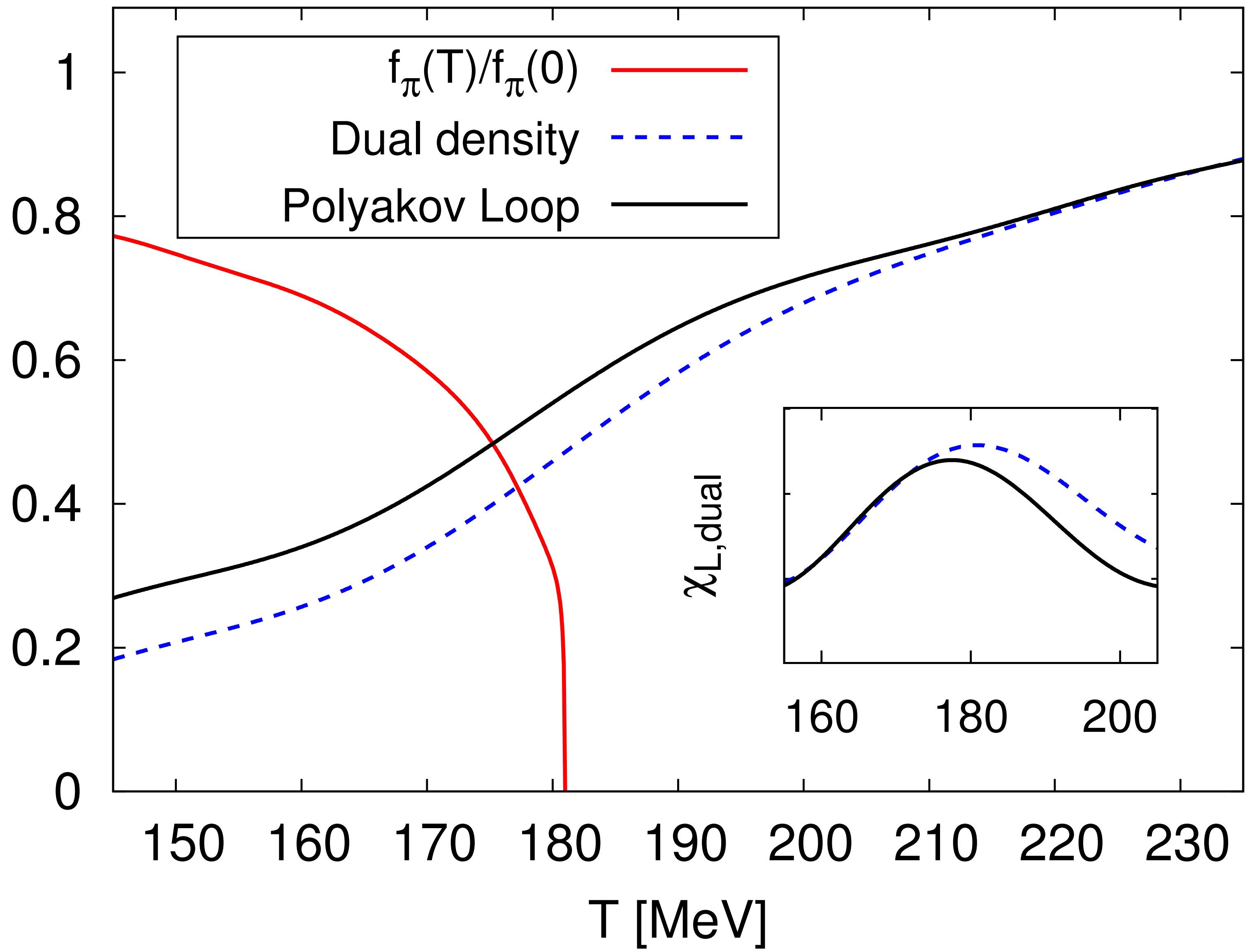
$\theta = 0:$



$$T_{\mathrm{conf},cr} \simeq T_{\chi,cr} \approx 180\ \mathrm{MeV}$$

Phase diagram for imaginary μ

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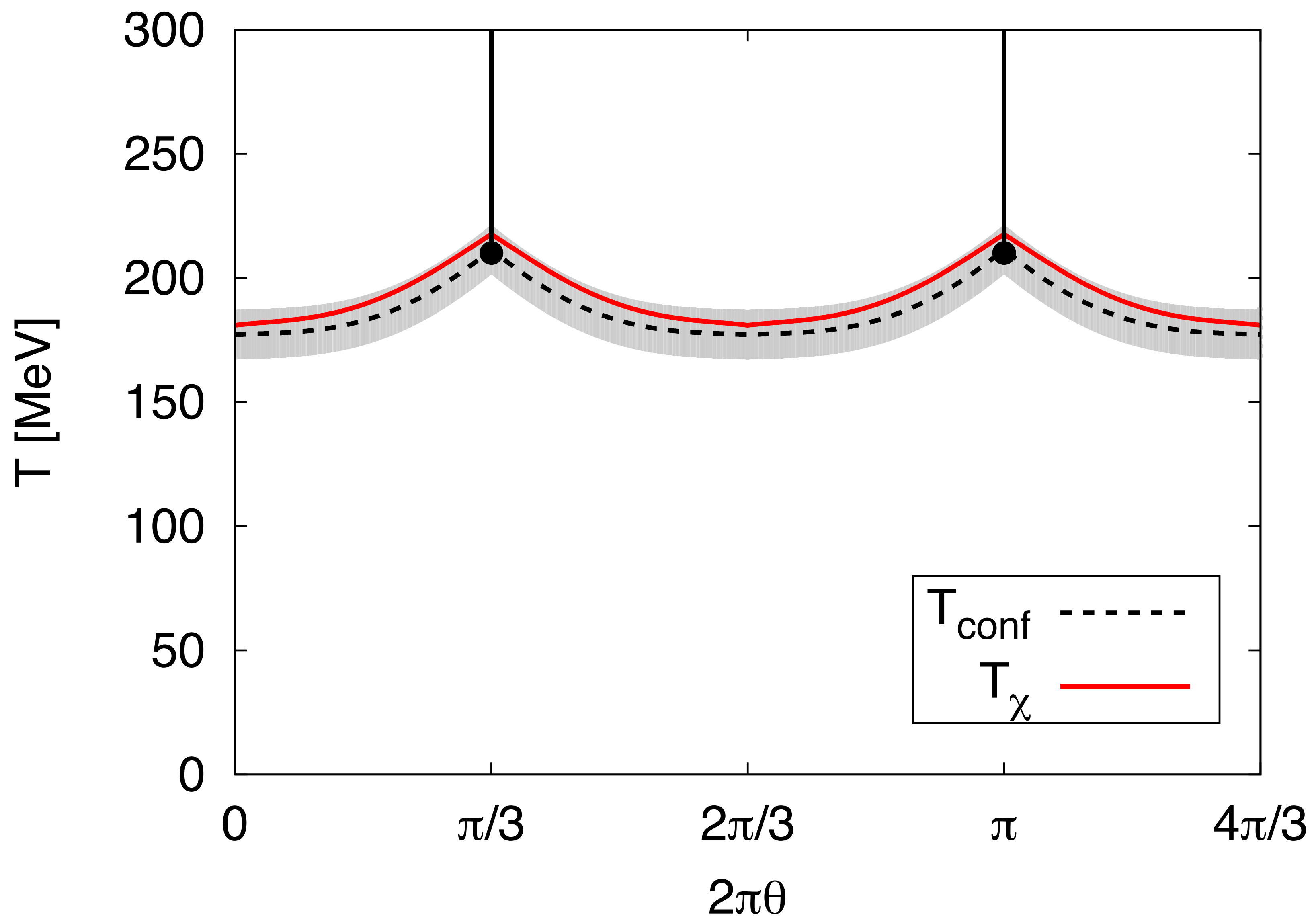
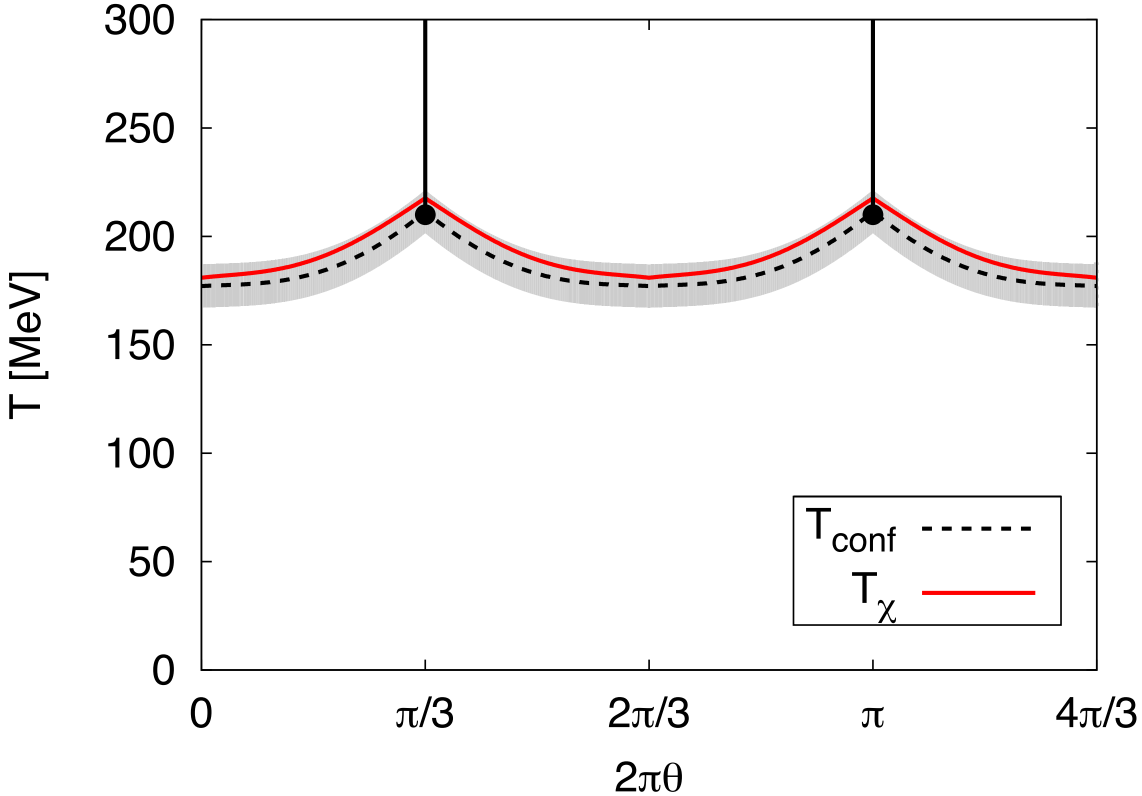


