

Lattice Study of Dense Baryonic Matter in

QC₂D



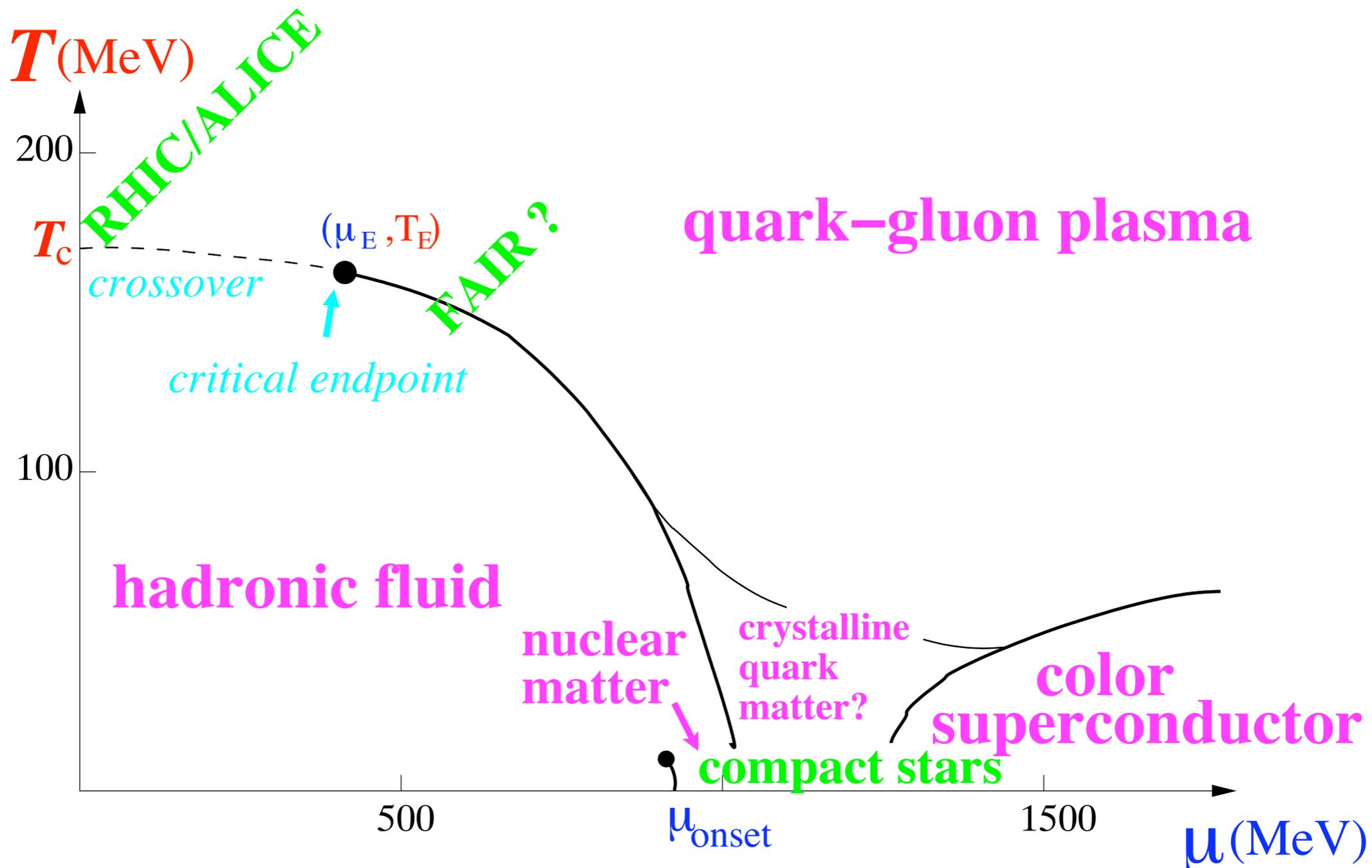
Simon Hands (Swansea U.)

- Why two colors?
- Equation of state for $\mu \neq 0$
- Hadron Spectrum
- Topology
- Monopoles & Stars

Collaborators: Seyong Kim, Jon-Ivar Skullerud, Phil Kenny, Peter Sitch
Ernst-Michael Ilgenfritz, Sebastian Schubert, James Tonkin

Norwegian Winter Workshop 25th February 2010

The QCD Phase Diagram



The Sign Problem for $\mu \neq 0$

In Euclidean metric the QCD Lagrangian reads

$$\mathcal{L}_{QCD} = \bar{\psi}(M + m)\psi + \frac{1}{4}F_{\mu\nu}F_{\mu\nu}$$

with $M(\mu) = \not{D}[A] + \mu\gamma_0$

Straightforward to show

$$\gamma_5 M(\mu) \gamma_5 \equiv M^\dagger(-\mu) \Rightarrow \det M(\mu) = (\det M(-\mu))^*$$

ie. Path integral measure is not positive definite for $\mu \neq 0$

Fundamental reason is explicit breaking of time reversal symmetry

Monte Carlo importance sampling, the mainstay of lattice QCD, is ineffective

What goes wrong with the usual positive HMC measure?

$$\det M^\dagger M \begin{cases} M & \text{describes} & \text{quarks } q \in \mathbf{3} \\ M^\dagger & \text{describes conjugate quarks } q^c \in \bar{\mathbf{3}} \end{cases}$$

In general $\exists qq^c$ gauge singlet bound states with $B > 0$

In QCD some qq^c states degenerate with the pion

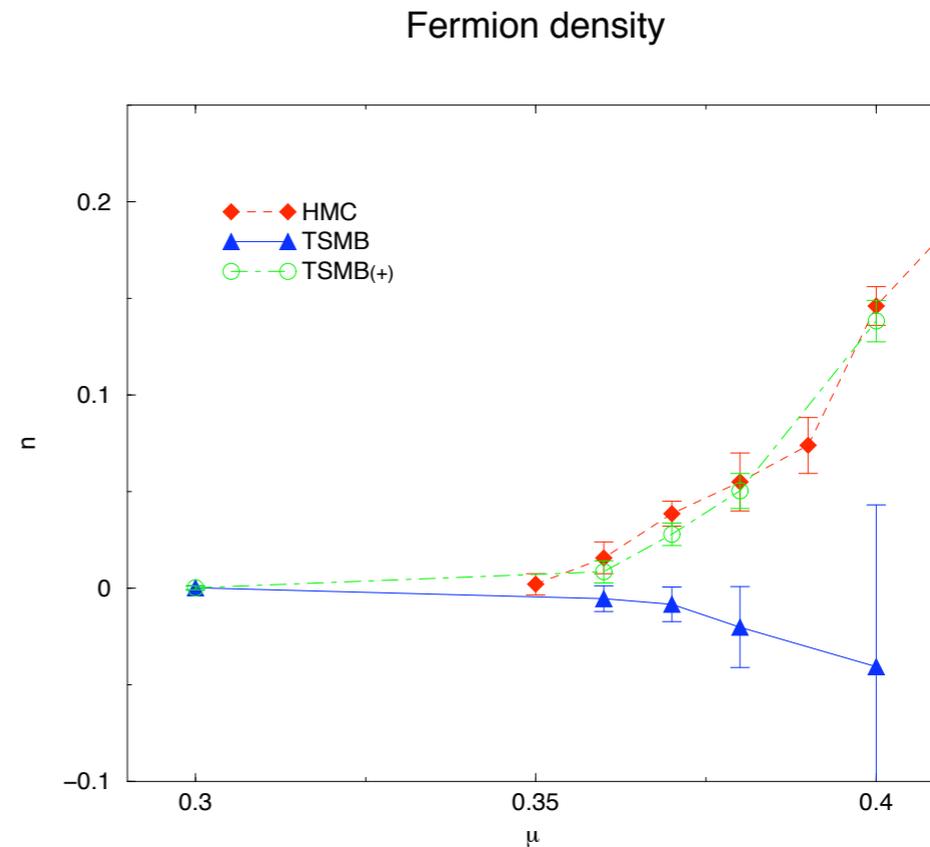
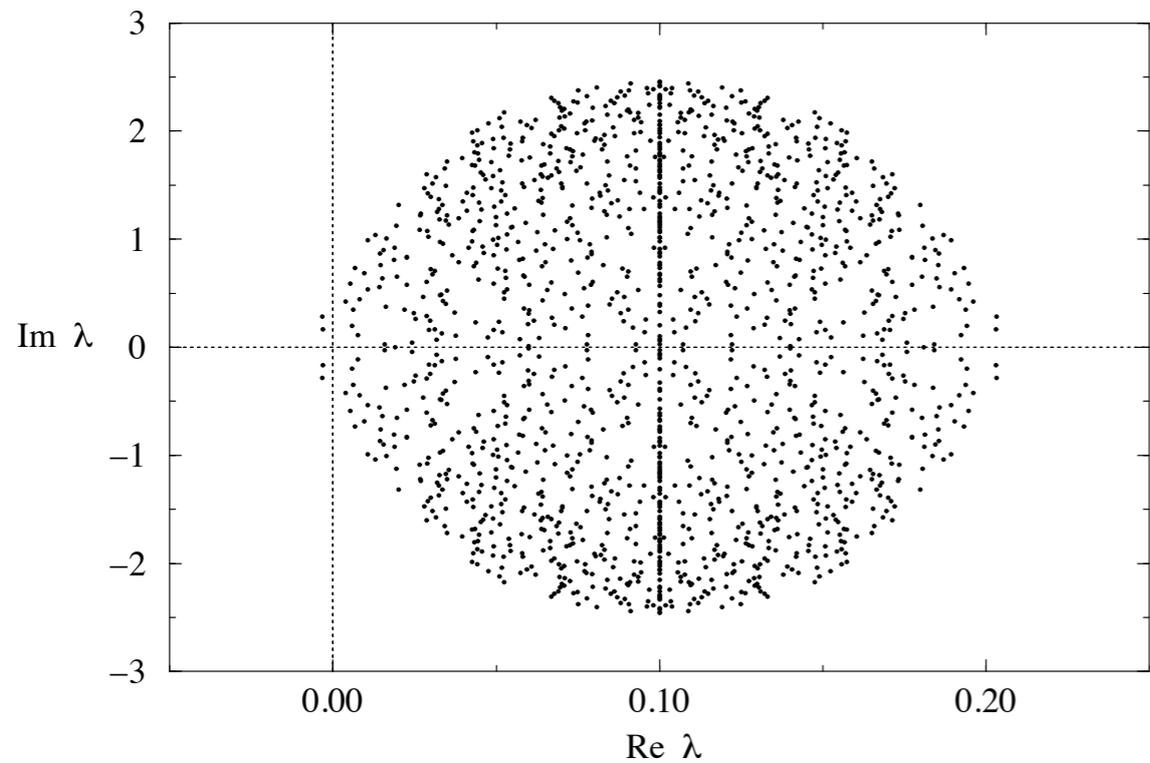
\Rightarrow unphysical onset of “nuclear matter” at $\mu_0 \simeq \frac{1}{2}m_\pi$.

Goldstone baryons: bug for QCD, feature for QC₂D...

Calculations with the true complex measure $\det^2 M$ nullify effects of qq^c states for the vacuum with $T = 0$,

$\frac{1}{2}m_\pi < \mu \lesssim \frac{1}{3}m_N$ by cancellations among configurations with different signs/phases

The *Silver Blaze* Problem...



This has been numerically verified, eg. in TSMB simulations of Two Color QCD with $N = 1$ adjoint staggered quarks.

SJH, Montvay, Scorzato, Skullerud, EurPJ C22 (2001) 451

The fake transition to a superfluid phase, forbidden by the Pauli Principle, at $\mu_0 a \simeq 0.35$ disappears once configurations with $\det M < 0$ are included with the correct weight.

QC₂D – the large N_c^{-1} limit

QCD with gauge group SU(2) and non-zero quark chemical potential μ has a real functional measure; it remains the *simplest* dense matter system with long-range interactions amenable to study with standard LGT methods.

