

Phase transitions and spectral functions from Dyson-Schwinger equations

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Work together with Jens Mueller and Axel Maas

C.F., PRL **103** (2009) 052003.

C.F. and J. A. Mueller, PRD **80** (2009) 074029.

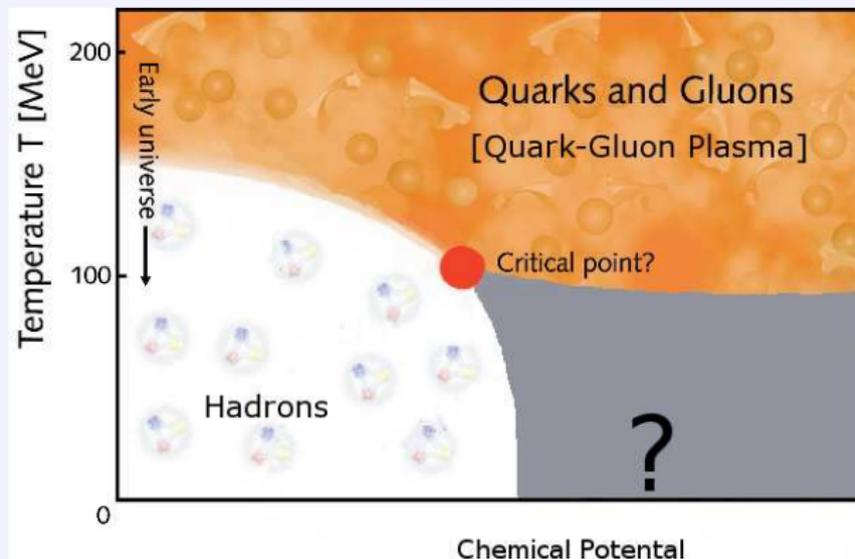
C.F., A. Maas and J. A. Mueller, almost written

J. A. Mueller, C.F. and D. Nickel in preparation

- 1 Introduction
- 2 The chiral and deconfinement transitions
- 3 Quark spectral functions

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QCD phase transitions



- Chiral limit ($m \rightarrow 0$): order parameter chiral condensate
- Static quarks ($m \rightarrow \infty$): order parameter Polyakov-loop

Lattice QCD vs. DSE/FRG: Complementary!

- Lattice simulations
 - ▶ Ab initio
 - ▶ Gauge invariant
- Functional approaches:
 - Dyson-Schwinger equations (DSE)
 - Functional renormalisation group (FRG)
 - ▶ Analytic solutions at small momenta
 - ▶ Space-Time-Continuum
 - ▶ Chiral symmetry: light quarks and mesons
 - ▶ Chemical potential: no sign problem

QCD in covariant gauge

$$\mathcal{Z}_{\text{QCD}} = \int \mathcal{D}[\Psi, A, c] \exp \left\{ - \int_0^{1/T} dt \int d^3x \left(\bar{\Psi} (i\not{D} - m) \Psi \right. \right. \\ \left. \left. - \frac{1}{4} (F_{\mu\nu}^a)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Landau gauge ($\xi = 0$) propagators in momentum space, $q = (\vec{q}, \omega_q)$:



$$D_{\mu\nu}^{\text{Gluon}}(q) = \frac{Z_T(q)}{q^2} P_{\mu\nu}^T(q) + \frac{Z_L(q)}{q^2} P_{\mu\nu}^L(q)$$



$$S^{\text{Quark}}(q) = \frac{1}{-i \vec{\gamma} \vec{q} A(q) - i \gamma_4 \omega_n C(q) + B(q)}$$

The Goal:

Gauge invariant information from **gauge fixed functional approach**

- 1 Introduction
- 2 The chiral and deconfinement transitions**
- 3 Quark spectral functions

The ordinary chiral condensate

$$\text{---} \circ \text{---}^{-1} = \text{---}^{-1} + \text{---} \circ \text{---}$$

- input: gluon propagator and quark-gluon vertex
- quarks: **antiperiodic** boundary conditions
- Order parameter for **chiral transition**:

$$\langle \bar{\psi} \psi \rangle = Z_2 N_c T \sum_{n_p} \int \frac{d^3 p}{(2\pi)^3} \text{Tr}_D S(p_{\vec{p}, \omega_p})$$