T_{conf} and T_χ lie close together, end in critical point (RW phase transition) at $(\mathrm{T_{RW}}, 2\pi\theta) = (210\,\mathrm{MeV}, \frac{\pi}{3}(2n+1))$

Phase diagram for imaginary μ



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coinciding T_{cr} result from interplay of quantum fluctuations & are not adjusted by hand

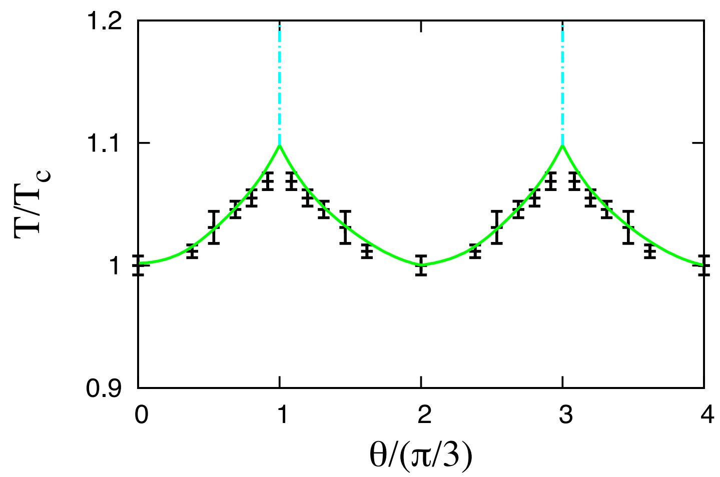
agrees with lattice:

Kratochvila et al '06;
Wu et al '06

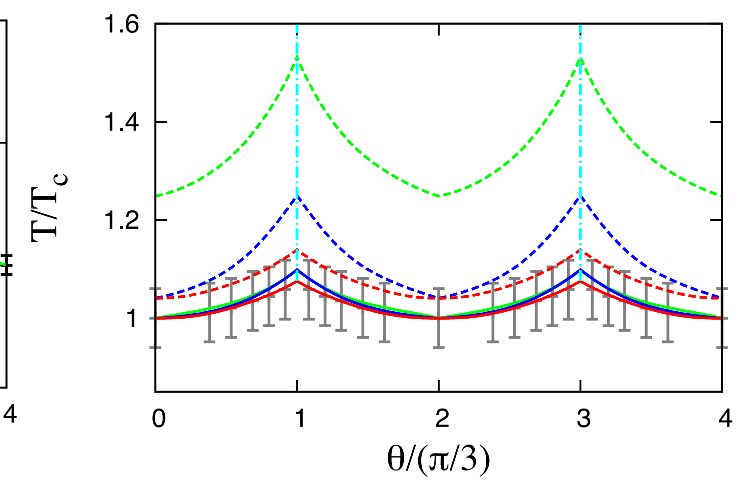
PNJL model agrees if 8-quark interaction is adjusted:

Sakai et al '08

$T_{\mathrm{conf},cr}$



$T_{\chi,cr}$



Dual order parameters

imaginary chemical potential → C. Gattringer '06; F. Synatschke, A. Wipf, C. Wozar '07; F. Bruckmann, C. Hagen, C. Gattringer '08; C. S. Fischer '09; C. S. Fischer, J. Mueller '09; J. Braun, LMH, F. Marhauser, J. M. Pawlowski '09

Observable $\mathcal{O}_\theta$: transforms non-trivially under center transformations $z = e^{2\pi i \theta_z}$ with $\theta_z = 0, 1/3, 2/3$ for SU(3)

is an oder parameter for confinement

$$\tilde{\mathcal{O}} = \int_0^1 d\theta \, e^{-2\pi i \theta} \mathcal{O}_\theta$$

$\tilde{\mathcal{O}}$ sensitive w.r.t. center transformations $\tilde{\mathcal{O}} \to z\tilde{\mathcal{O}}$