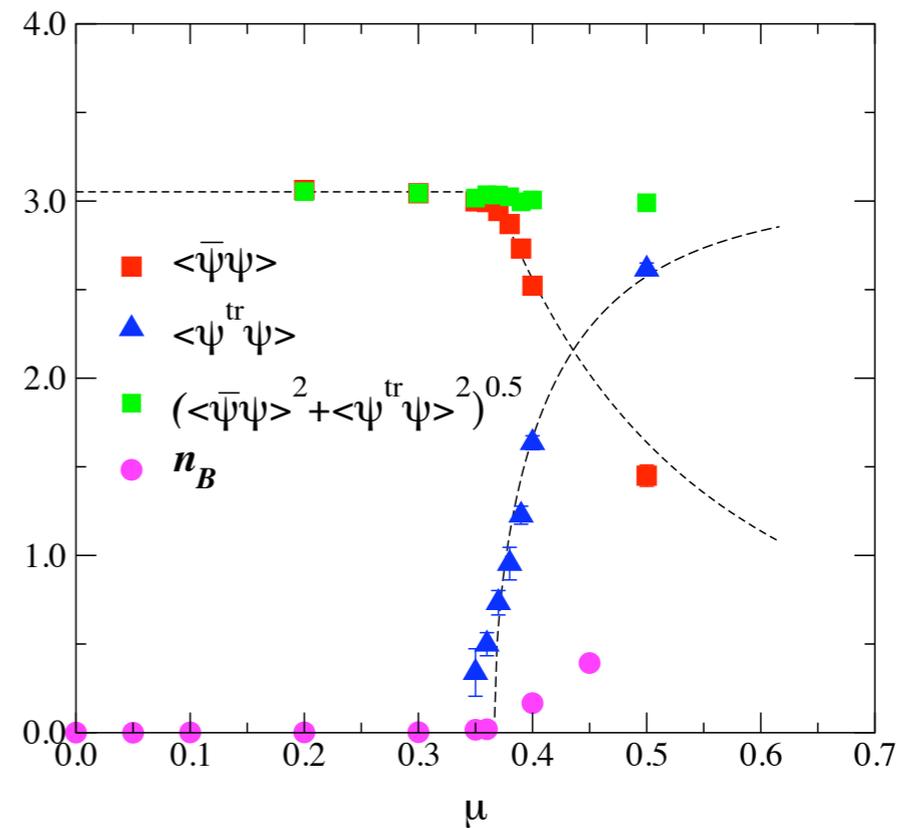
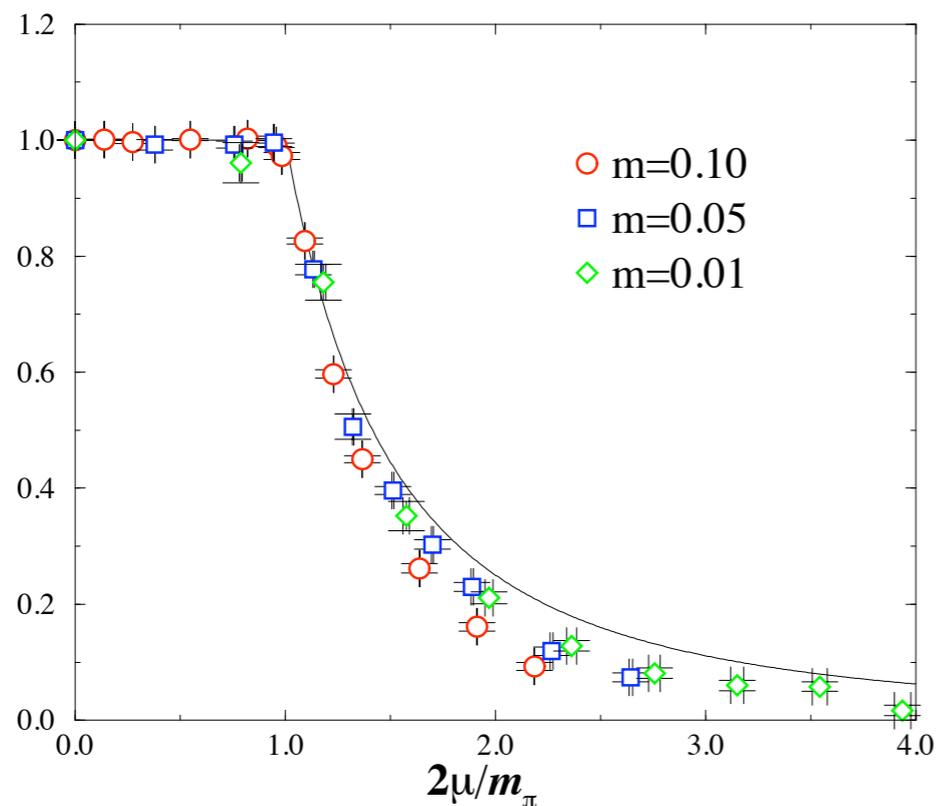
Since q and \bar{q} live in equivalent reps. of the color group, chiral multiplets contain both $q\bar{q}$ mesons and qq baryons. For $m_\pi \ll m_\rho$ the behaviour as μ is varied can be studied using chiral perturbation theory (χ PT)

Key result: for $\mu \geq \mu_o = \frac{1}{2}m_\pi$ a baryon charge density develops, $n_q > 0$, along with a gauge invariant superfluid condensate $\langle qq \rangle \neq 0$. For $\mu \gtrsim \mu_o$, the system is a dilute Bose Einstein Condensate (BEC) consisting of weakly interacting scalar qq baryons.

Quantitatively, for $\mu \gtrsim \mu_0$ χ PT predicts

$$\frac{\langle \bar{\psi}\psi \rangle}{\langle \bar{\psi}\psi \rangle_0} = \left(\frac{\mu_0}{\mu} \right)^2 ; \quad n_q = 8N_f f_\pi^2 \mu \left(1 - \frac{\mu_0^4}{\mu^4} \right) ; \quad \frac{\langle qq \rangle}{\langle \bar{\psi}\psi \rangle_0} = \sqrt{1 - \left(\frac{\mu_0}{\mu} \right)^4}$$

[Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl.Phys.B582(2000)477]
 confirmed by QC₂D simulations with staggered fermions



[SJH, I. Montvay, S.E. Morrison, M. Oevers, L. Scorzato J.I. Skullerud,
 Eur.Phys.J.C17(2000)285, *ibid* C22(2001)451]

Thermodynamics at $T = 0$ from χ PT

quark number density $n_{\chi PT} = 8N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right)$ [KSTVZ]

pressure $p_{\chi PT} = -\frac{\Omega}{V} = \int_{\mu_o}^{\mu} n_q d\mu = 4N_f f_\pi^2 \left(\mu^2 + \frac{\mu_o^4}{\mu^2} - 2\mu_o^2\right)$

energy density $\varepsilon_{\chi PT} = -p + \mu n_q = 4N_f f_\pi^2 \left(\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 2\mu_o^2\right)$

conformal anomaly

$$(T_{\mu\mu})_{\chi PT} = \varepsilon - 3p = 8N_f f_\pi^2 \left(-\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 4\mu_o^2\right)$$

NB $(T_{\mu\mu})_{\chi PT} < 0$ for $\mu > \sqrt{3}\mu_o$

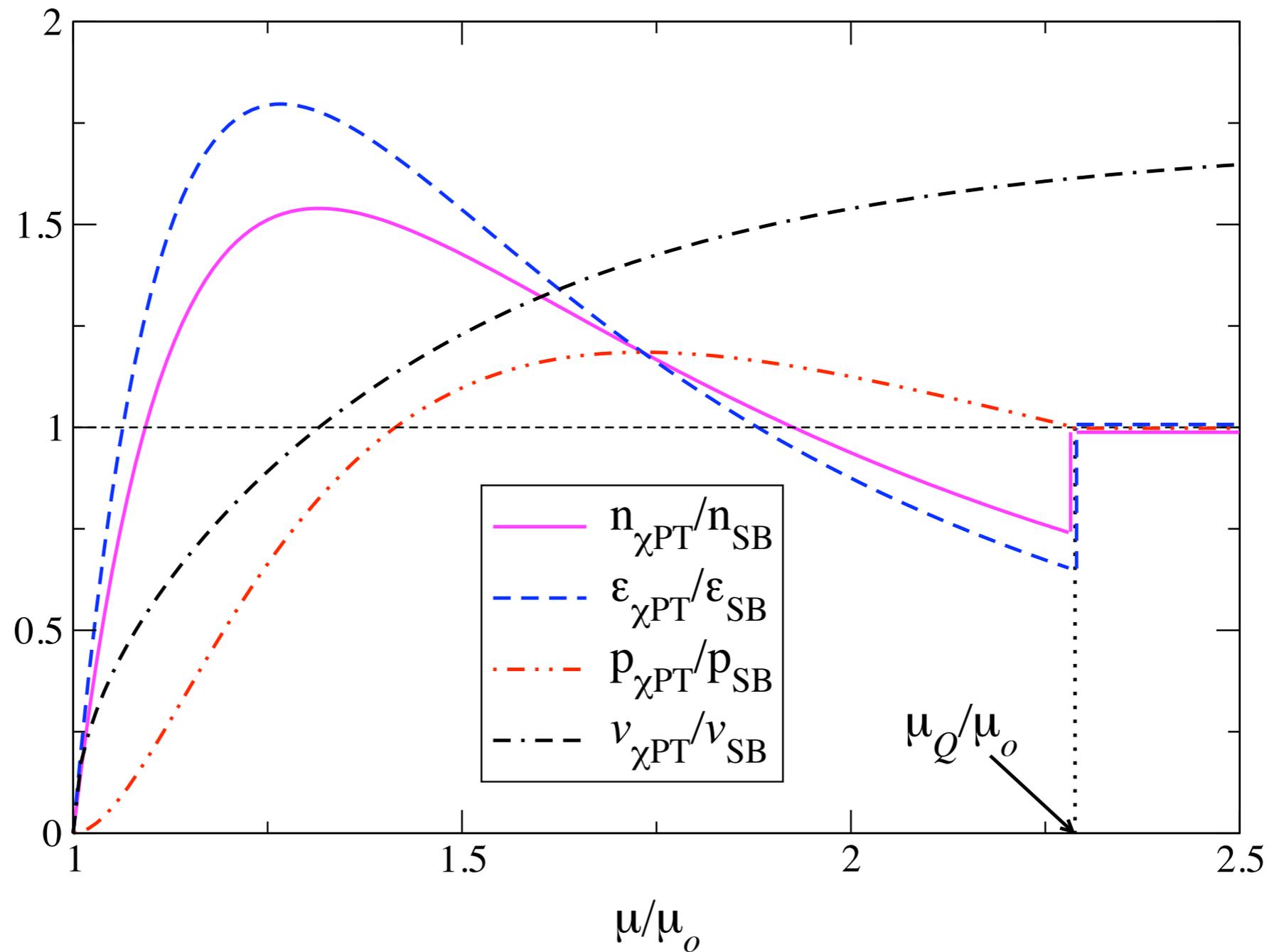
speed of sound $v_{\chi PT} = \sqrt{\frac{\partial p}{\partial \varepsilon}} = \left(\frac{1 - \frac{\mu_o^4}{\mu^4}}{1 + 3\frac{\mu_o^4}{\mu^4}}\right)^{\frac{1}{2}}$

This is to be contrasted with another paradigm for cold dense matter, namely a degenerate system of weakly interacting (deconfined) quarks populating a Fermi sphere up to some maximum momentum $k_F \approx E_F = \mu$

$$\Rightarrow n_{SB} = \frac{N_f N_c}{3\pi^2} \mu^3; \quad \varepsilon_{SB} = 3p_{SB} = \frac{N_f N_c}{4\pi^2} \mu^4;$$
$$\delta_{SB} = 0; \quad v_{SB} = \frac{1}{\sqrt{3}}$$

Superfluidity arises from condensation of diquark Cooper pairs from within a layer of thickness Δ centred on the Fermi surface:

$$\Rightarrow \langle qq \rangle \propto \Delta \mu^2$$



By equating free energies, we naively predict a first order deconfining transition from BEC to quark matter;

eg. for $f_\pi^2 = N_c/6\pi^2$, $\mu_d \approx 2.3\mu_0$.

Simulation Details ($N_f=2$ Wilson flavors)

Coarse Lattice: $8^3 \times 16$ $\beta=1.7$ $\kappa=0.178$

$\Rightarrow a=0.230(5)\text{fm}; m_\pi a=0.79(1); m_\pi/m_\rho=0.779(4); T=54(1)\text{MeV}$

Fine Lattice: $12^3 \times 24$ $\beta=1.9$ $\kappa=0.168$

$\Rightarrow a=0.186(8)\text{fm}; m_\pi a=0.68(1); m_\pi/m_\rho=0.80(1); T=44(2)\text{MeV}$

O(300) HMC trajectories of mean length 0.5 on coarse lattice

SJH, S. Kim and J.I Skullerud, Eur. Phys. J. C48 (2006) 193

O(500) HMC trajectories of mean length 0.5 on fine lattice

SJH, S. Kim and J.I Skullerud, arXiv:1001.1682

To counter IR fluctuations and maintain HMC ergodicity, we introduce a diquark source term $j\kappa(\psi_2^{tr} C\gamma_5\tau_2\psi_1 - \bar{\psi}_1 C\gamma_5\tau_2\bar{\psi}_2^{tr})$