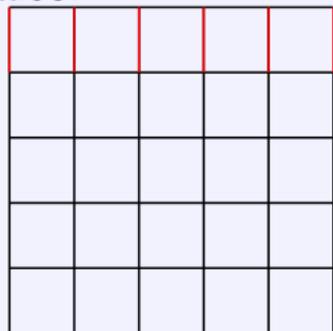
The dual condensate I

Consider general $U(1)$ -valued boundary conditions in temporal direction for quark fields ψ :

$$\psi(\vec{x}, 1/T) = e^{i\varphi} \psi(\vec{x}, 0)$$

Matsubara frequencies: $\omega_p(n_t) = (2\pi T)(n_t + \varphi/2\pi)$

Lattice:



$$\langle \bar{\psi} \psi \rangle_\varphi \sim \sum \frac{\exp[i\varphi n]}{m^n} \text{ Closed Loops}$$

E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).

The dual condensate II

Then define dual condensate Σ_n :

$$\Sigma_n = - \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi n} \langle \bar{\psi}\psi \rangle_\varphi$$

- $n = 1$ projects out loops with $n(l) = 1$: **dressed Polyakov loop**
- transforms under center transformation exactly like ordinary Polyakov loop: **order parameter for center symmetry breaking**
- **Σ_1 is accessible with functional methods**

C.F., PRL **103** (2009) 052003

C. Gattringer, PRL **97**, 032003 (2006)

F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).

E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** 094007 (2008).

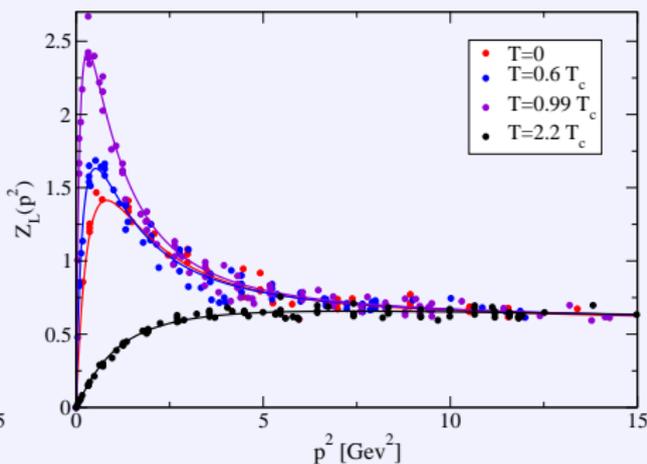
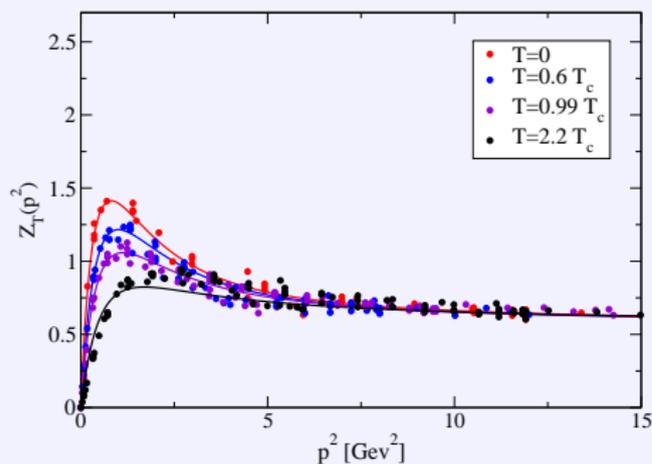
F. Synatschke, A. Wipf and K. Langfeld, PRD **77**, 114018 (2008).

J. Braun, L. Haas, F. Marhauser, J. M. Pawłowski, arXiv:0908.0008 [hep-ph]

T-dependent gluon

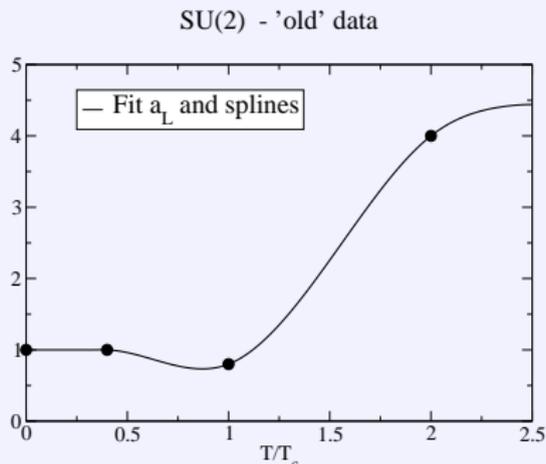
$$\text{---} \text{---} \text{---}^{-1} = \text{---} \text{---} \text{---}^{-1} + \text{---} \text{---} \text{---}^{-1}$$


• T-dependent gluon propagator from lattice data



C.F., Maas and Mueller, in preparation

T-dependent gluon screening mass (old!)