e.g. dual density $\tilde{n} \sim \log Z(\theta)$

Order parameters

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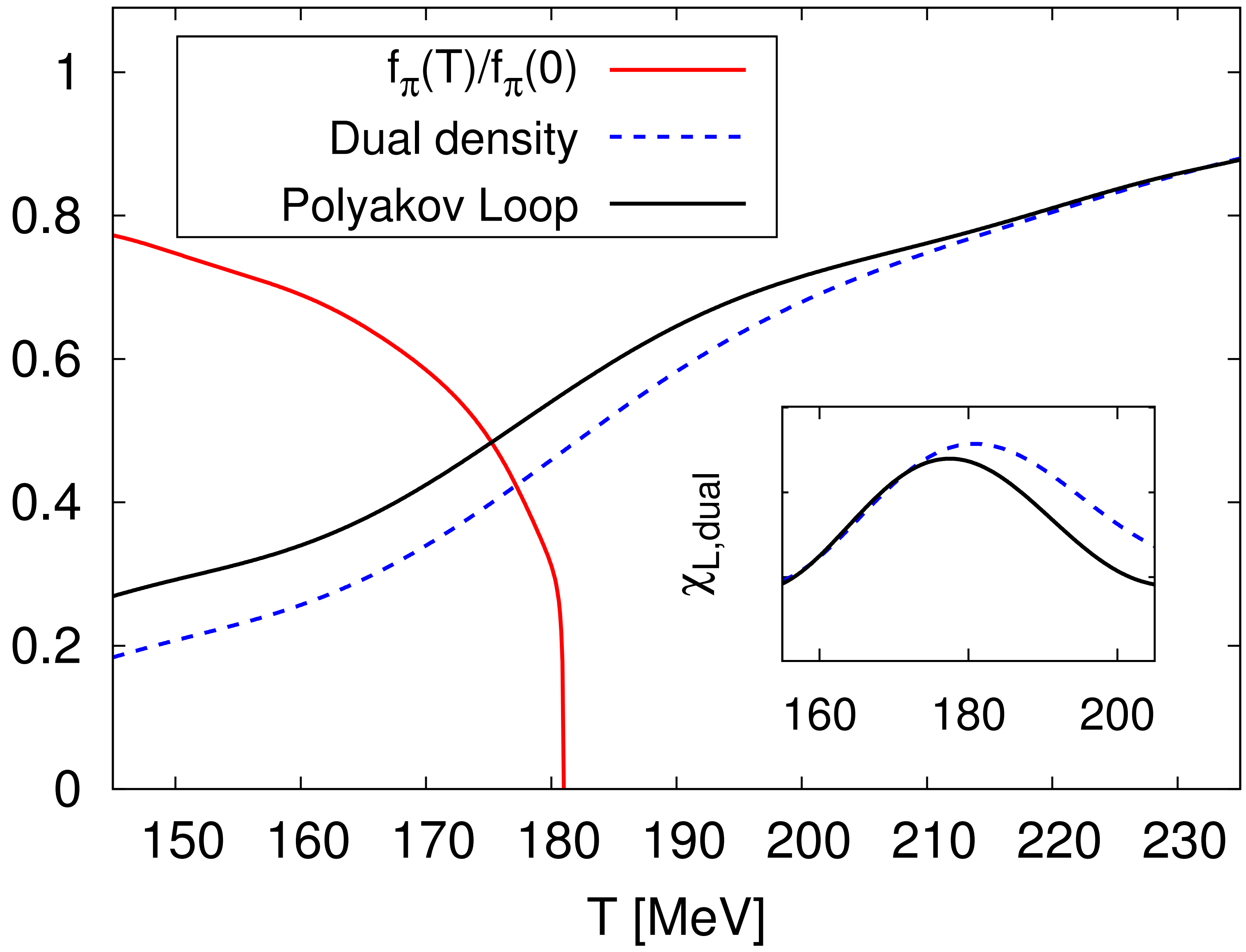
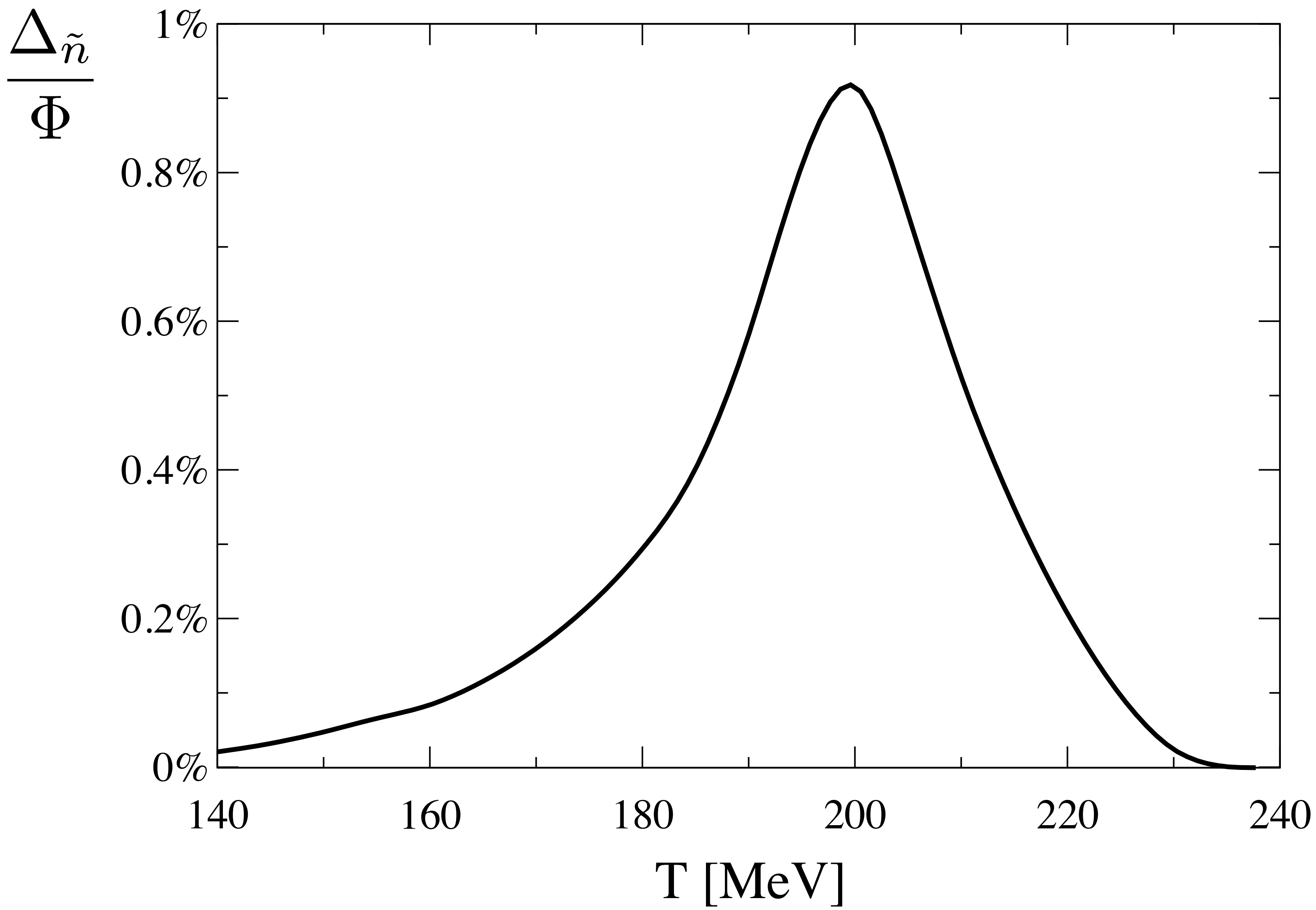


$$T_{\mathrm{conf},\Phi} = T_{\mathrm{conf},\tilde{n}} \longrightarrow$$ **consistency check**

Polyakov loop vs. dual density

J. Braun, LMH, J. M. Pawlowski, work in progress



$$\Delta_{\tilde{n}} = \frac{\tilde{n}[\langle A_0 \rangle]}{\tilde{n}[0]} - \Phi[\langle A_0 \rangle]$$

deviation of dual density from Polyakov loop

α_s at imaginary μ & finite T

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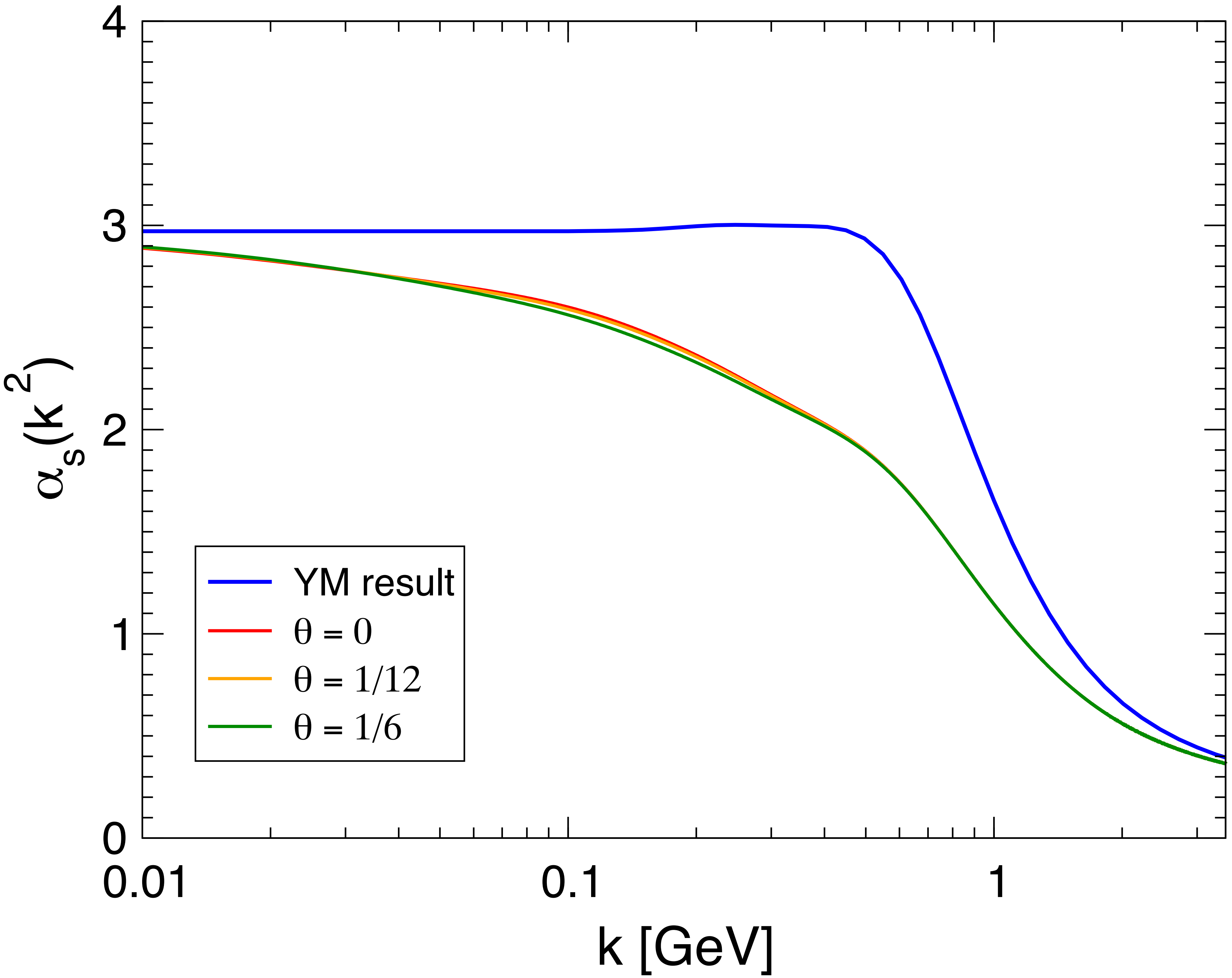