In most results presented here $ja=0.04$

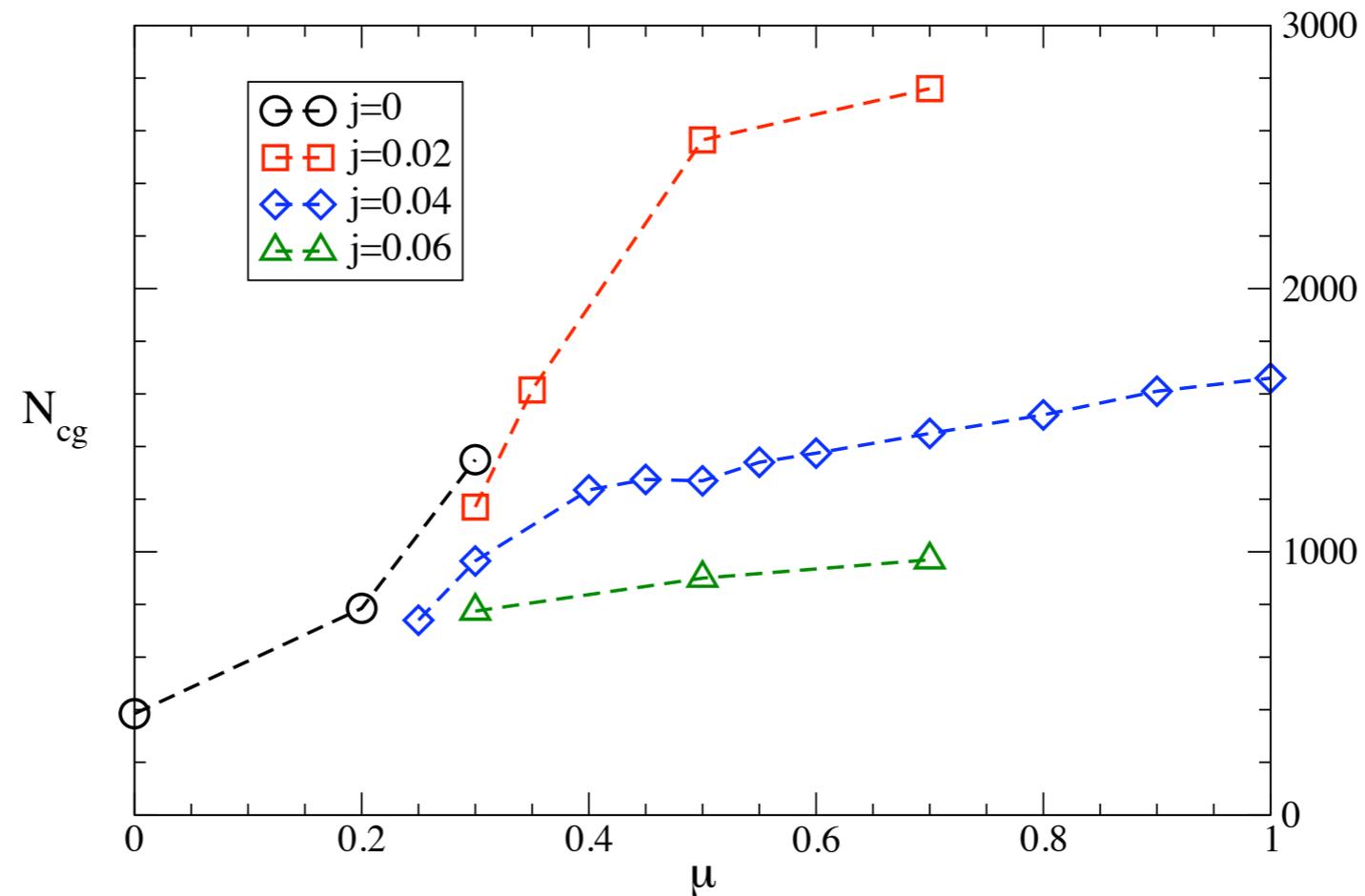
Why Wilson?

- retain a conserved baryon charge
- chiral problems sink to bottom of Fermi Sea?
- saturation artifacts postponed as Fermi Sea only filled from centre of Brillouin Zone
- eigenvalue spectrum governed by same Chiral Orthogonal Ensemble as continuum QC_2D
- permit exact ergodic HMC algorithm with positive real measure and $N_f = 2$ (no 4th roots!)

Note that $N_f = 2$ ensures theory is asymptotically free for all couplings, with a controllable continuum limit, and is confining for $\mu = T = 0$

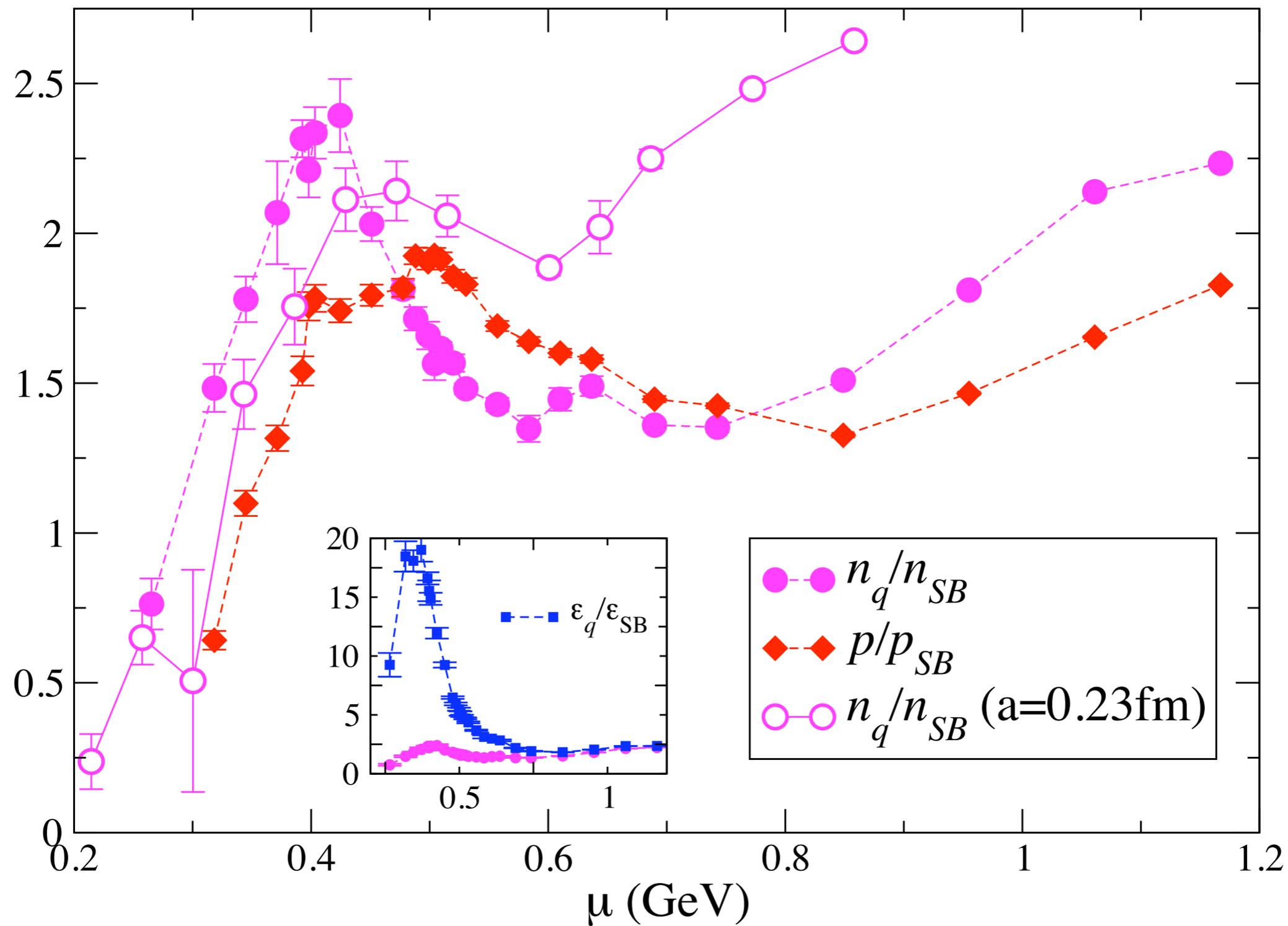
$N_f = 4$ uncomfortably close to Banks-Zaks threshold $N_f^{BZ} = \frac{34N_c^3}{13N_c^2 - 3} \simeq 5.6$

Computer effort



The number of `congrad` iterations required for convergence during HMC guidance rises as μ increases \Leftrightarrow accumulation of small eigenvalues of M .

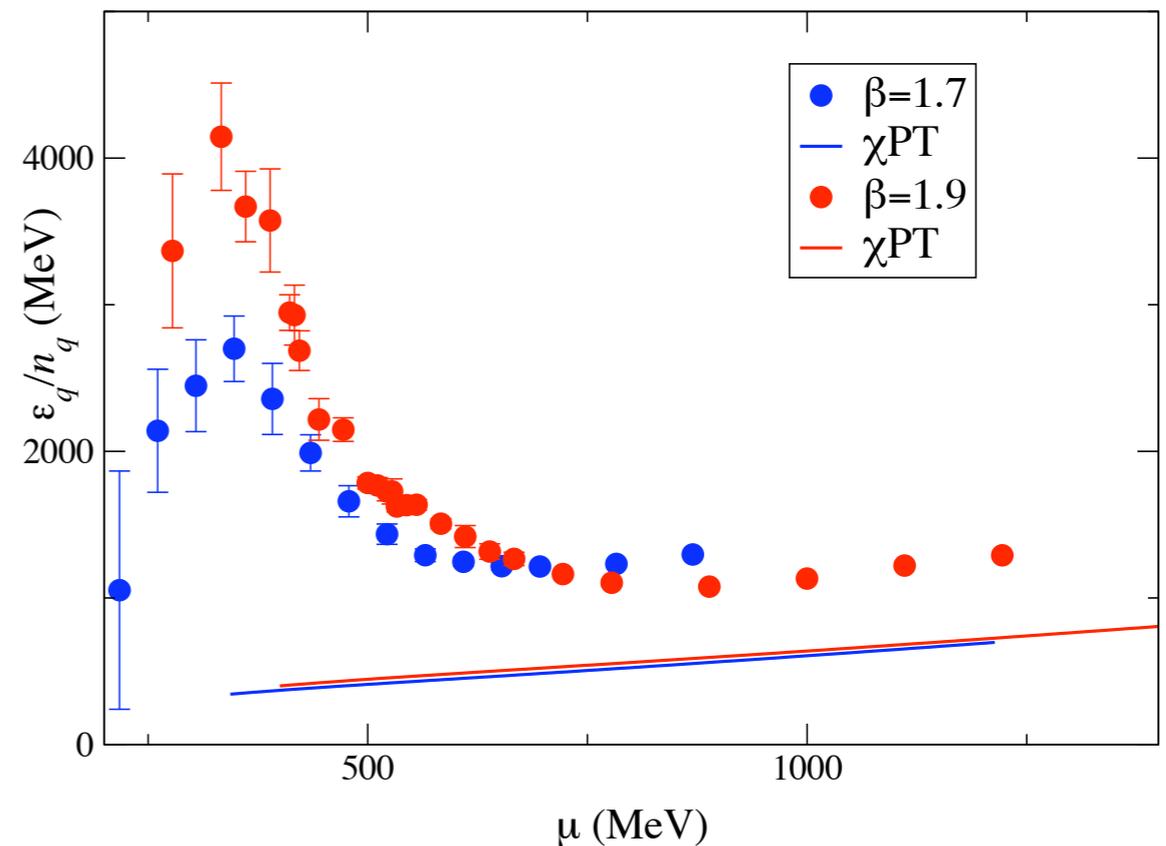
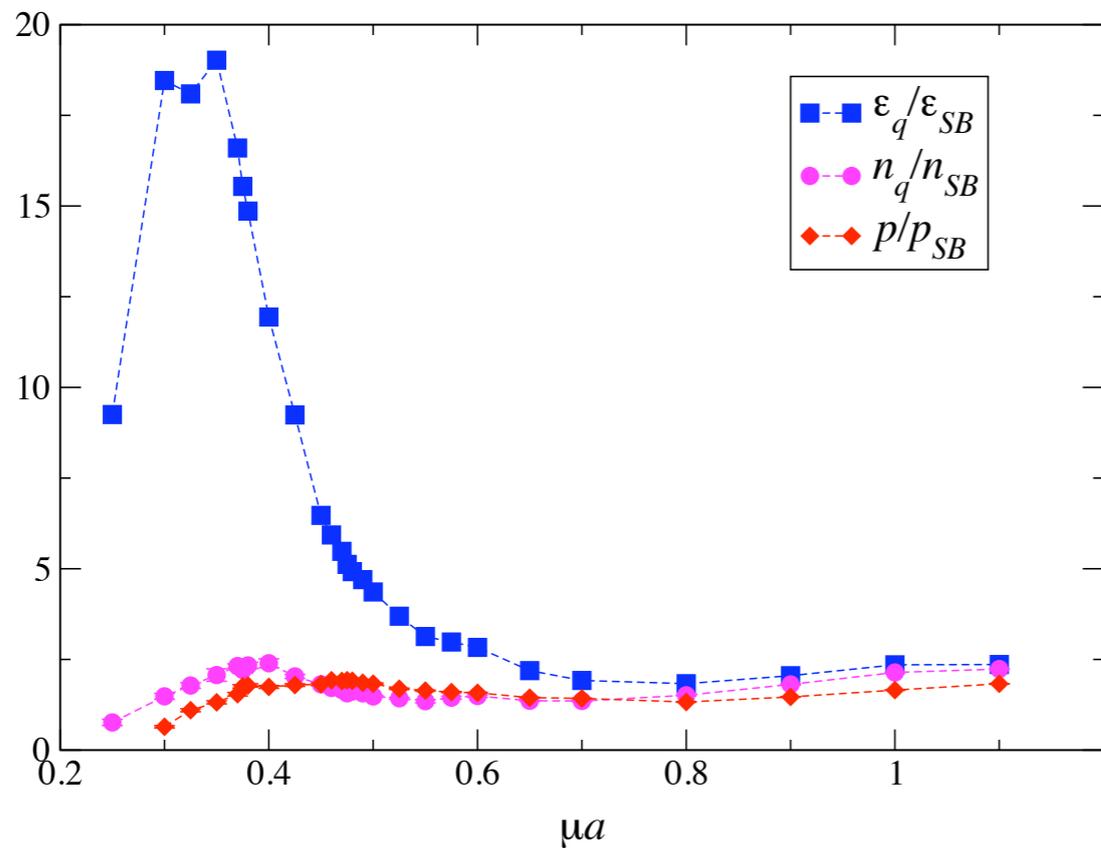
Equation of State on Fine Lattice ($12^3 \times 24$)



Identify onset at $\mu_o \approx 360$ MeV

Transition/crossover to “quark matter” at $\mu_Q \approx 530$ MeV $n_q \approx 4 - 5$ fm $^{-3}$