Cucchieri, Maas, Mendes, PRD75 (2007)

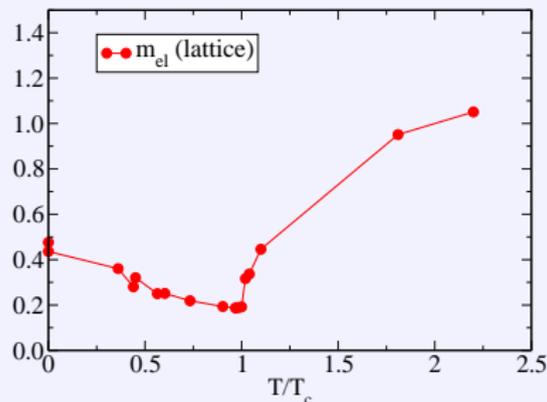
C.F., PRL **103** (2009) 052003.

C.F. and J. A. Mueller, PRD **80** (2009) 074029.

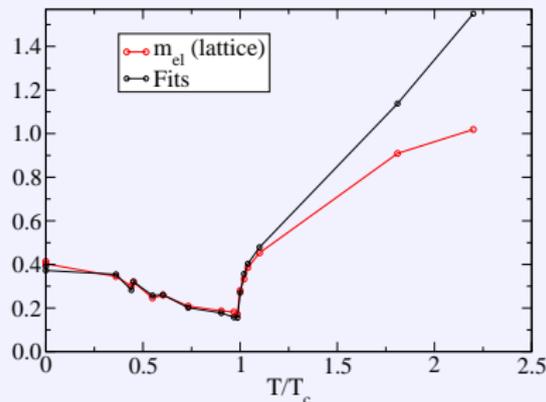
$$Z_{T,L}(q, T) = \frac{q^2 \Lambda^2}{(q^2 + \Lambda^2)^2} \left\{ \left(\frac{c}{q^2 + \Lambda^2 a_{T,L}(T)} \right)^2 + \frac{q^2}{\Lambda^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2 + 1]}{4\pi} \right)^\gamma \right\}$$

T-dependent gluon screening mass (new!)

SU(2) - 'new' data



SU(3) - 'new' data

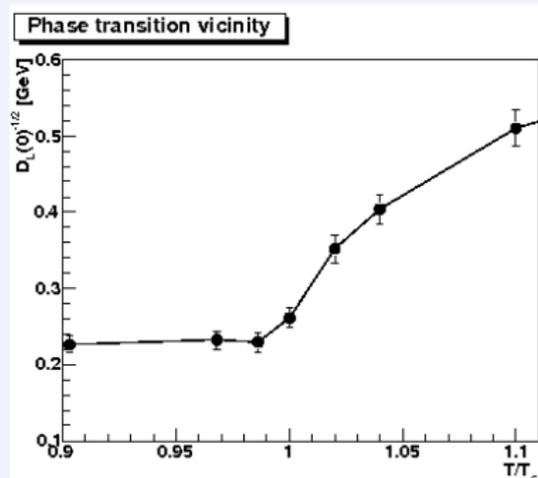


C.F., Maas, Mueller in preparation

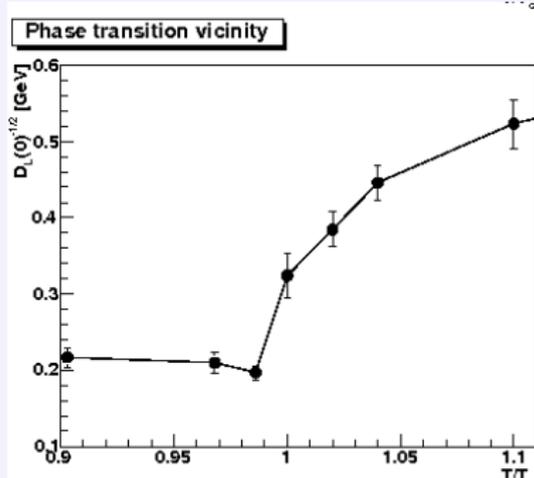
$$Z_{T,L}(q, T) = \frac{q^2 \Lambda^2}{(q^2 + \Lambda^2)^2} \left\{ \left(\frac{c}{q^2 + \Lambda^2 a_{T,L}(T)} \right)^{b_{T,L}(T)} + \frac{q^2}{\Lambda^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2 + 1]}{4\pi} \right)^\gamma \right\}$$