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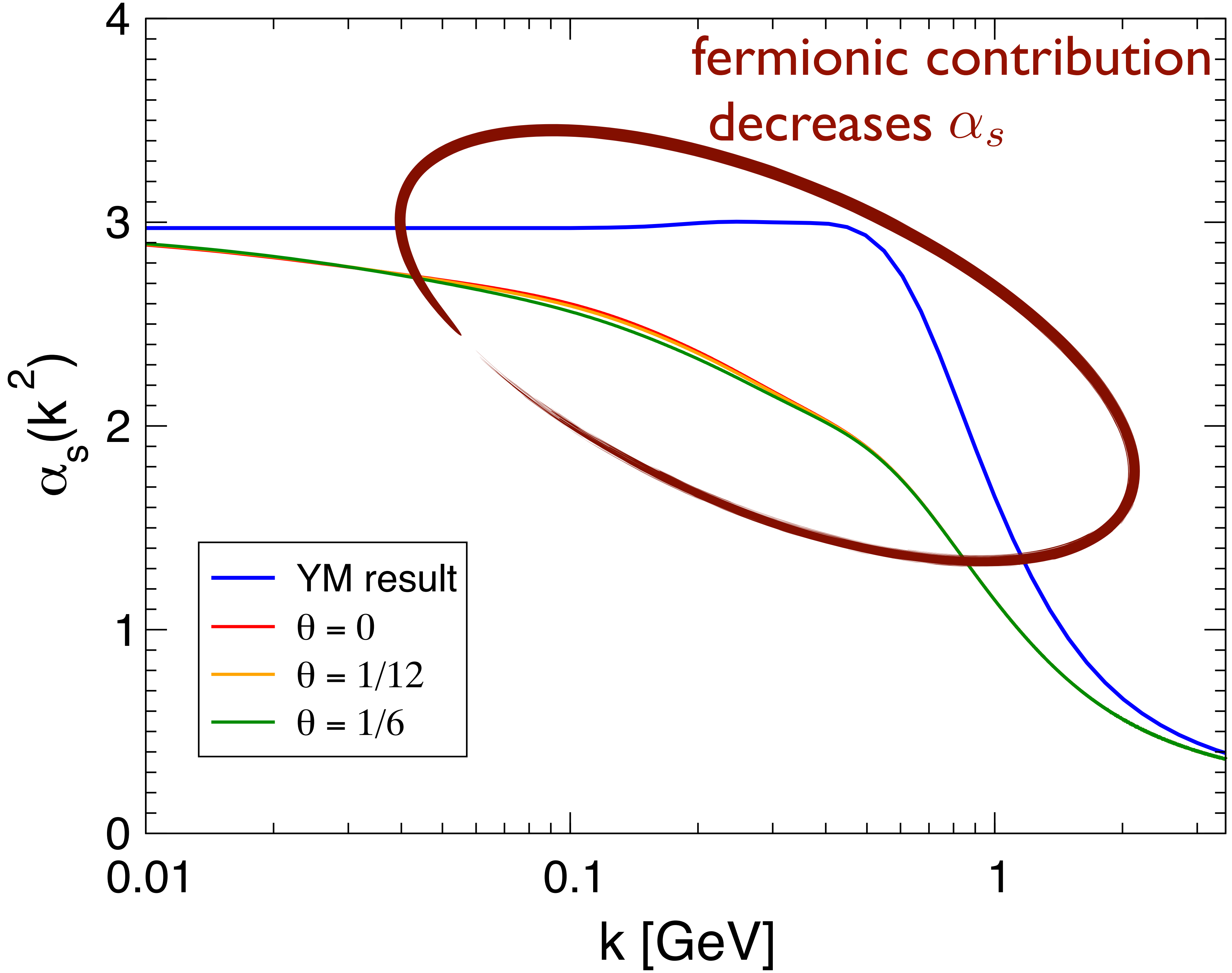
J. Braun, LMH, J. M. Pawlowski, work in progress



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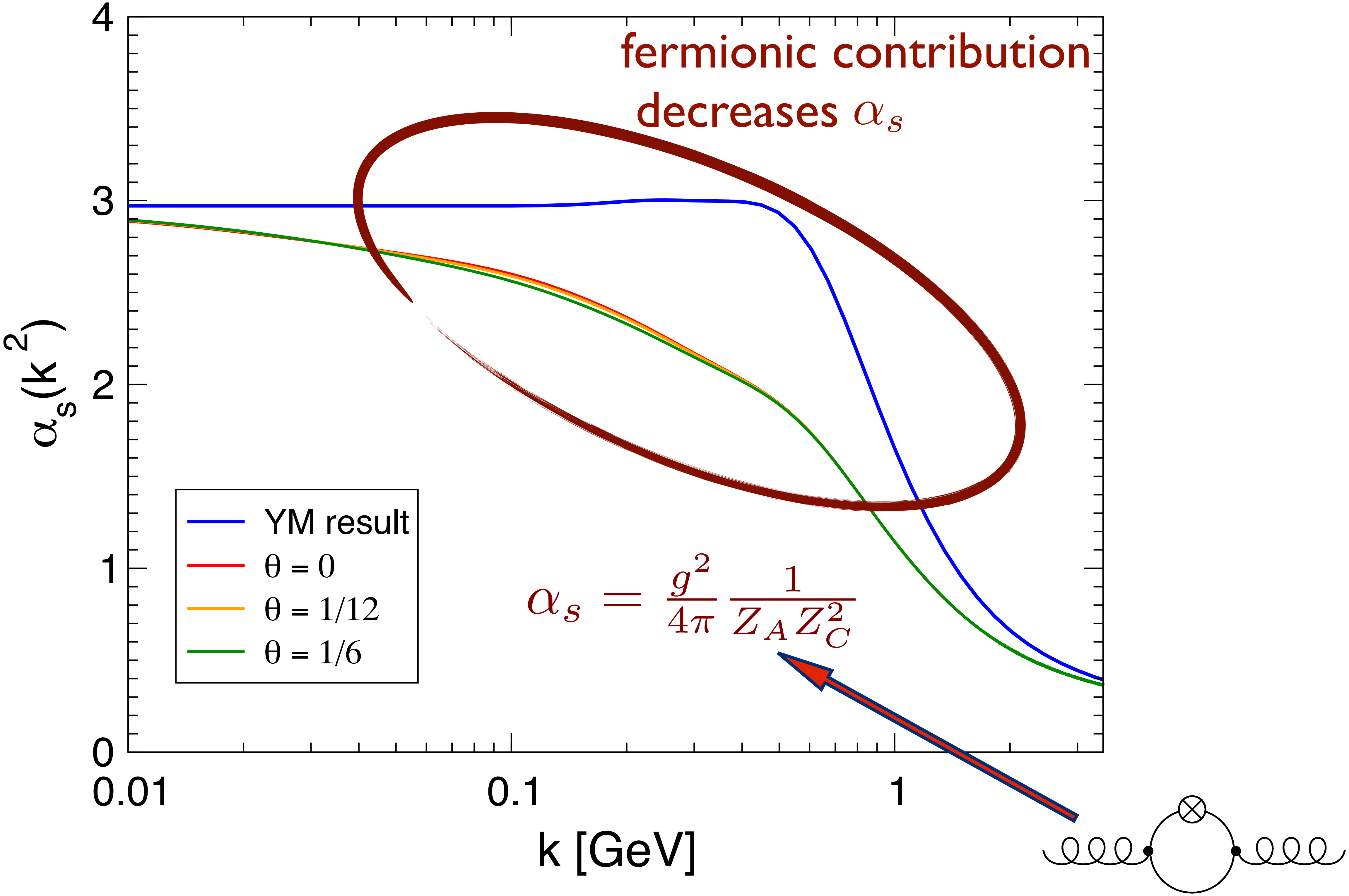