Quark Energy Density

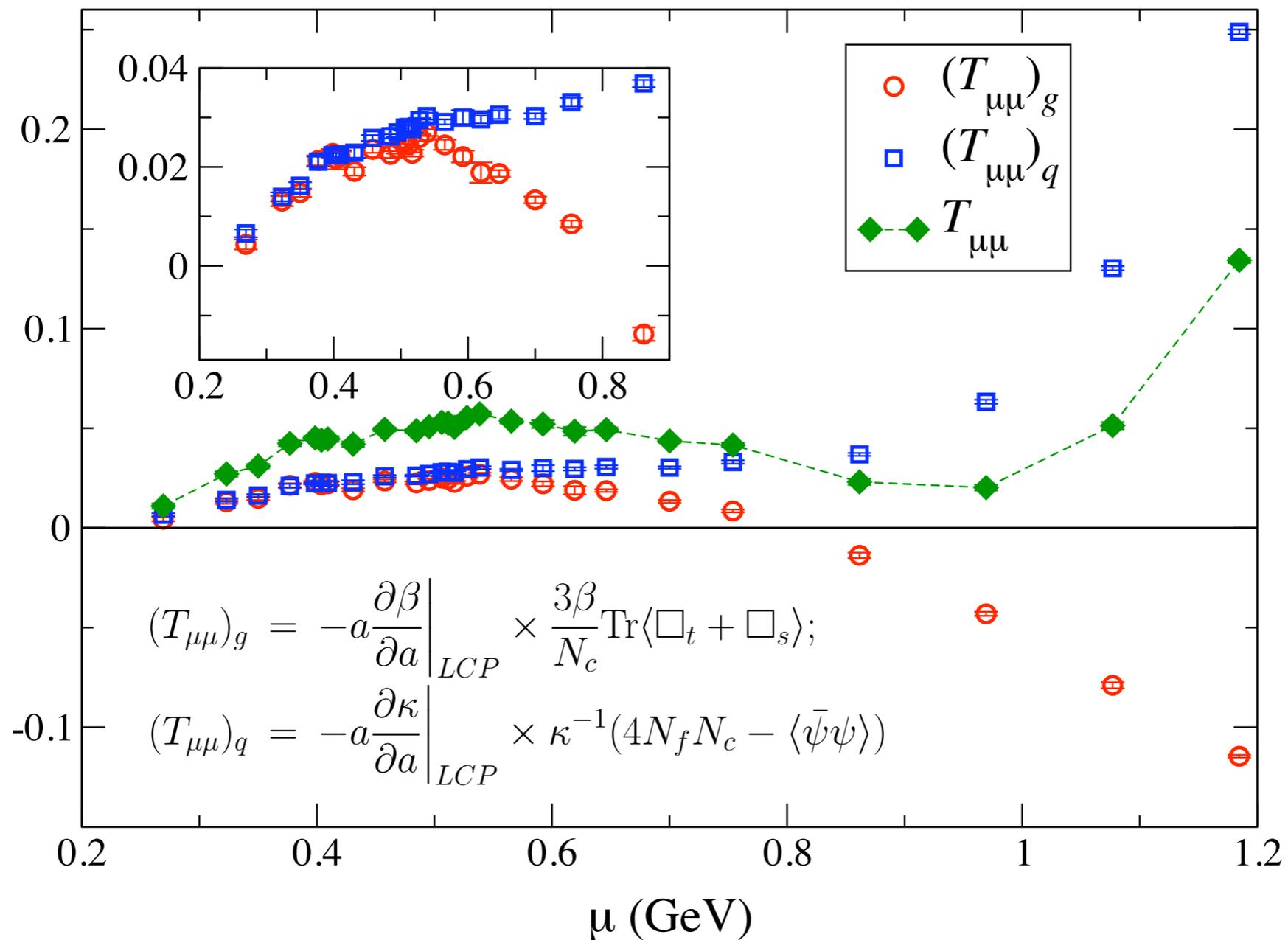


In contrast to χ PT prediction, (unrenormalised) quark energy density ε_q greatly exceeds SB value as $\mu \searrow \mu_{o+}$



Energy per quark ε_q/n_q has shallow minimum for $\mu > \mu_Q$
 NB in Grand Canonical Ensemble $p \neq 0$ at this minimum

Conformal Anomaly $T_{\mu\mu}$



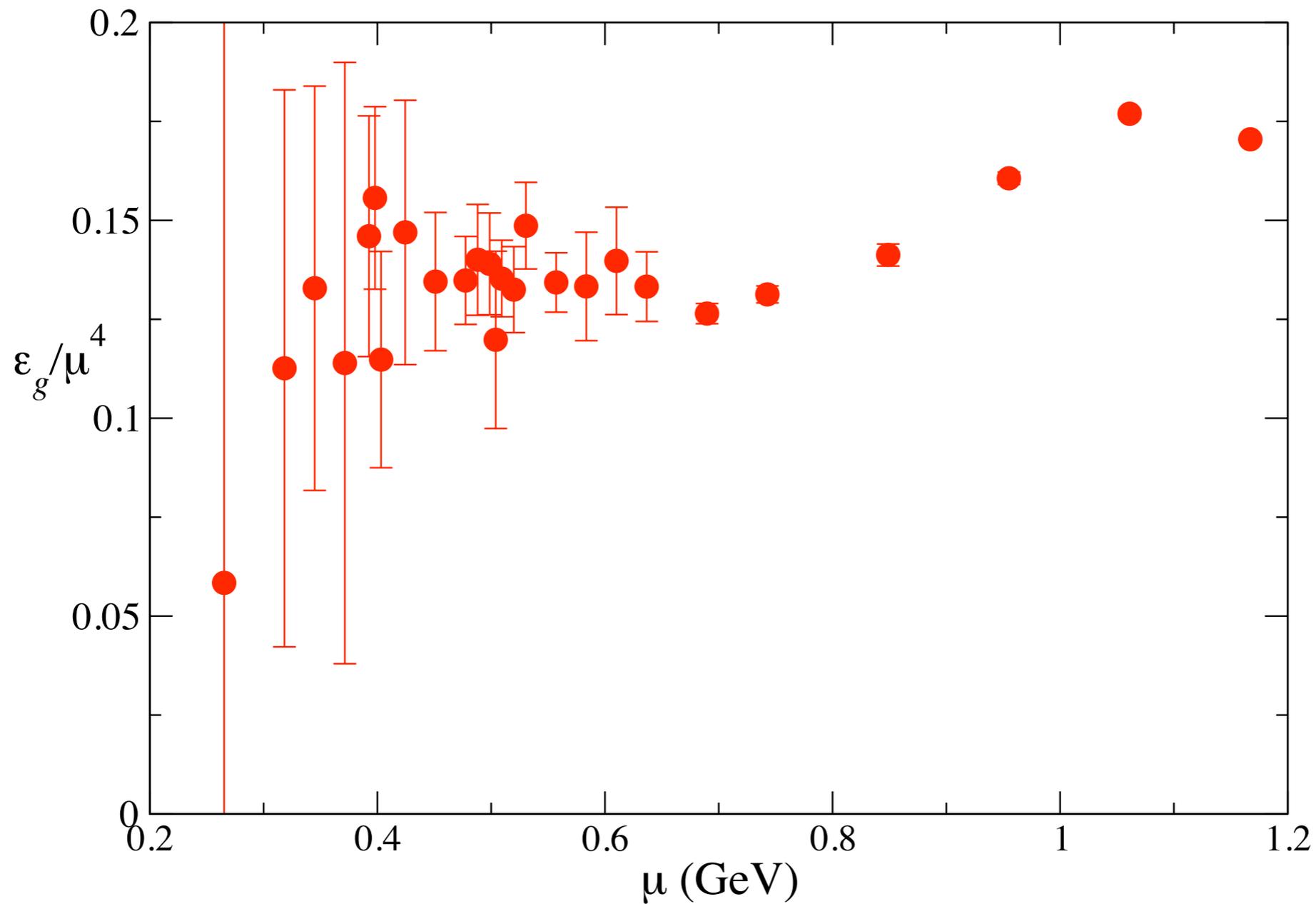
Quark and gluon contributions very similar for $\mu < \mu_Q$, but differ for $\mu > \mu_Q$

$(T_{\mu\mu})_g$ well-approximated by a parabola

$(T_{\mu\mu})_q$ changes sharply at $\mu_D \approx 850 \text{ MeV}$

Note $\varepsilon > 3p$ in limit $\mu \rightarrow \infty$

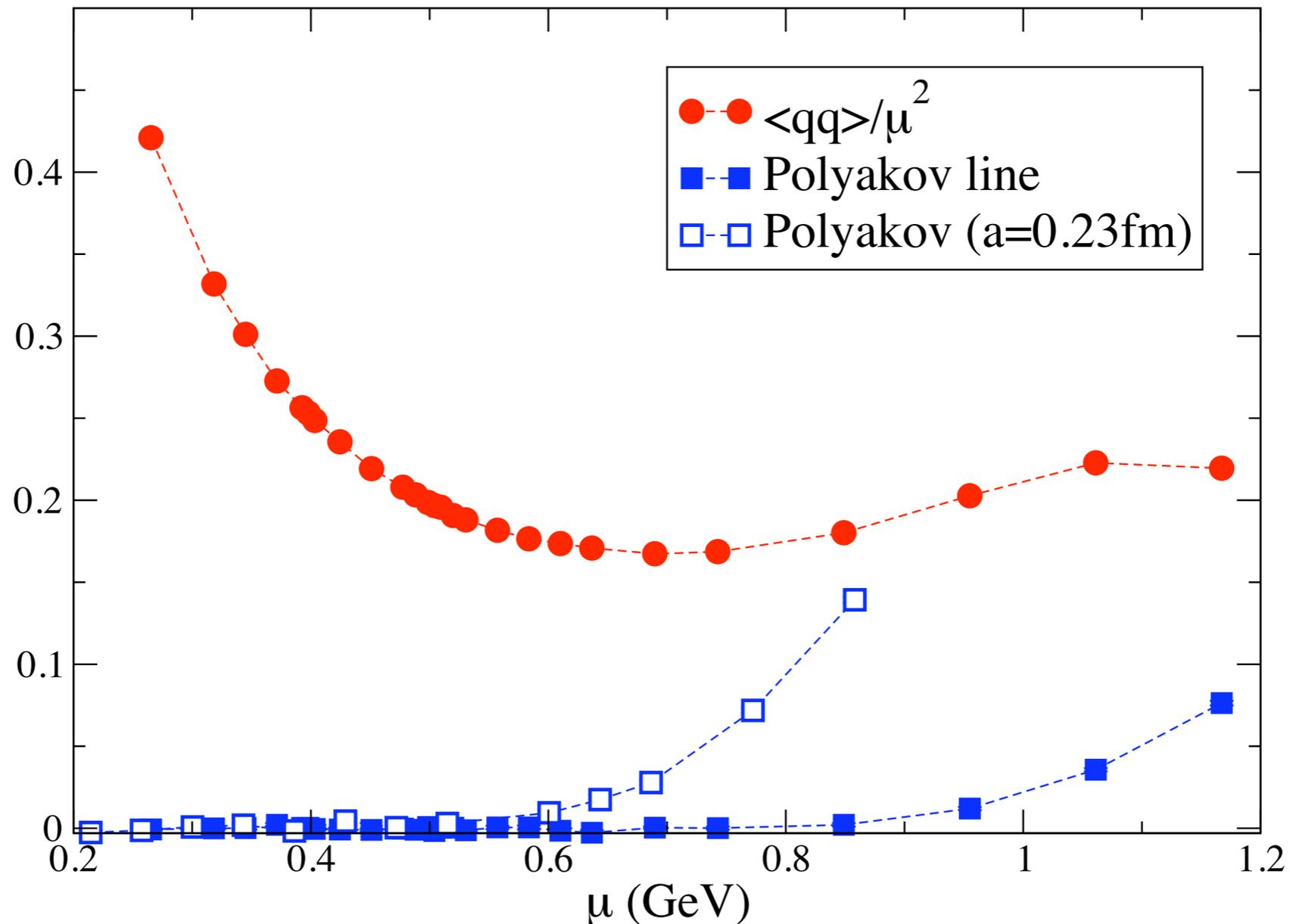
Gluon Energy Density



Gluon energy density $\propto \langle \square_t - \square_s \rangle$ scales according to dimensional analysis $\epsilon_g \propto \mu^4$ for $\mu < \mu_D$

No singular behaviour at $\mu = \mu_Q$

Order parameters



Superfluid condensate $\langle qq \rangle$ scales à la BCS for $\mu_Q \leq \mu \leq \mu_D$