T-dependent gluon screening mass at T_c

SU(2)



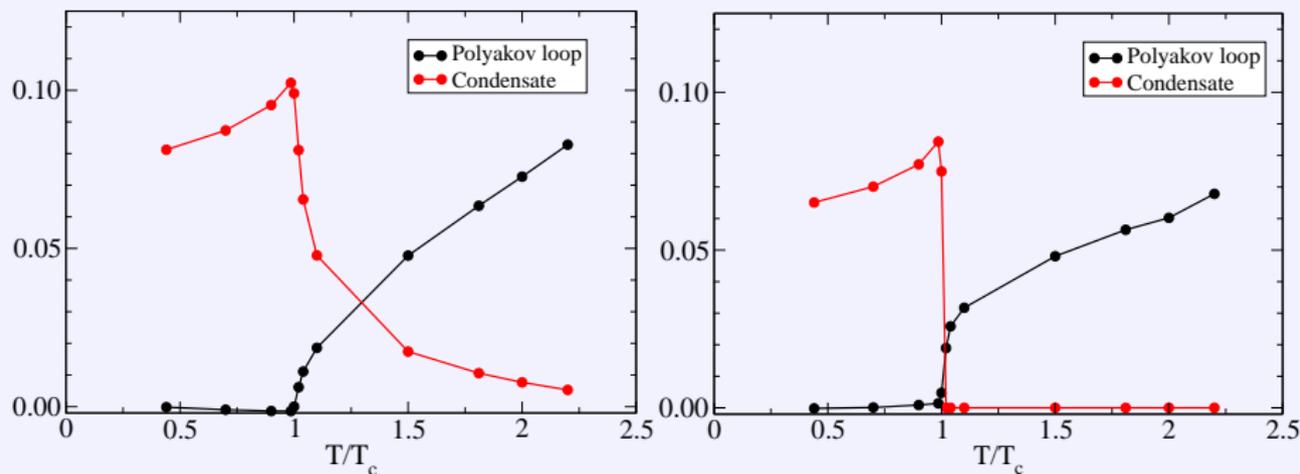
SU(3)



C.F., Maas, Mueller in preparation

- first or second order ? refined calculation necessary!

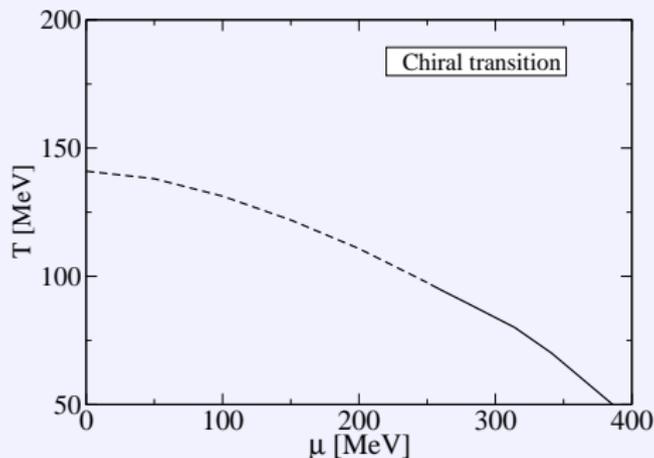
Transition temperatures



C.F., Maas, Mueller in preparation

- similar transition temperatures for chiral and deconfinement
- SU(2): $T \approx 305$ MeV SU(3): $T \approx 270$ MeV
- increasing chiral condensate due to electric screening masses

A first exploration of the phase diagram (mean field)



C.F., Luecker, Mueller, work in progress

- only here: T and μ -independent ansatz for gluon and vertex
- Similar result than models
- Work in progress: Include T and μ -dependence of gluon

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Idea: Fit spectral representation to quark propagator