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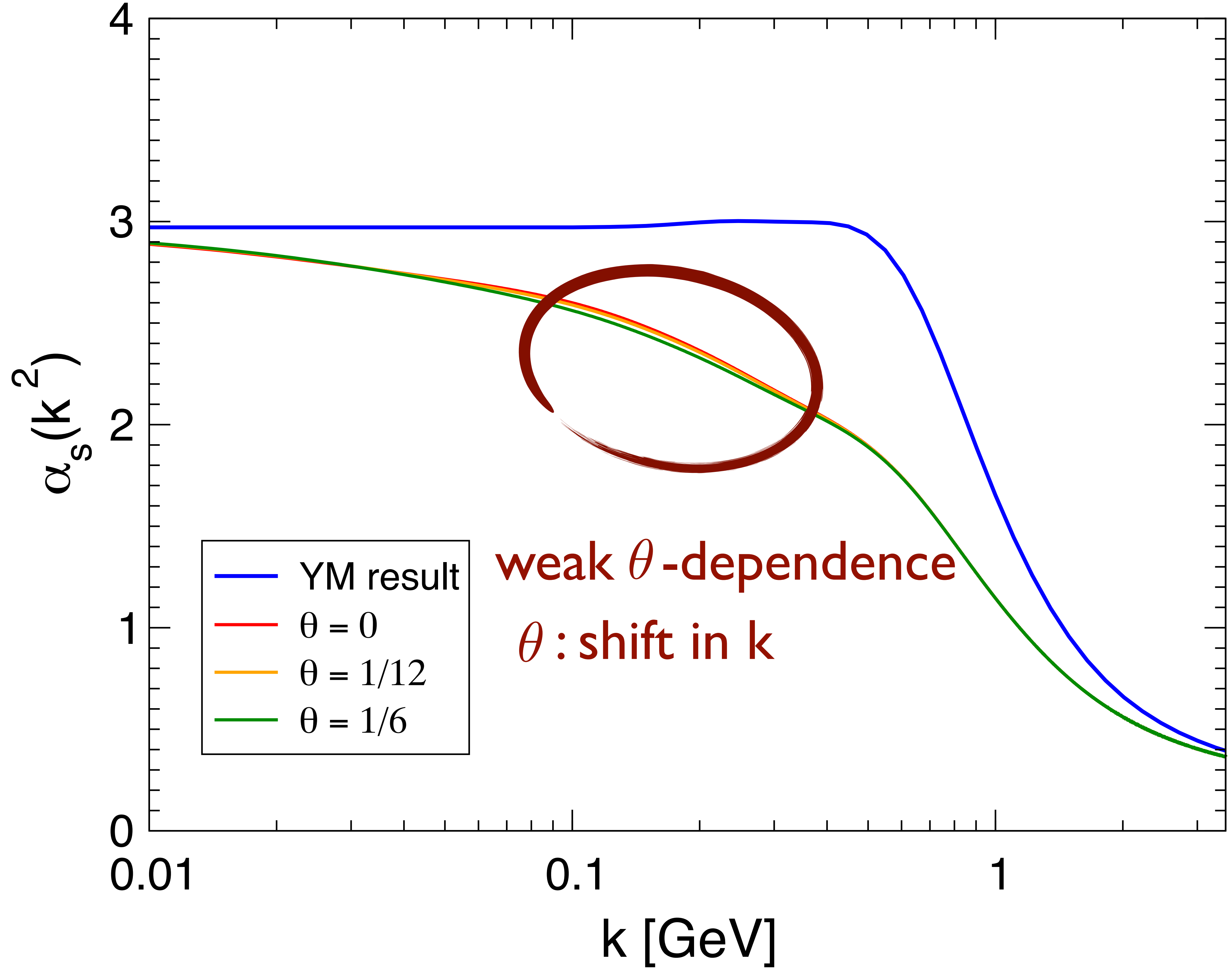


On the chiral phase transition with real chemical potential

(1st step : $T=0$, $N_f=1$)

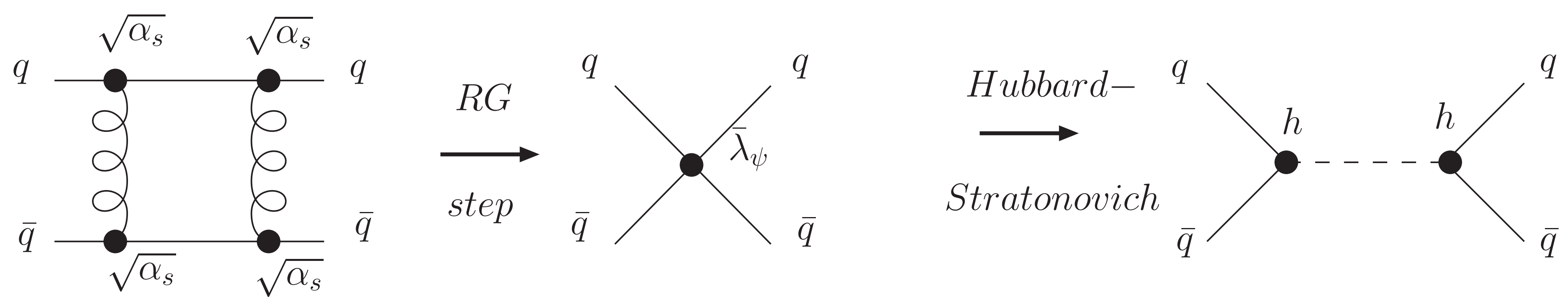
Dynamical hadronisation

QCD flow $\xrightarrow{\text{dynamically}}$ hadronic effective theory

Dynamical hadronisation

H. Gies, C. Wetterich '02; J. M. Pawlowski '05; S. Flörchinger, C. Wetterich '09

QCD flow $\xrightarrow{\text{dynamically}}$ hadronic effective theory



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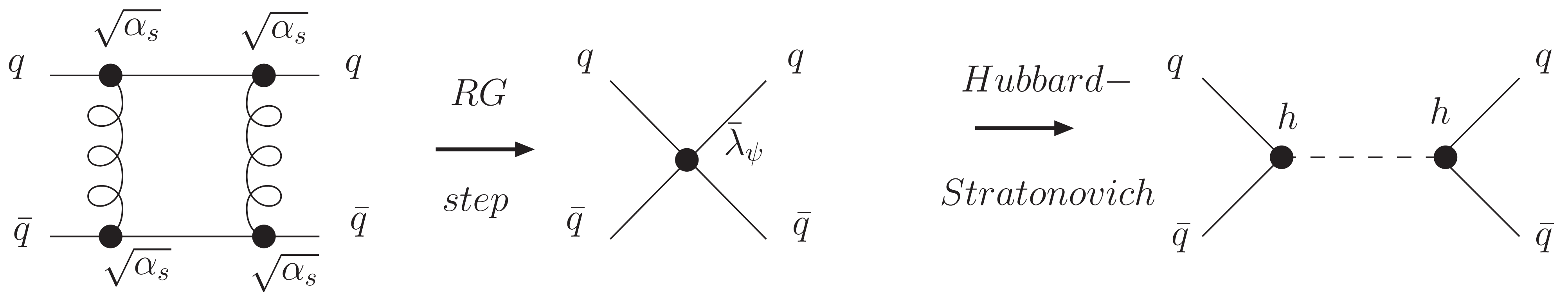

chiral condensate: $\langle\bar{\psi}\psi\rangle \sim h\langle\phi\rangle \sim m$