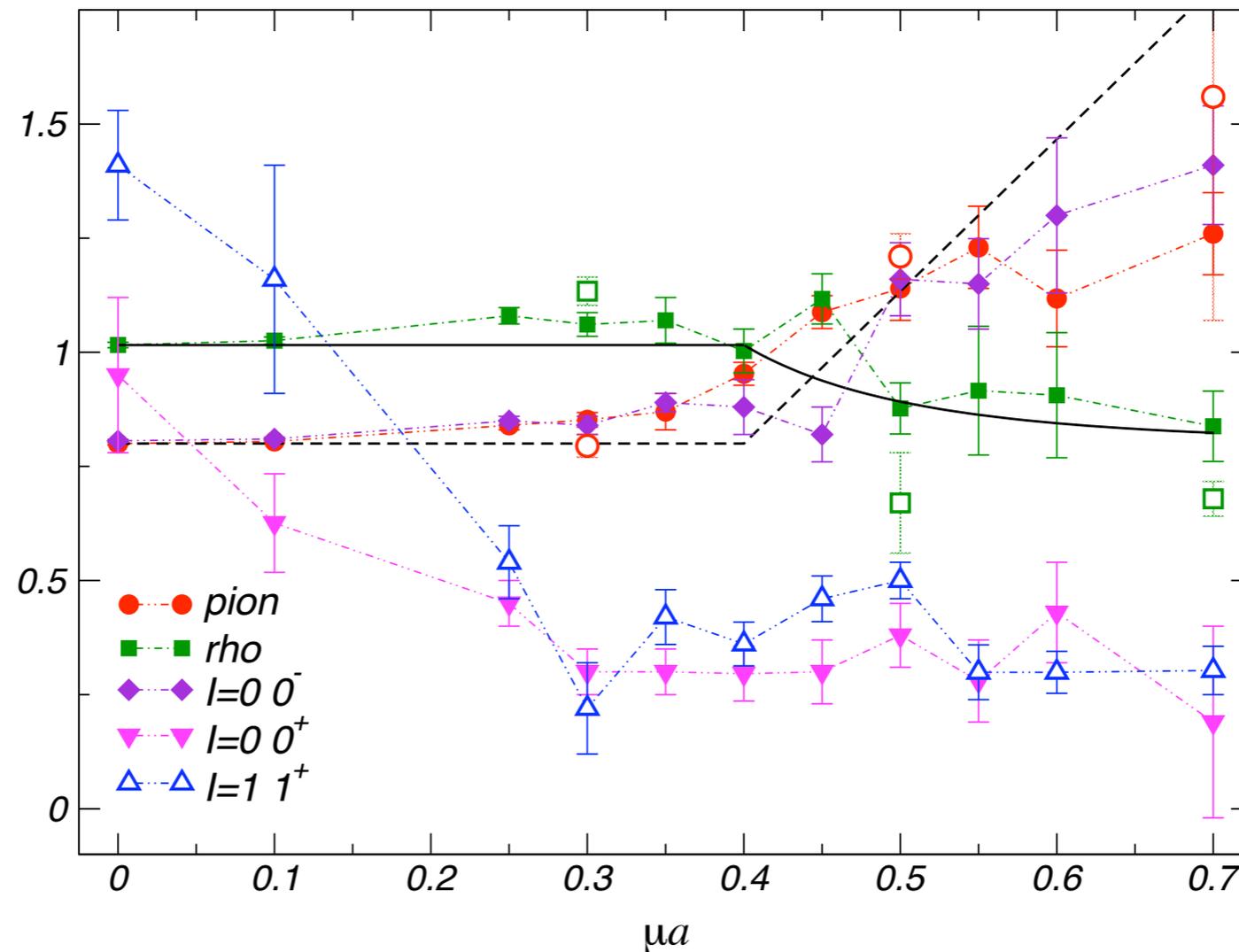
Polyakov line rises from zero at $\mu \approx \mu_D$

\Rightarrow Deconfinement at $\mu_D \approx 850 \text{ MeV}$ $n_q \approx 16 - 32 \text{ fm}^{-3}$

Value of μ_D very T -dependent?

Mesons on $8^3 \times 16$

SJH, P. Sitch, J.I. Skullerud PLB662 405 (2008)



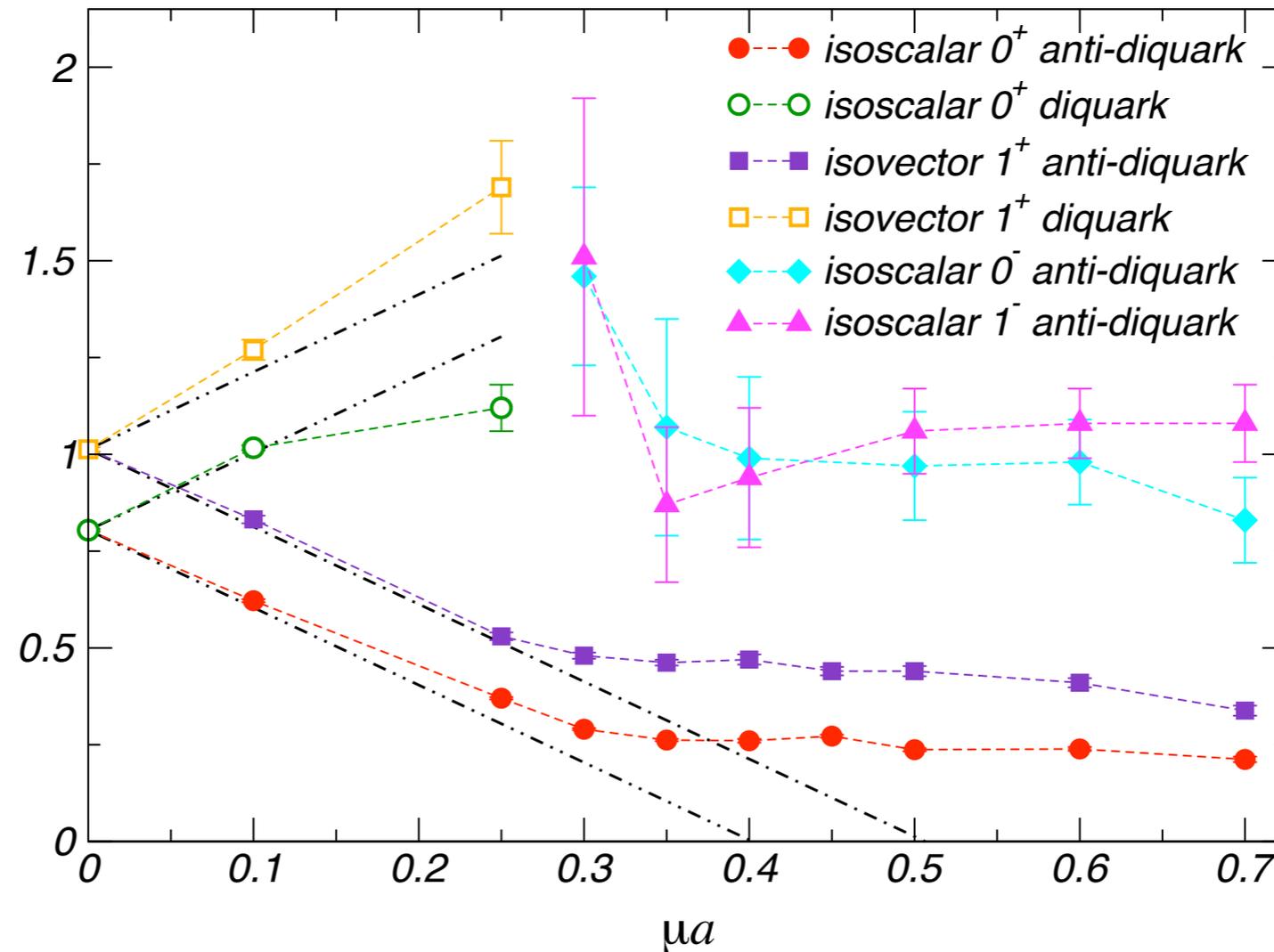
Meson spectrum roughly constant up to onset. Then $m_\pi \approx 2\mu$ in accordance with χ PT, while m_ρ decreases once $n_q > 0$, in accordance with effective spin-1 action

[Lenaghan, Sannino & Splittorff PRD65:054002(2002)]

Cf. Hiroshima group

[Muroya, Nakamura & Nonaka PLB551(2003)305]

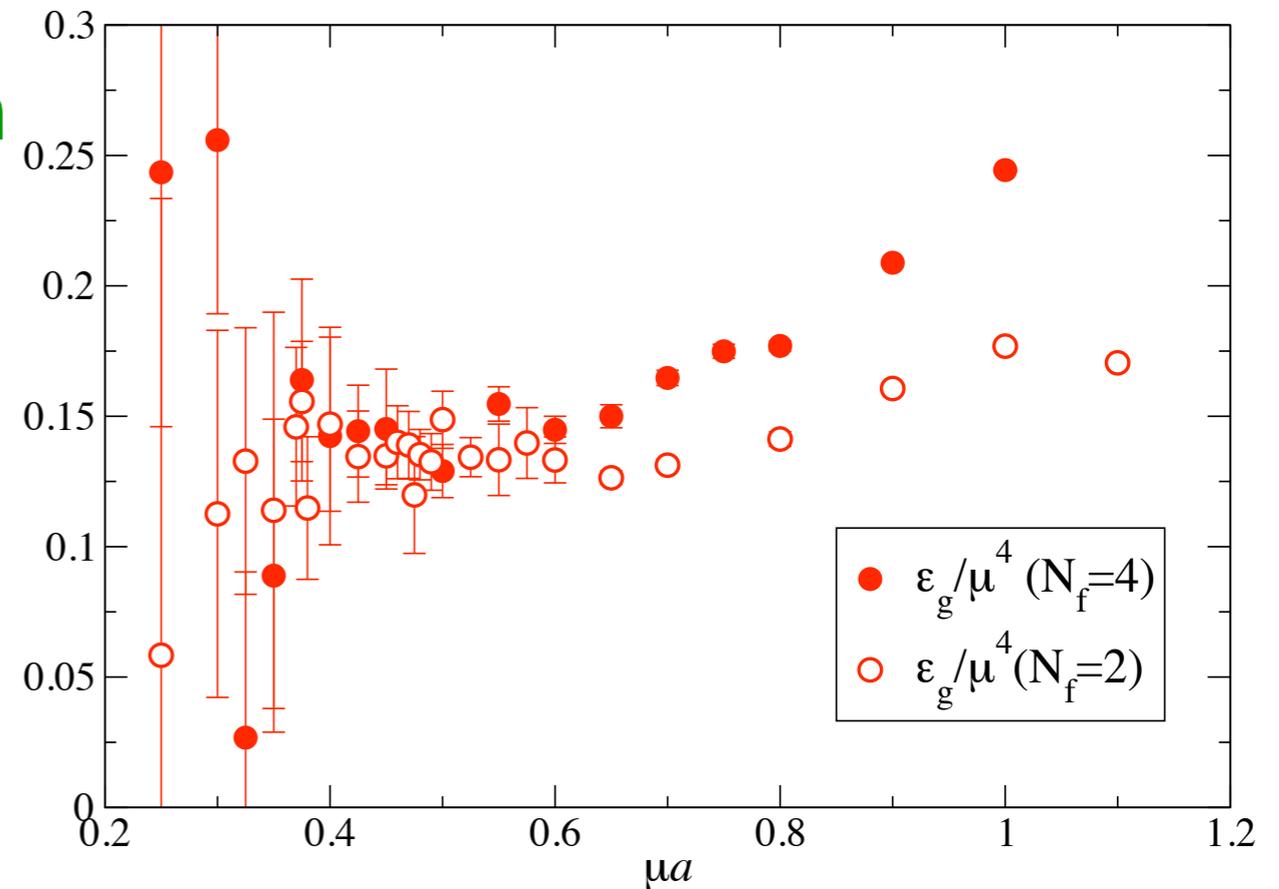
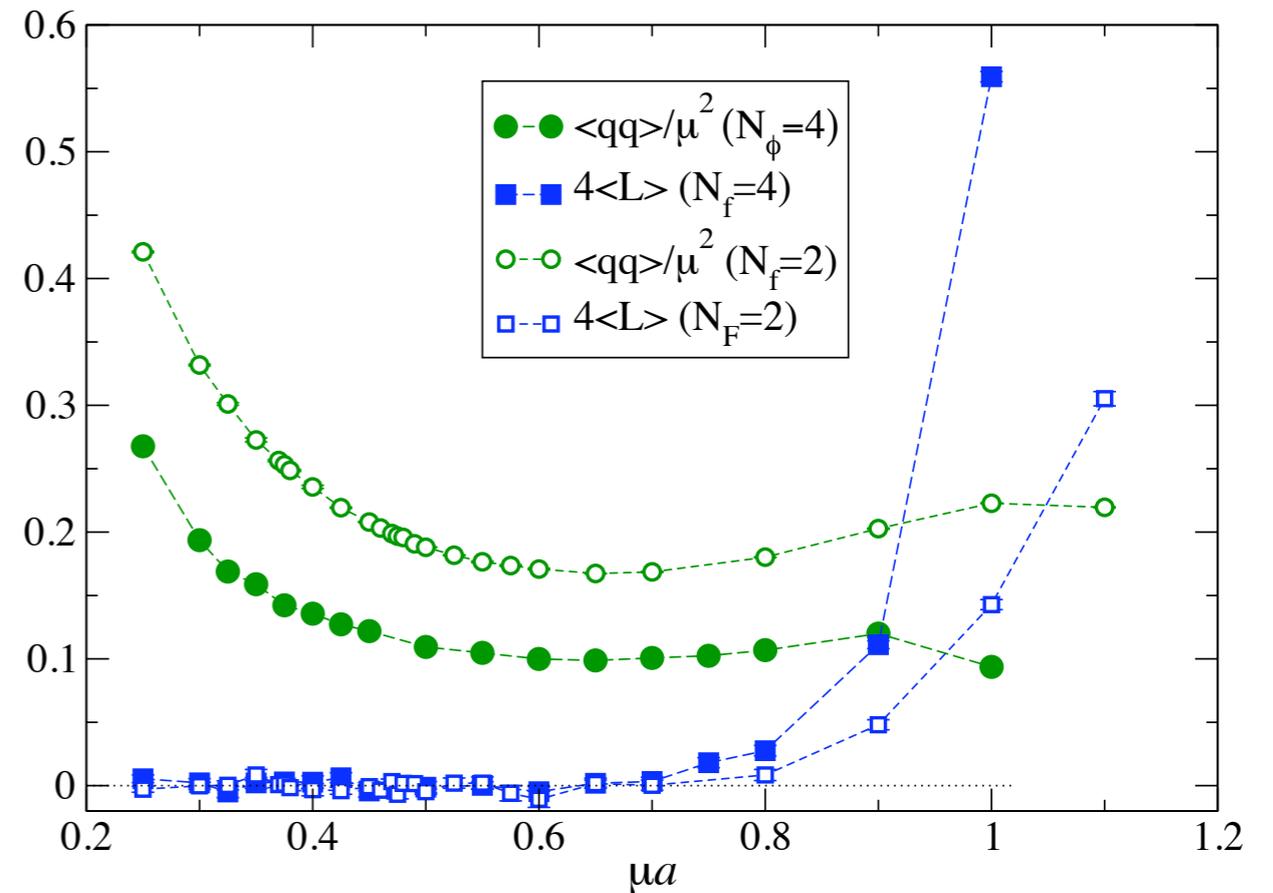
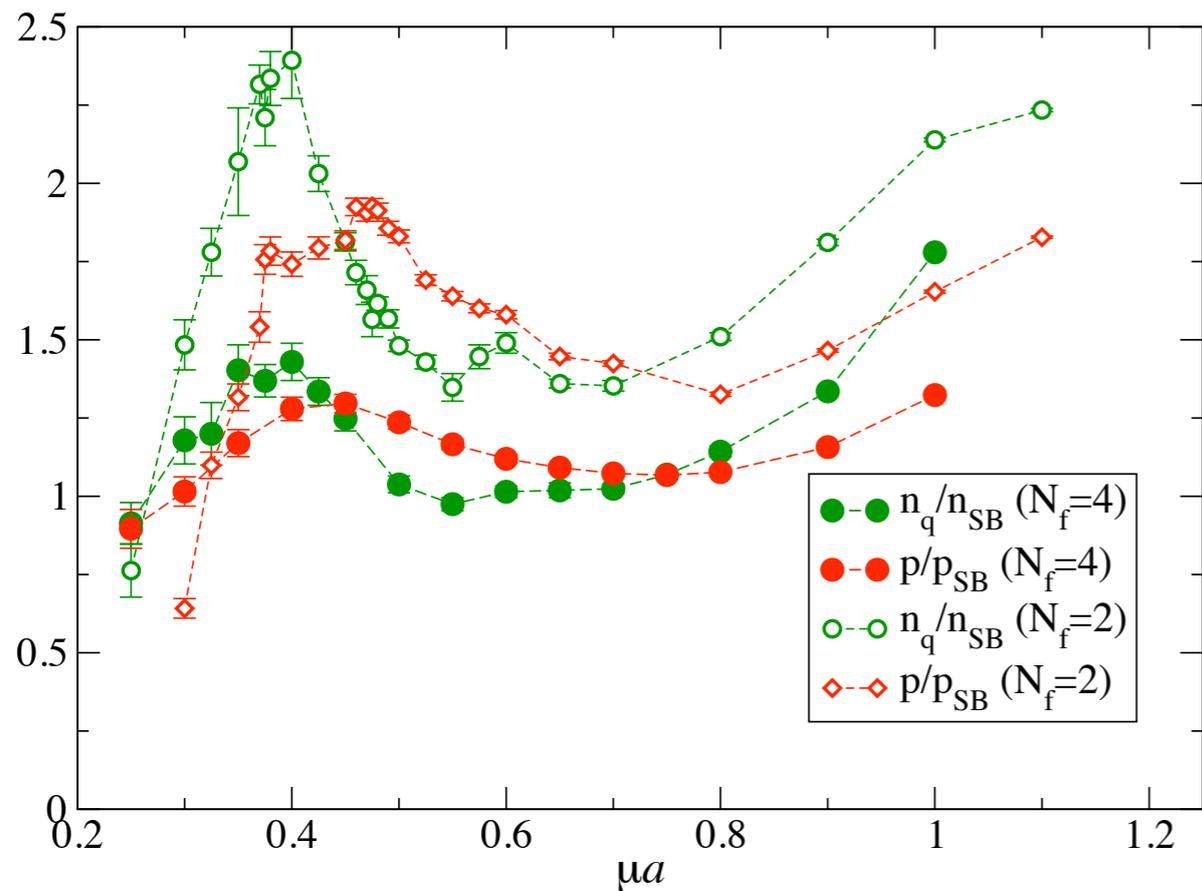
Diquark Spectrum on $8^3 \times 16$