F. Karsch and M. Kitazawa, PRD **80**, 056001 (2009).

F. Karsch and M. Kitazawa, PLB **658**, 45 (2007).

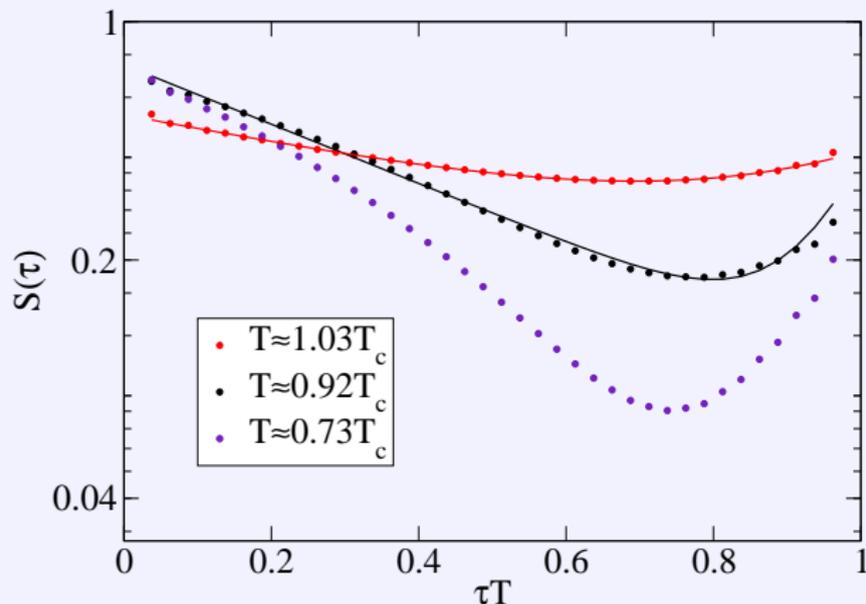
$$S(\omega_p, \vec{p}) = \int d\omega' \frac{\rho(\omega', \vec{p})}{\omega_p - \omega'}$$

Use ansatz for spectral function:

$$\rho(\omega) = Z_1 \delta(\omega - E_1) + Z_2 \delta(\omega + E_2)$$

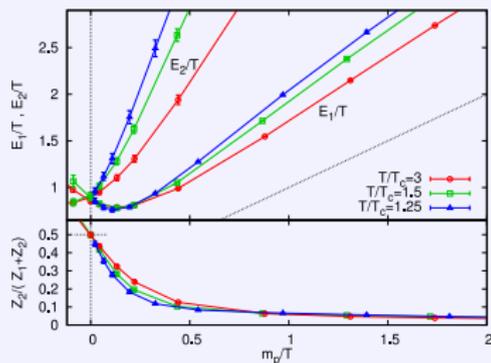
Pseudoparticles: Quark and Plasmino

Results I



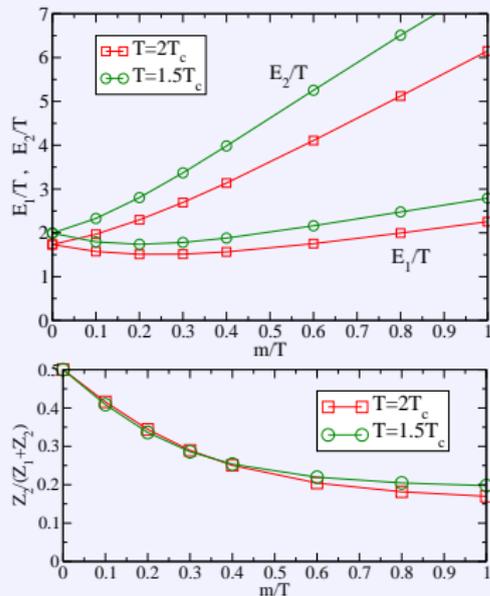
- $T > T_c$: Two pole ansatz works: quark and plasmino
- $T < T_c$: Positivity violations; two pole fit does not work at all

Lattice



Karsch and Kitazawa, PRD **80**, 056001 (2009).