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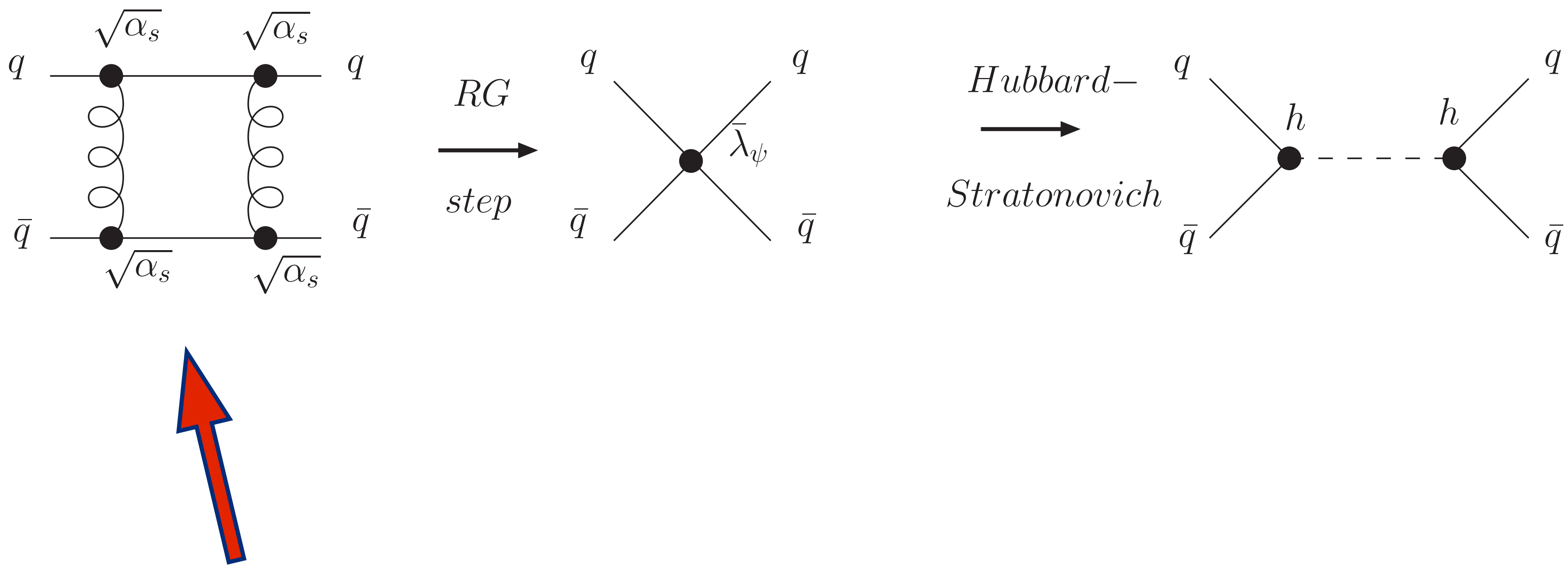
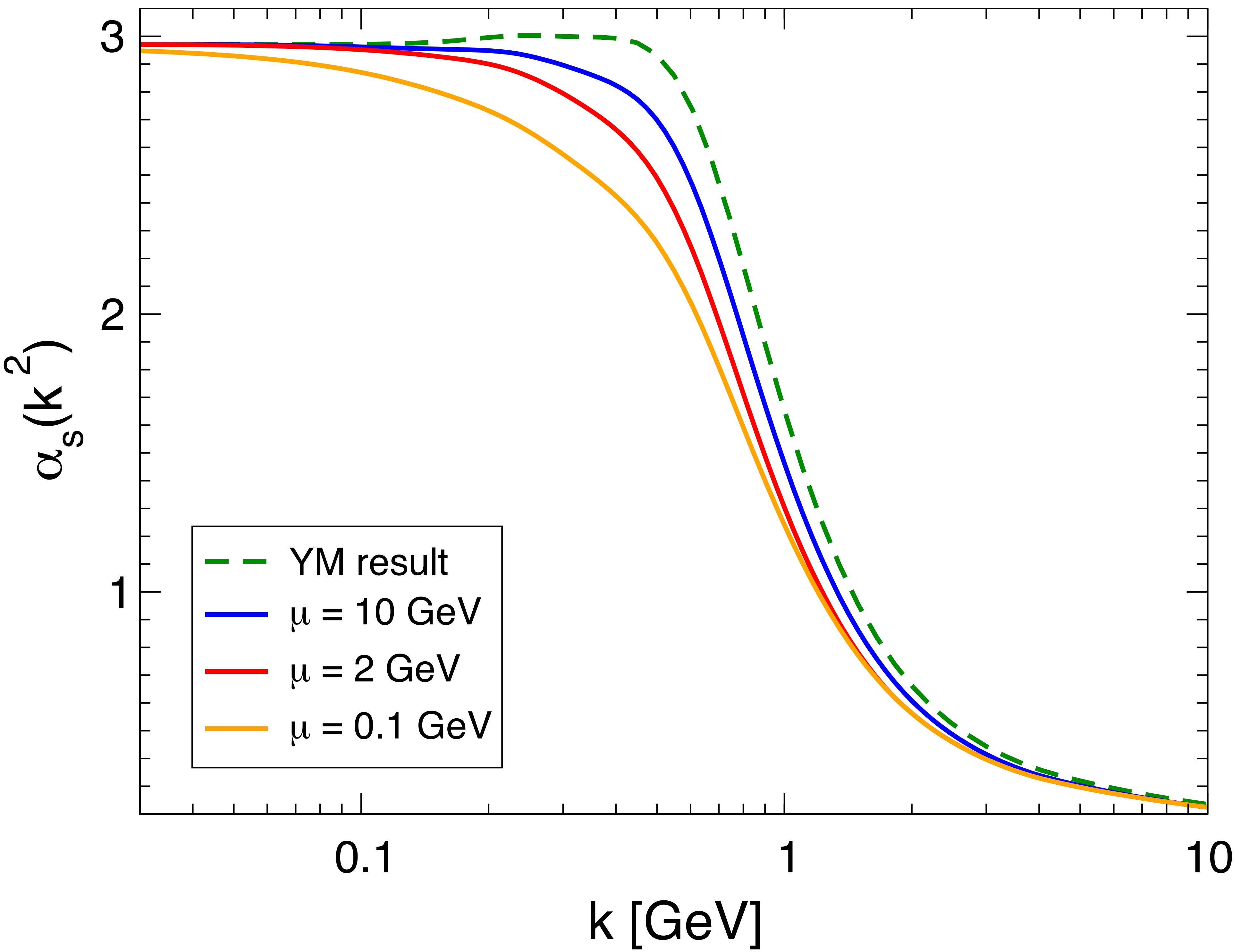


will be generated at next step ⟶ bosonise at each RG step

chiral condensate: $\langle \bar{\psi}\psi \rangle \sim h\langle \phi \rangle \sim m$