Diquark spectrum modelled by $m_{\pi,\rho} \pm 2\mu$ up to onset, while post-onset:

- Splitting of “Higgs/Goldstone” degeneracy in $I = 0 0^+$ channel
- Meson/Baryon degeneracy in $I = 0 0^+$ and $I = 1 1^+$ channels

And $N_f=4$? (preliminary, w/ Phil Kenny)



Same distinct physical regimes can be identified

maybe slightly closer to continuum?

maybe lattice slightly hotter?

Partial Summary

QC₂D has several distinct phases as μ is increased

- a vacuum phase with $n_q \approx 0$ for $\mu < \mu_0$
- a superfluid BEC phase described by χ PT for $\mu_0 < \mu < \mu_Q$
- a superfluid confined quark matter phase for $\mu_Q < \mu < \mu_D$
- deconfined quark matter for $\mu > \mu_D$

Behaviour for $\mu_Q < \mu < \mu_D$ resembles the **quarkyonic** phase postulated by McLerran and Pisarski based on large- N_c considerations

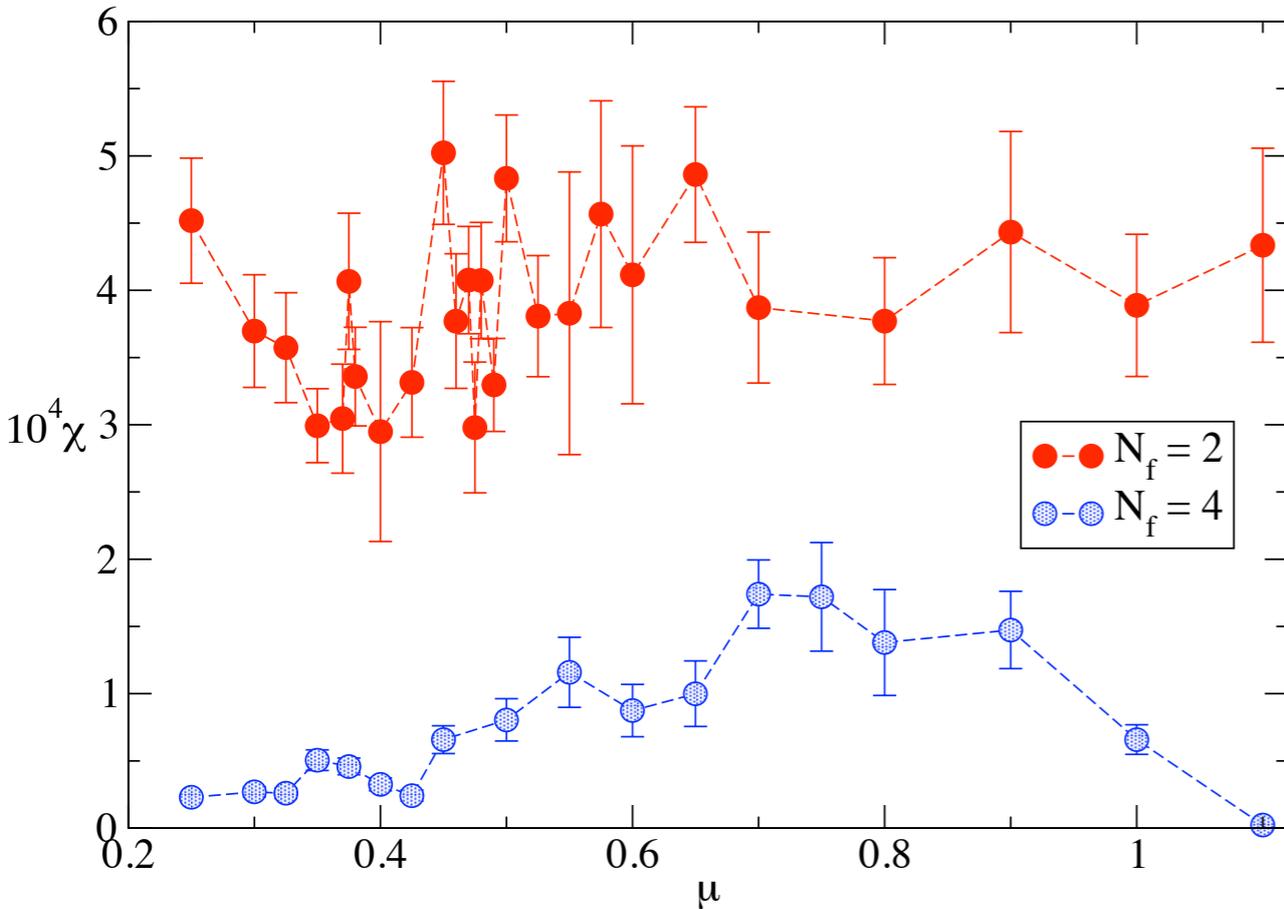
[L. McLerran and R.D. Pisarski Nucl. Phys. A796 (2007) 83]

NB: we are unable to determine whether the "quarkyonic" phase is chirally symmetric

The deconfining transition at μ_D appears very T-dependent (could even be a lattice artifact)

Topological Susceptibility and Instanton Size

preliminary, w/ Phil Kenny

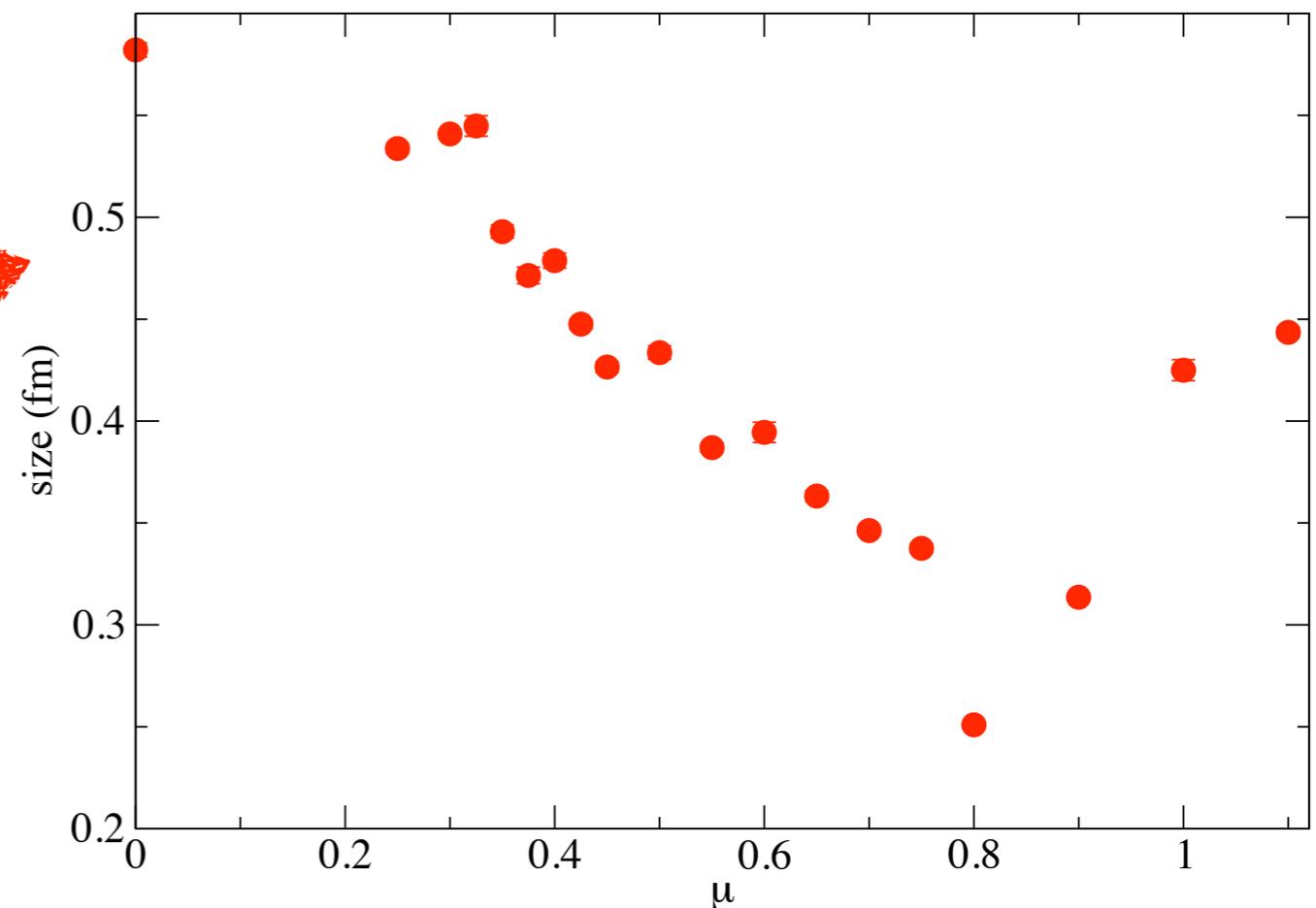


Topological susceptibility shows little structure with μ for $N_f=2$, but appears enhanced in quarkyonic region for $N_f=4$