DSE

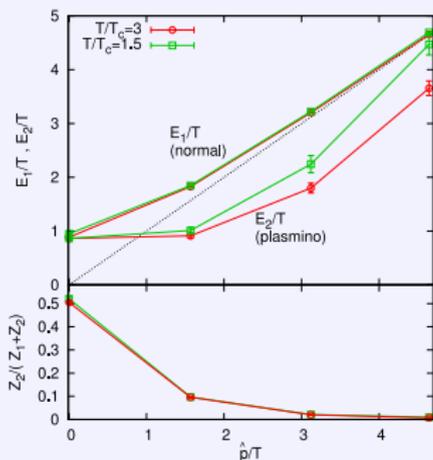


Mueller, C.F., Nickel in preparation

- Qualitative agreement with lattice results
- Large quark masses: plasmino disappears

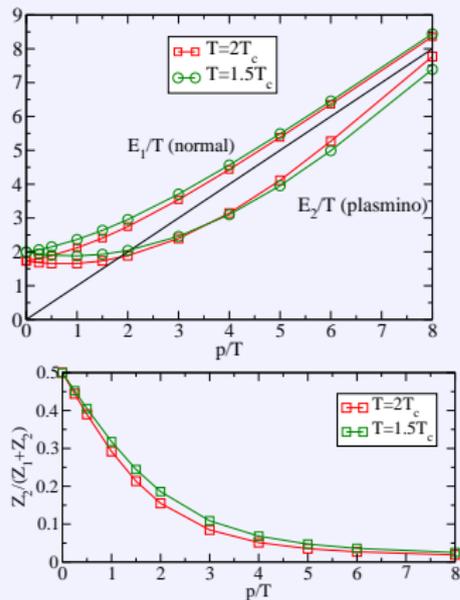
Results III: Dispersion Relation

Lattice



Karsch and Kitazawa, PRD **80**, 056001 (2009).

DSE



Mueller, C.F., Nickel in preparation

- Min for Plasmino at $p \neq 0$
- Plasmino enters spacelike region (artefact !?)

To do:

- Unquenching
- Finite chemical potential beyond mean field
- Thermodynamic observables

Further recent results from functional methods (non-exhausting):

- Gluon at $T = 0$:

C.F., Maas, Pawłowski, *Annals Phys.* **324** (2009) 2408

- Chiral and deconfinement transition using the FRG:

Braun and Gies, arXiv:0912.4168.

Braun, arXiv:0908.1543.

Braun, Haas, Marhauser and Pawłowski, arXiv:0908.0008.

Braun, *Eur. Phys. J. C* **64** (2009) 459.

Marhauser and Pawłowski, arXiv:0812.1144.

Braun, Gies and Pawłowski, *PLB* in press; arXiv:0708.2413.

Schaefer, Wambach, *Phys. Part. Nucl.* **39**, 1025 (2008) [arXiv:hep-ph/0611191].

Schaefer, Wambach, *Phys. Rev. D* **75**, 085015 (2007) [arXiv:hep-ph/0603256].

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 **LOEWE** – Landes-Offensive zur Entwicklung
Wissenschaftlich-ökonomischer Exzellenz



Helmholtz-Alliance: Extremes of density and
temperature; cosmic matter in the laboratory

Ansatz for Quark-Gluon-Vertex:

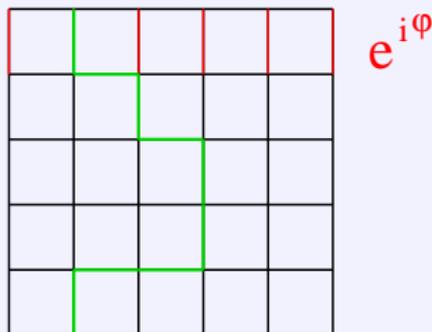
$$\Gamma_\nu(q, k, p) = \tilde{Z}_3 \left(\delta_{4\nu} \gamma_4 \frac{C(k) + C(p)}{2} + \delta_{j\nu} \gamma_j \frac{A(k) + A(p)}{2} \right) \times \\ \times \left(\frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right).$$

The dual condensate II

Relation of condensate to loops of link variables $U_\mu(x)$:

$$\langle \bar{\psi} \psi \rangle_\varphi = \text{Tr} [m + D_\varphi]^{-1} = \frac{1}{V m} \sum_{l \in \mathcal{L}} \frac{e^{i\varphi n(l)}}{(2am)^{|l|}} \text{Tr}_c \prod_{(x,\mu) \in l} s(l) U_\mu(x).$$

- geometric series of inverse staggered Dirac operator
- winding number $n(l)$ of loop l around temporal direction



E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).