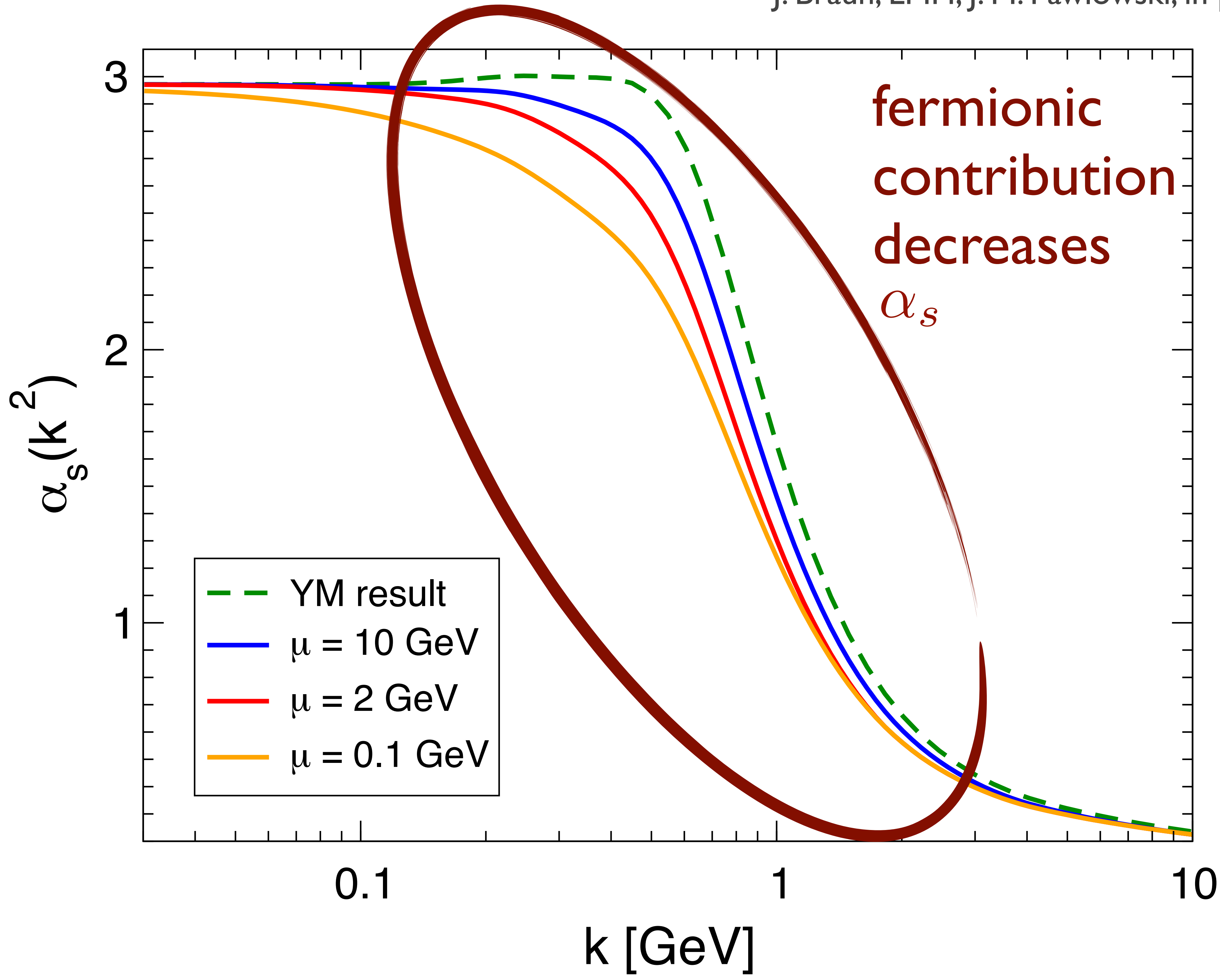
α_s at finite μ & T=0, N_f=1

J. Braun, LMH, J. M. Pawlowski, in preparation

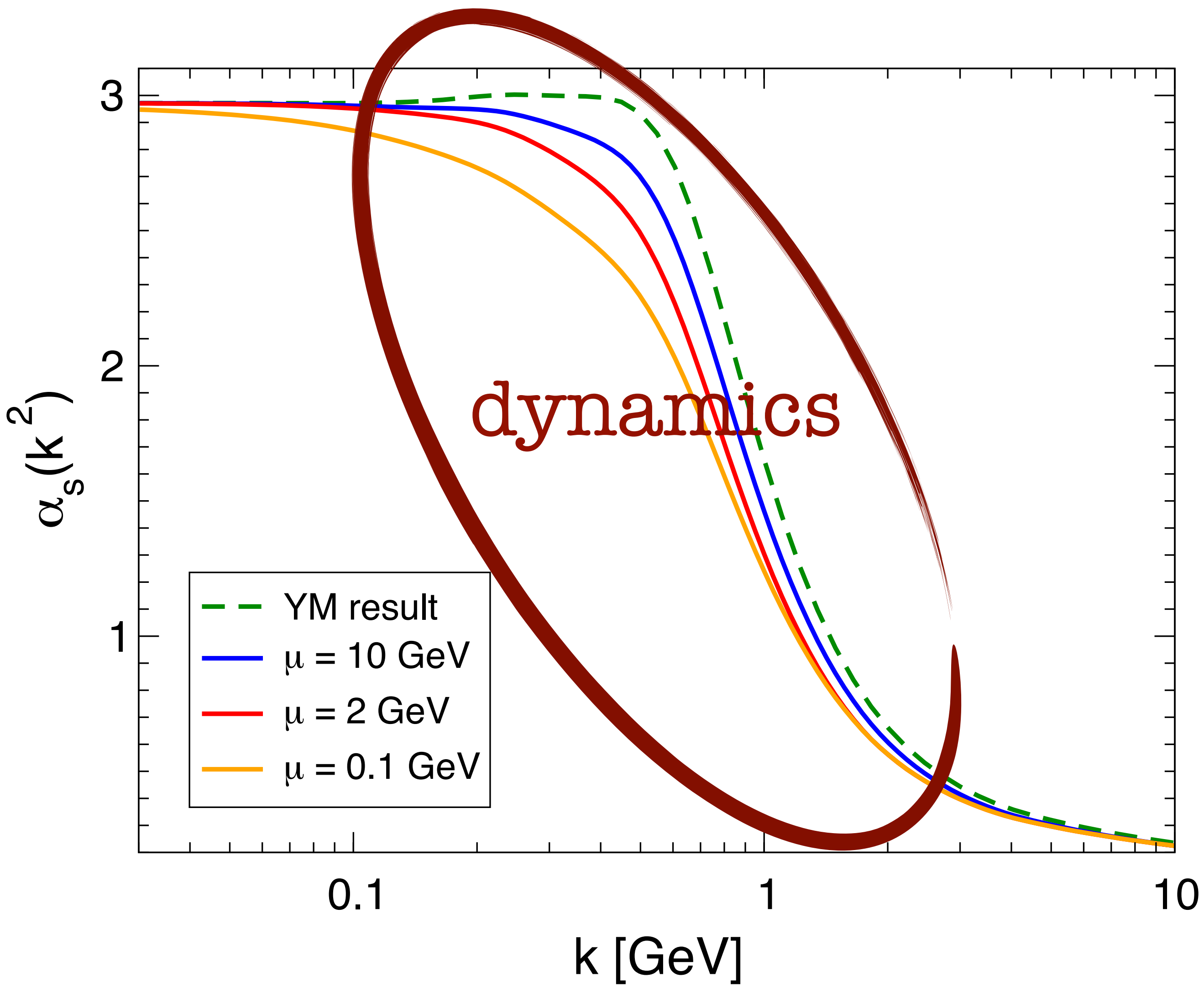


α_s at finite μ & T=0, N_f=1

J. Braun, LMH, J. M. Pawlowski, in preparation



α_s at finite μ & T=0, N$_f$=1



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real μ contributes mass-like
$\longrightarrow$ strong μ-dependence

chiral phase transition at
$\mu_{\rm cr} \approx 350$ MeV

Summary & Outlook

Summary

strong coupling α_s: weak θ-dependence
strong μ-dependence

phase diagram $N_f = 2$: imaginary chemical potential

Roberge-Weiss periodicity
$T_{\mathrm{conf},cr} \simeq T_{\chi,cr}$

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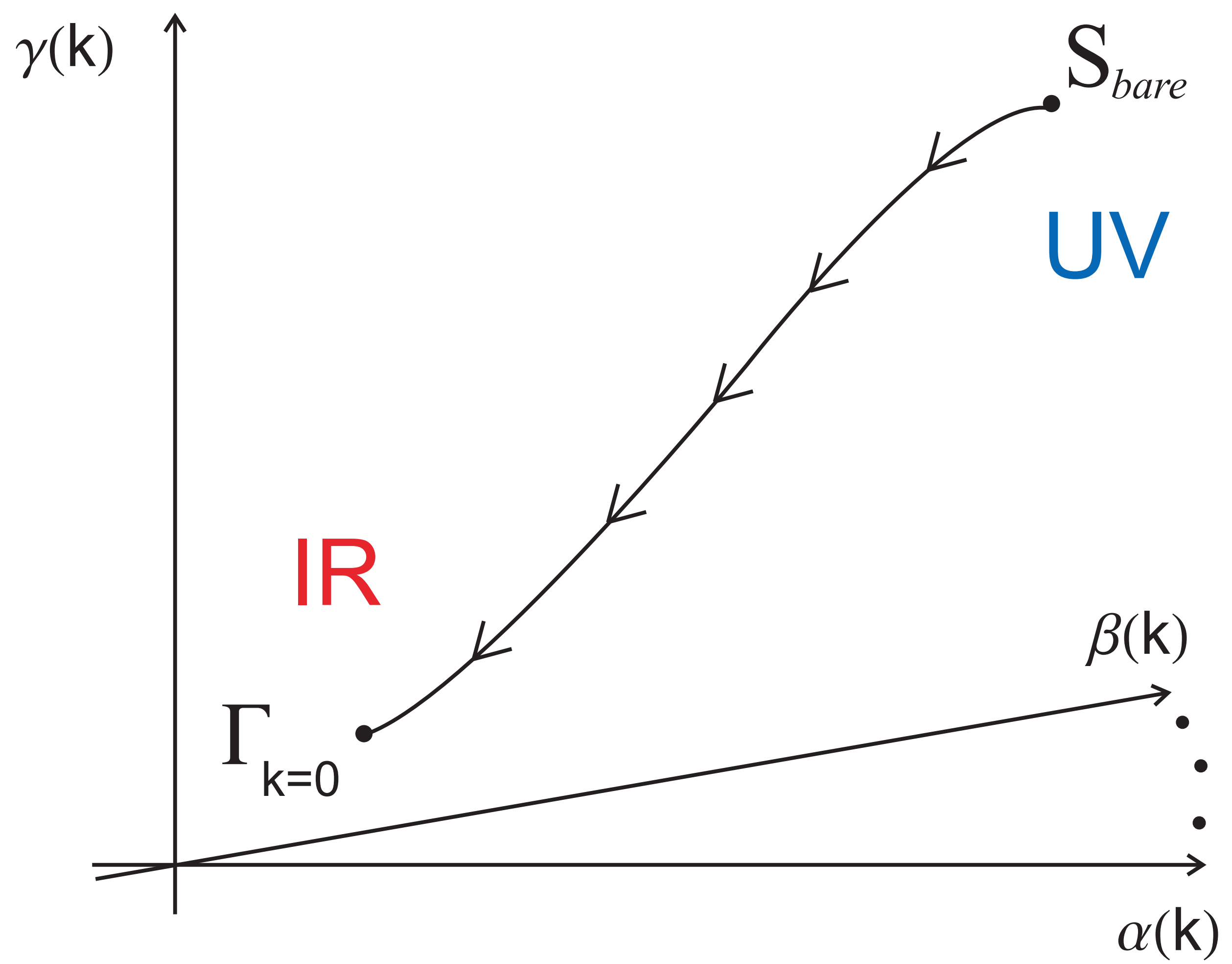
Outlook