Cf. suppression in superfluid phase for $N_f=8$ with staggered fermions

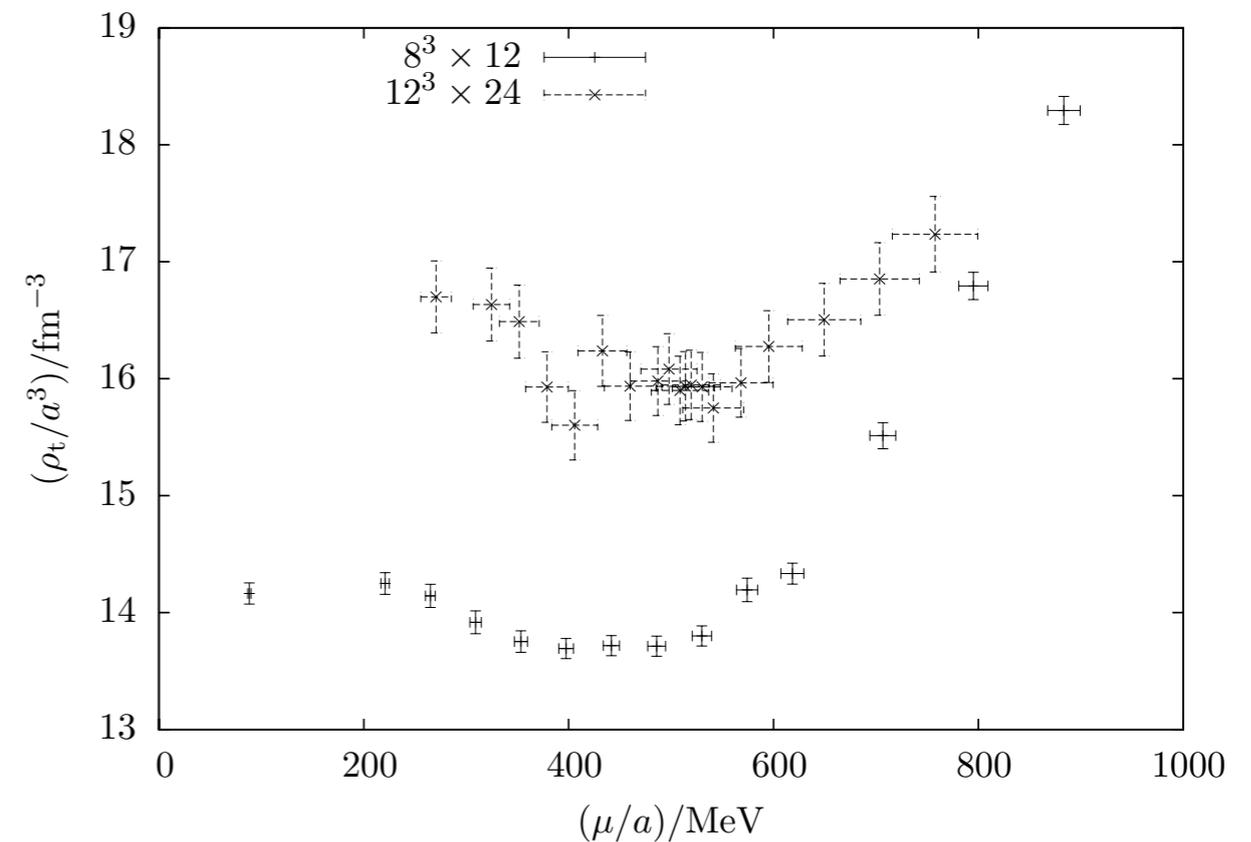
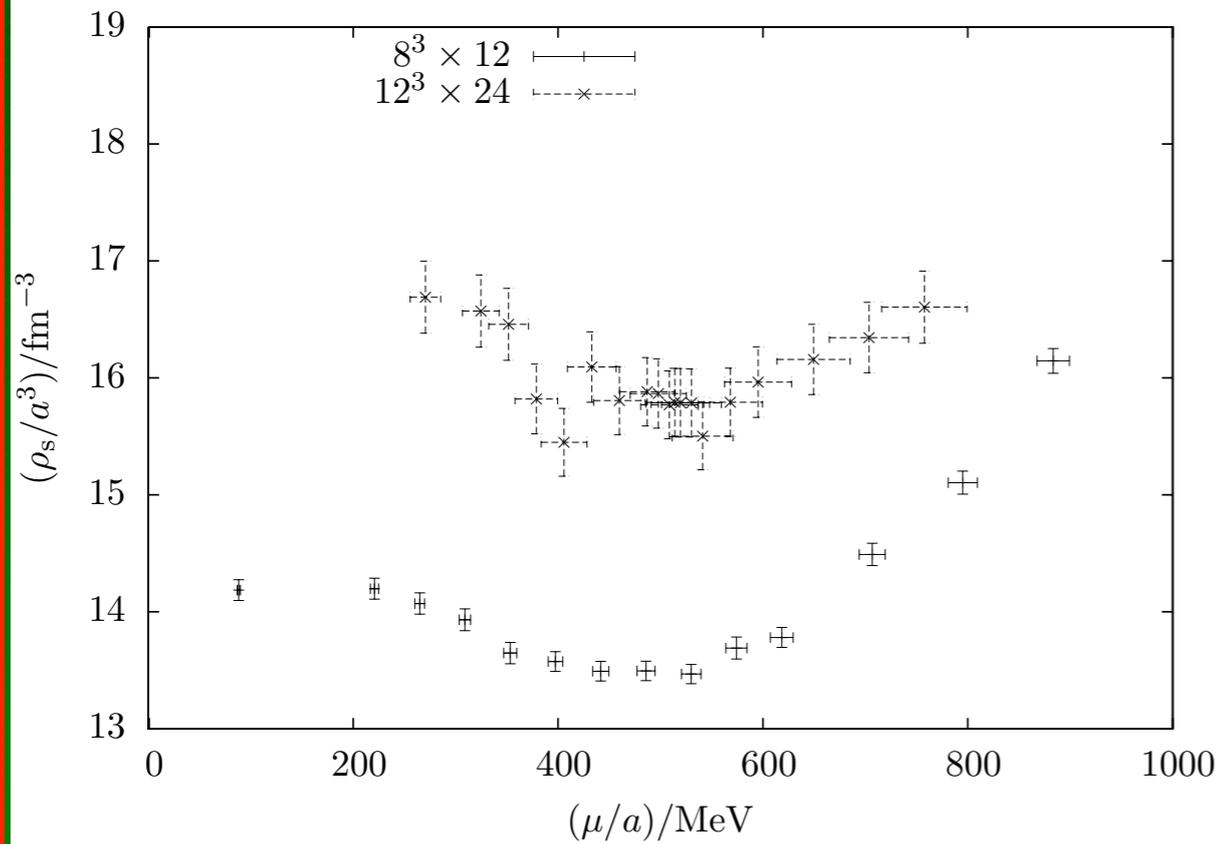
B. Alles, M. D'Elia & M.P. Lombardo, NPB752(2006)124

Average instanton size decreases systematically with μ throughout confined regime ($N_f=4$)



w/ E.M. Ilgenfritz, S. Schubert

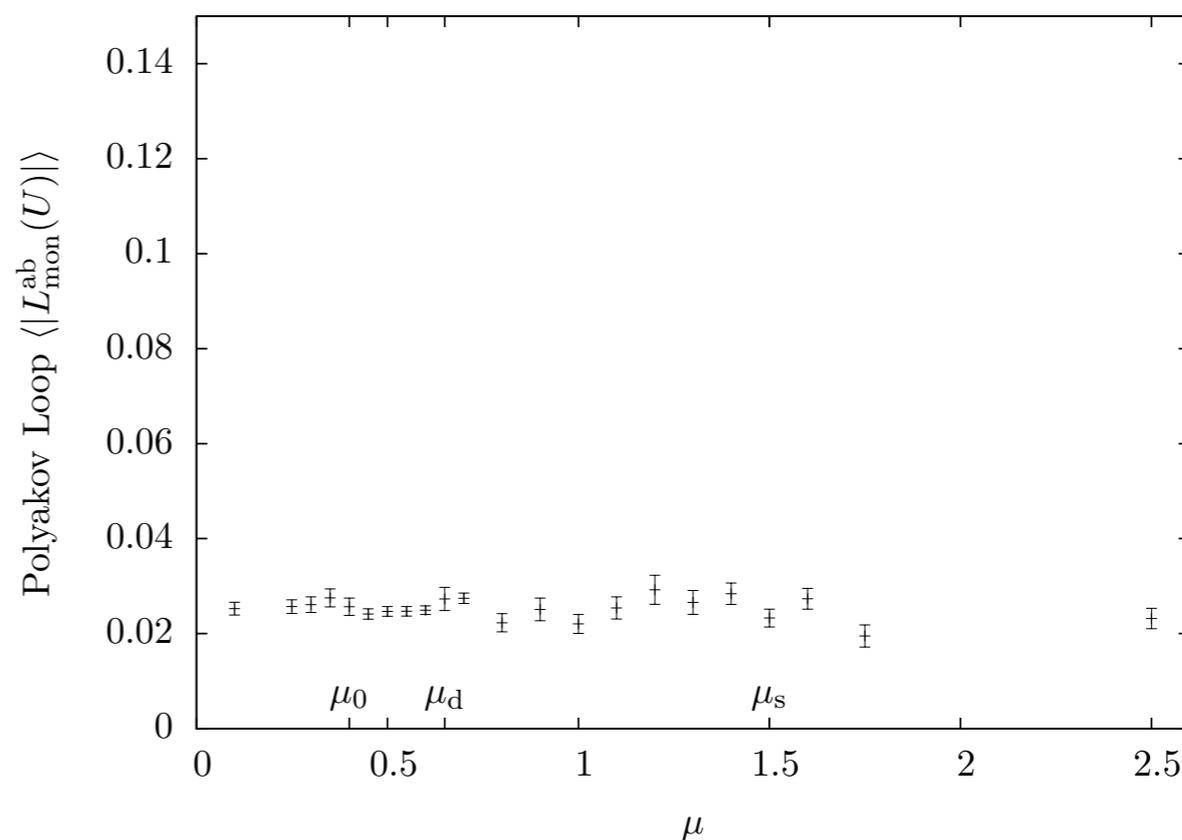
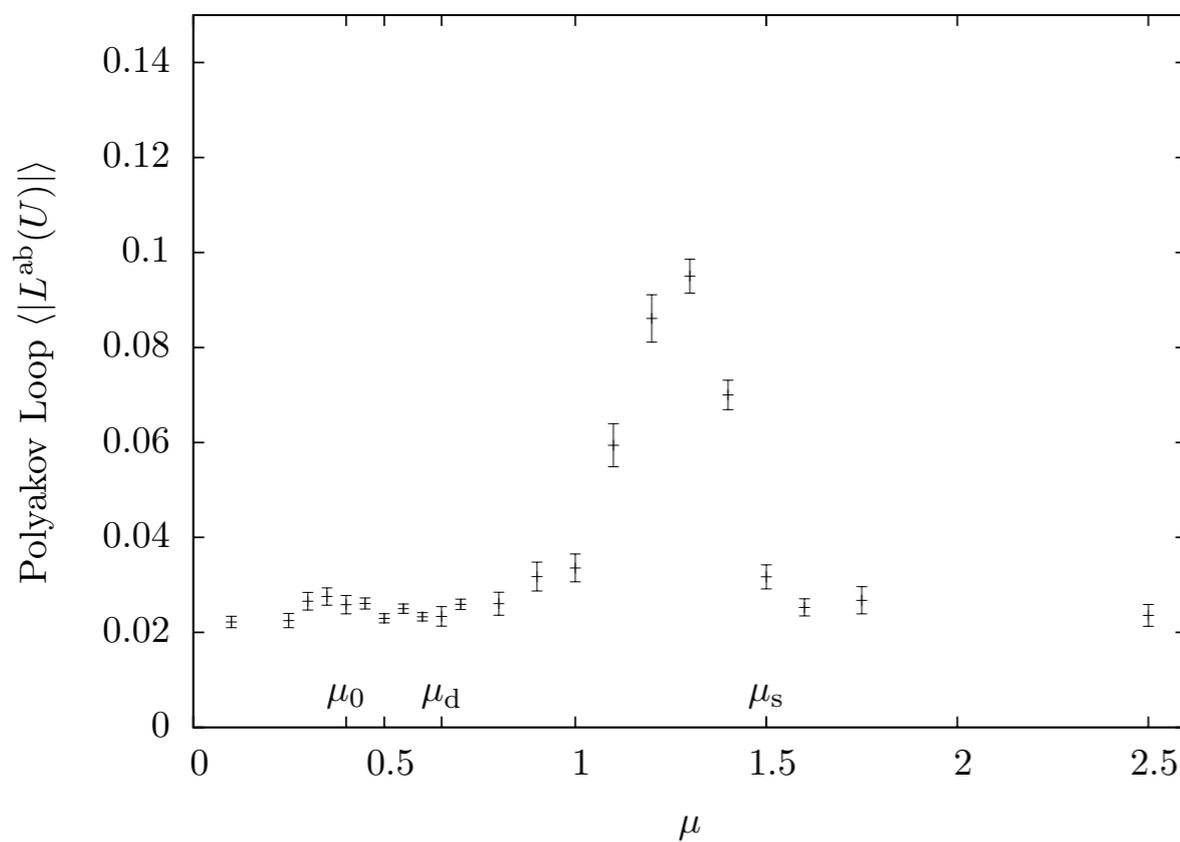
Magnetic Monopoles in MAG Gauge



We have studied the distribution of magnetic monopoles found using the DeGrand-Toussaint procedure following abelian projection from the MAG gauge

The monopole densities ρ_s and ρ_t are approximately equal, and show structure both at $\mu \approx \mu_0$ and $\mu \approx \mu_{QM}$

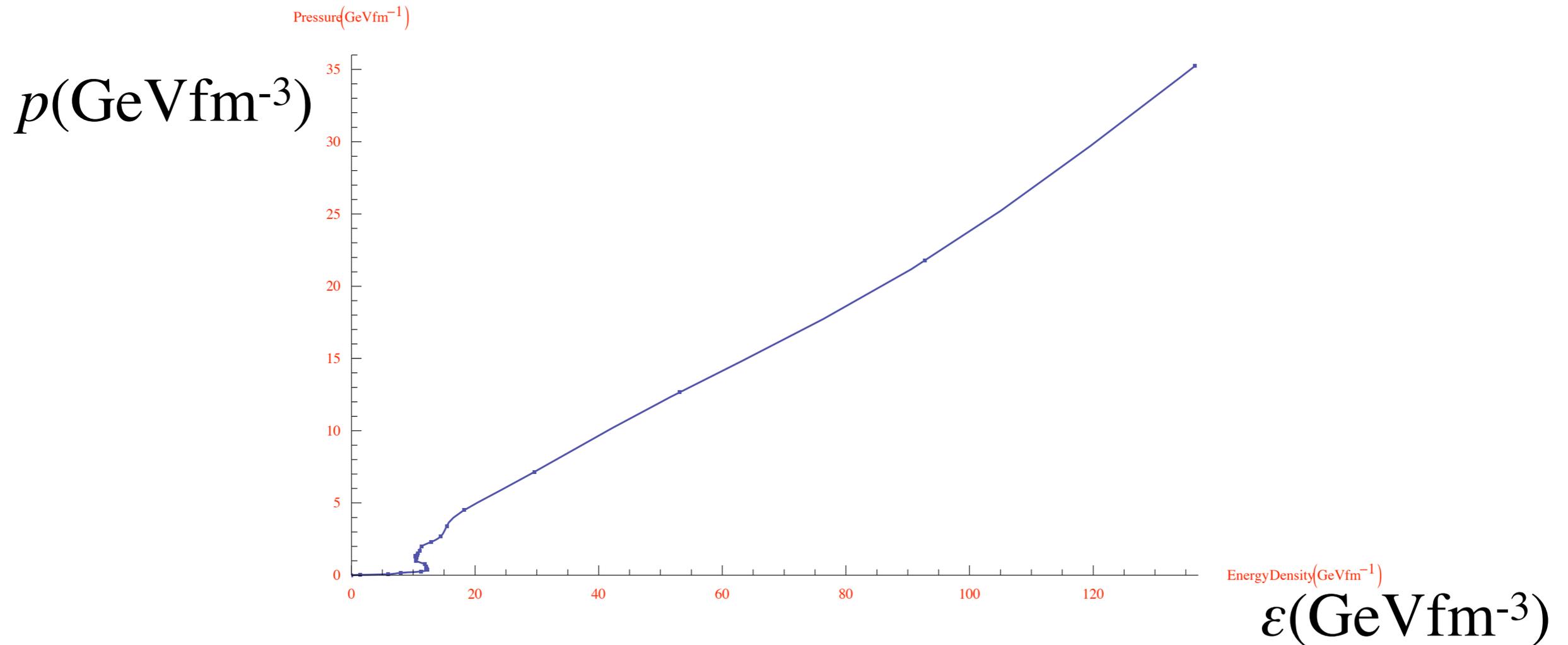
The Polyakov Loop is Non-Abelian?



After abelian projection we can decompose the abelian link fields as $\theta_\mu = \theta_\mu^{\text{mon}} + \theta_\mu^{\text{phot}}$ with $\theta_{\mu x}^{\text{mon}} \equiv -2\pi \sum_y (\partial^+ \partial^-)_{xy}^{-1} m_{\mu y}$

The abelian Polyakov loop on $8^3 \times 16$ is about 25% of the full loop, and remarkably, the signal completely vanishes in the $\{\theta^{\text{mon}}\}$ configuration, showing that “deconfinement” is NOT due to abelian monopoles in this case

Q2CD E.O.S

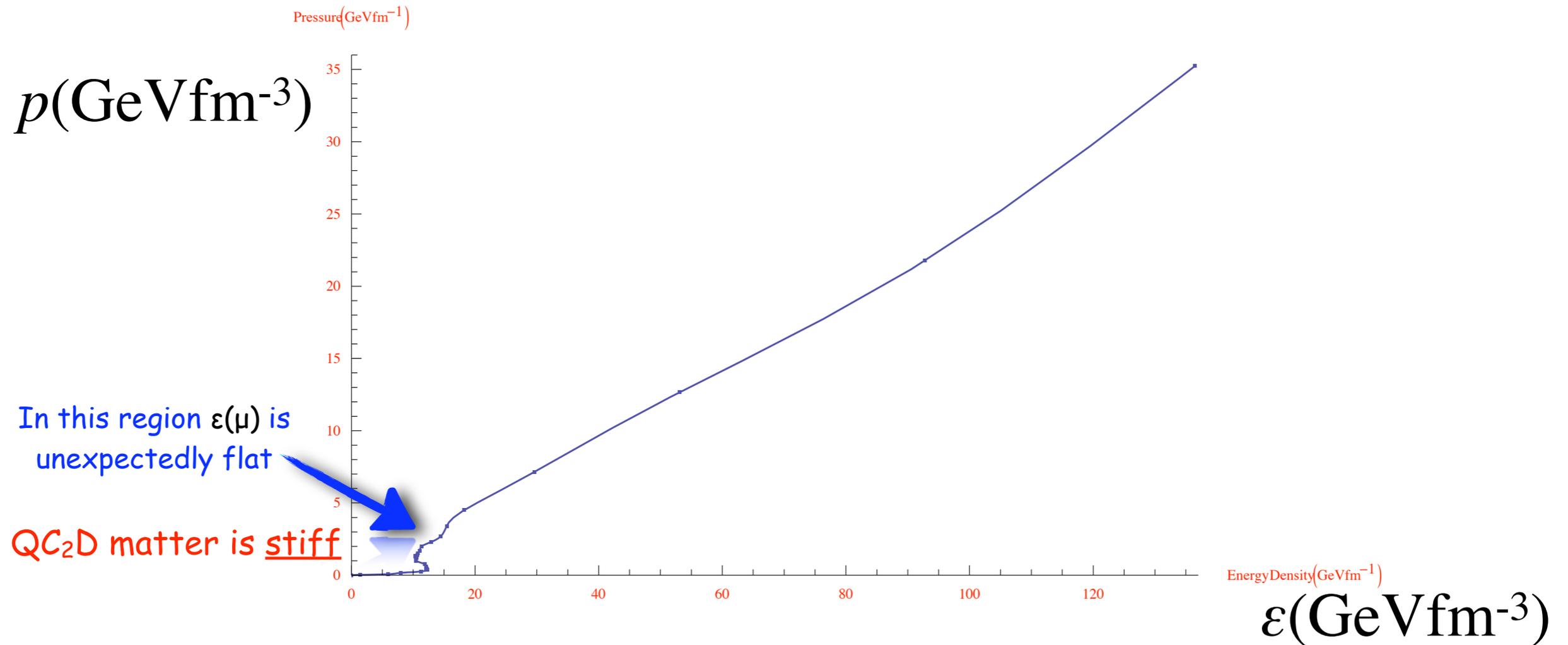


We can also use the EoS in the form $\varepsilon(p)$
as input to the Tolman-Oppenheimer-Volkoff equations

$$\frac{dp}{dr} = -\frac{(p + \varepsilon(p))(M(r) + 4\pi r^3 p)}{r(r - 2M)}$$
$$\frac{dM}{dr} = 4\pi r^2 \varepsilon(r)$$

$$G=c=1$$

Q2CD E.O.S



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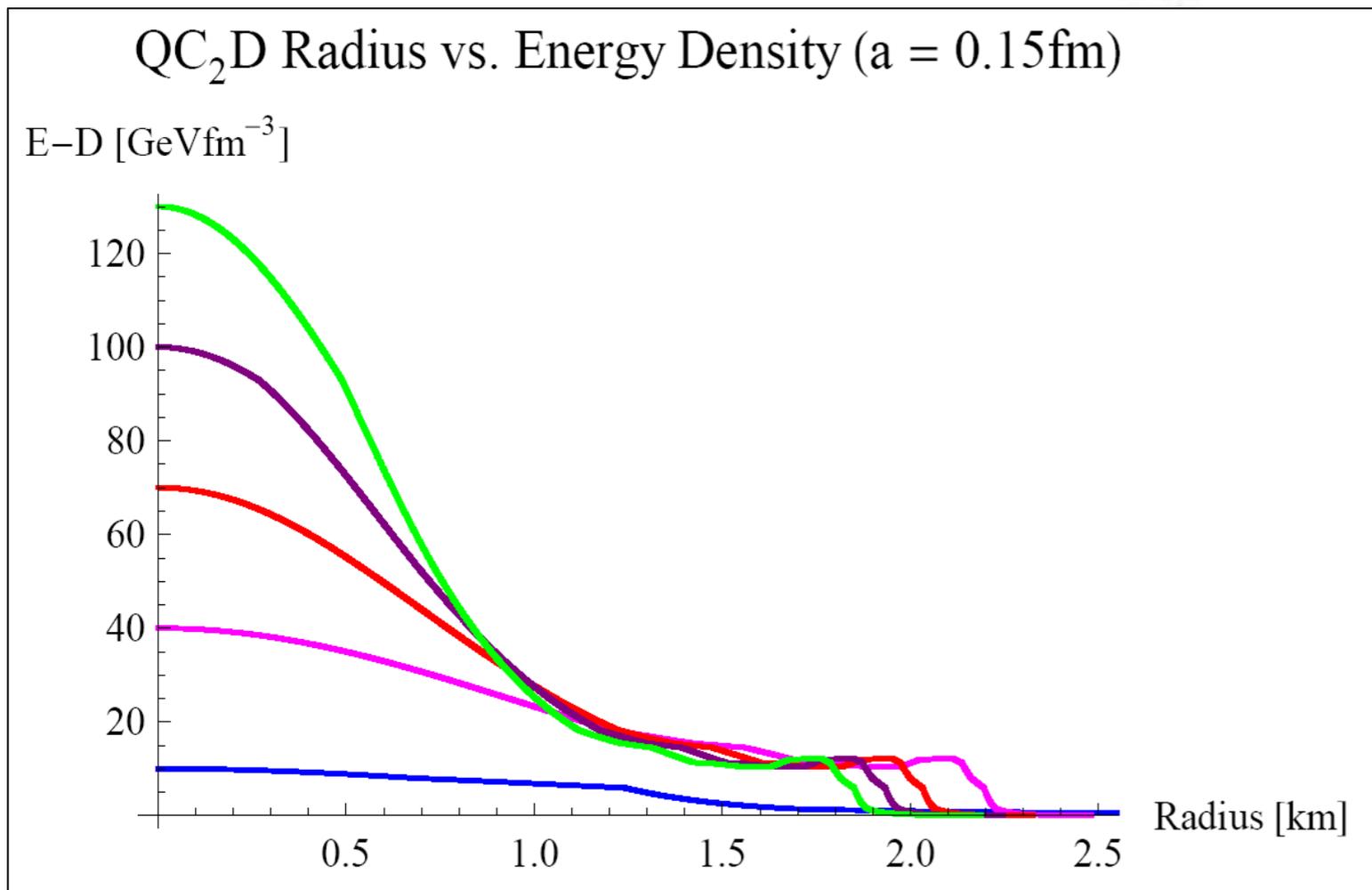
$$\frac{dp}{dr} = -\frac{(p + \varepsilon(p))(M(r) + 4\pi r^3 p)}{r(r - 2M)}$$

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon(r)$$

$$G=c=1$$

A Star is born?

w/ James Tonkin



Solution of TOV enables
modelling of "neutron stars"

$$M_{\max} = 0.8M_{\odot}; R = 3.3\text{km};$$
$$\epsilon_c = 12\text{GeVfm}^{-3} (a=0.17\text{fm})$$

need Karsch coefficient for definitive answer...



Summary

- ★ dense QC₂D has three distinct transitions/
crossovers at $\mu_0 < \mu_Q < \mu_D$:
 - * Vacuum for $\mu < \mu_0$
 - * BEC for $\mu_0 < \mu < \mu_Q$
 - * "Quarkyonic" phase for $\mu_Q < \mu < \mu_D$
(are the 2-body bound states of $N_c=2$ special?)
 - * Deconfined phase for $\mu > \mu_D$
- ★ It's deconfinement, Jim! but not as we know it
Very temperature sensitive - or maybe a lattice artifact?
- ★ Why topological structure only for $N_f=4$? Why
don't abelian-projected monopoles confine?
- ★ Time to dream of arXiv:astro-lat ?