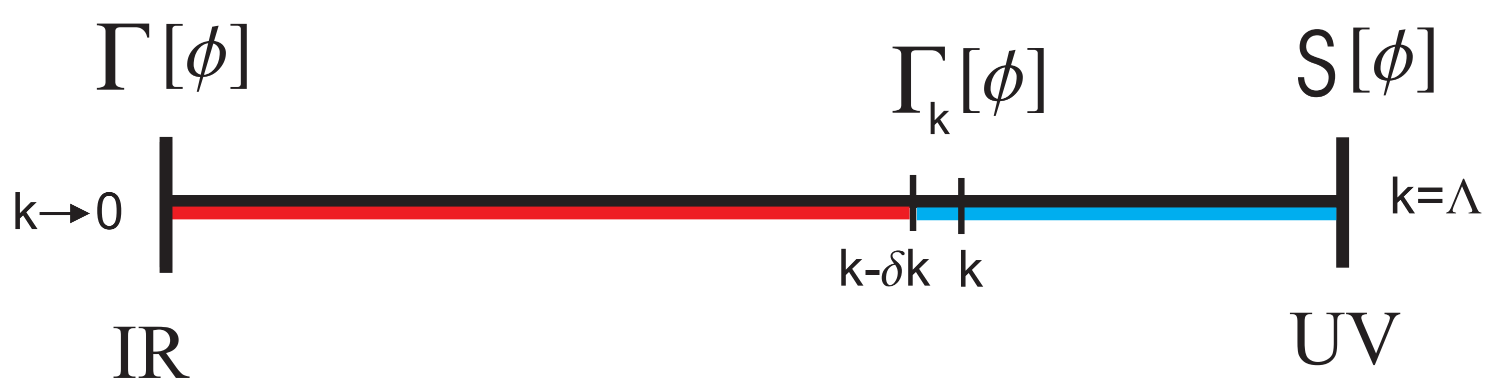
extend to $N_f = 2 + 1$: dynamical hadronisation &
real chemical potential

Additional slides

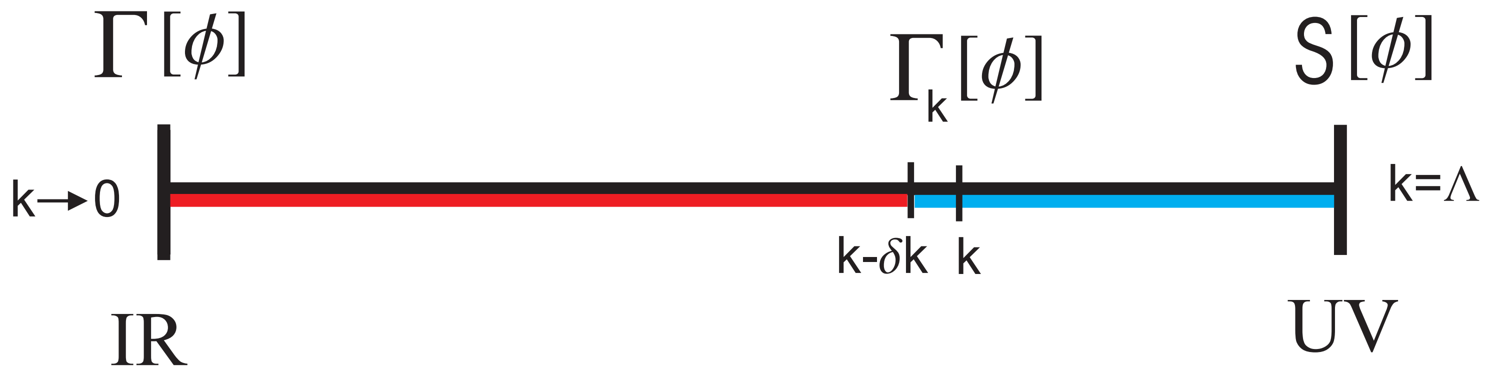
The flow of the effective action

Theory space:

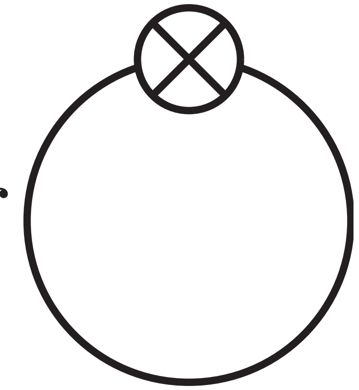




The flow equation



Flow equation for the effective action:

$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}$$ 

Wetterich equation, '93

Center symmetry

center of group G:
set of elements that commute with all elements of G

$z \in$ center of gauge group Z(G)

$U_z(t,x)$ gauge transformation

$$\Rightarrow \; U_z^{-1}(0,\vec{x})U_z(\beta,\vec{x}) = z$$

$z = \mathbb{1}\, e^{2\pi i \theta_z}$ where e.g. SU(2) $\theta_z = 0, 1/2$

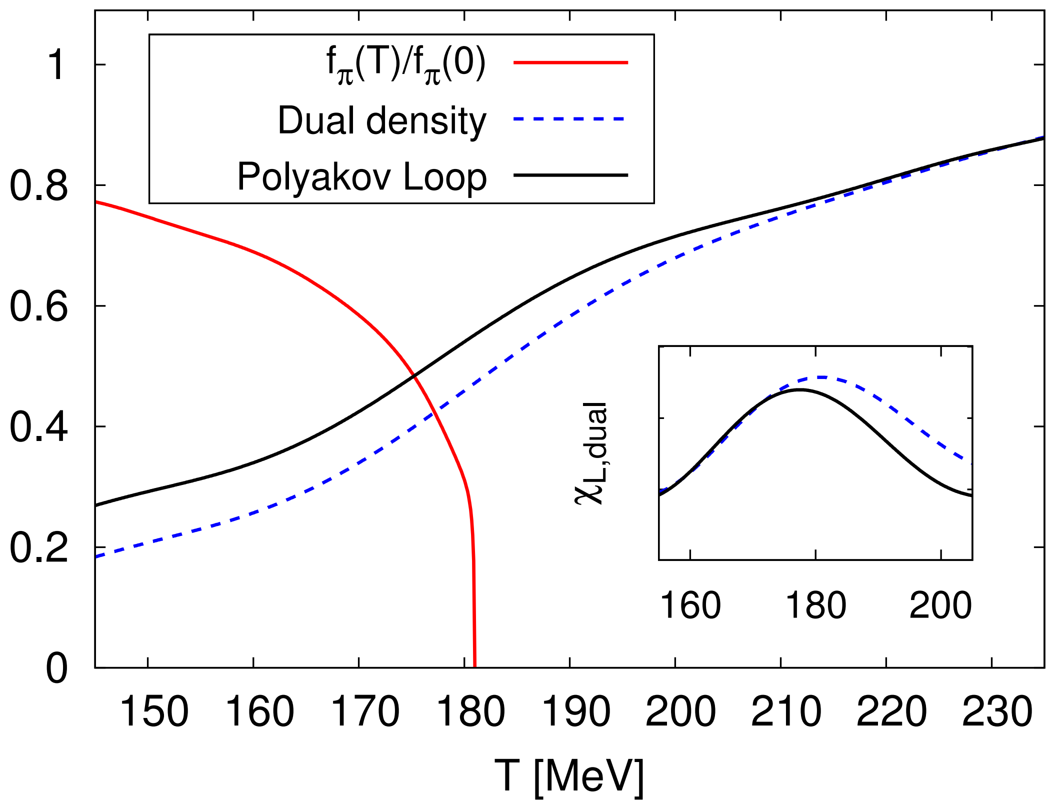
and SU(3) $\theta_z = 0, 1/3, 2/3$

and $\displaystyle\sum_{\theta_z} e^{2\pi i \theta_z} = 0$

Critical temperatures

$$N_f = 2$$

$$N_f = 2 + 1$$



J. Braun, LMH, F. Marhauser, J. M. Pawlowski '09

$$T_{\mathrm{conf},cr} \simeq T_{\chi,cr}$$

compatible with Karsch et al '08

$$T_{\mathrm{conf},cr} \simeq T_{\chi,cr}$$

compatible with Fodor et al '08

$$175\,\mathrm{MeV} \simeq T_{\mathrm{conf},cr} > T_{\chi,cr} \simeq 150\,\mathrm{MeV}$$