

# Quarkyonic Chiral Spirals

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In collaboration with

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# Contents

## 1, Introduction

- Quarkyonic matter, chiral pairing phenomena

## 2, How to Solve

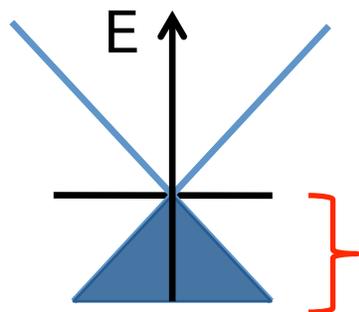
- Model with linear confinement
- Dimensional reduction from (3+1)D to (1+1)D

## 3, Two Dictionaries

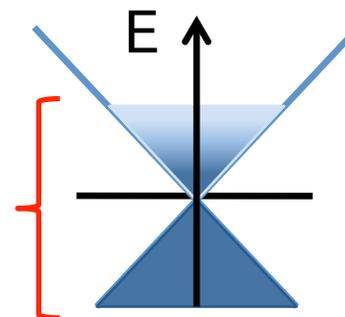
- Mapping (3+1)D onto (1+1)D: quantum numbers
- Chiral Spiral solutions

## 4, Summary

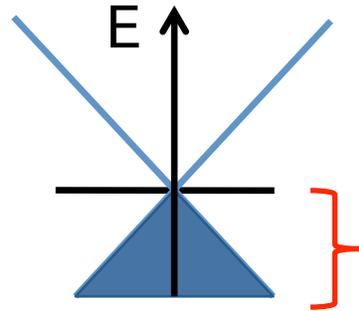
# Dense QCD at $T=0$ : Confining aspects



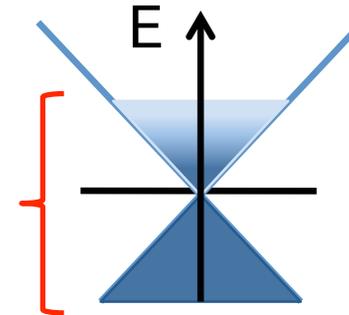
Colorless



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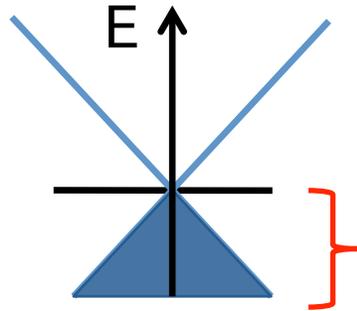
Allowed phase space differs  $\rightarrow$  Different screening effects

e.g.)

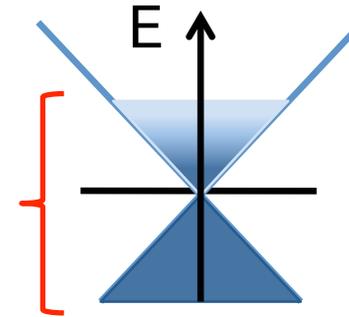


$$M_D \sim N_c^{-1/2} \times f(\mu)$$

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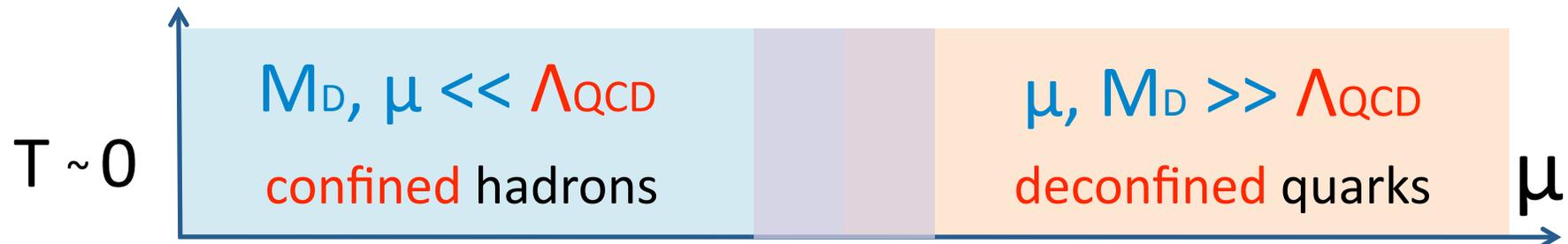


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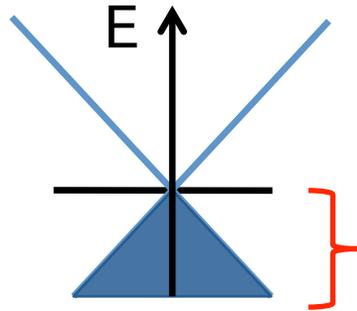
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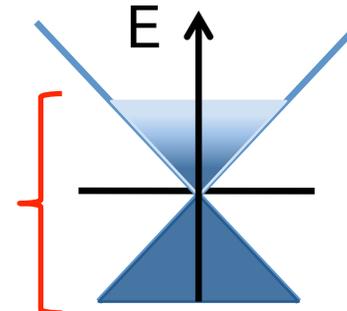
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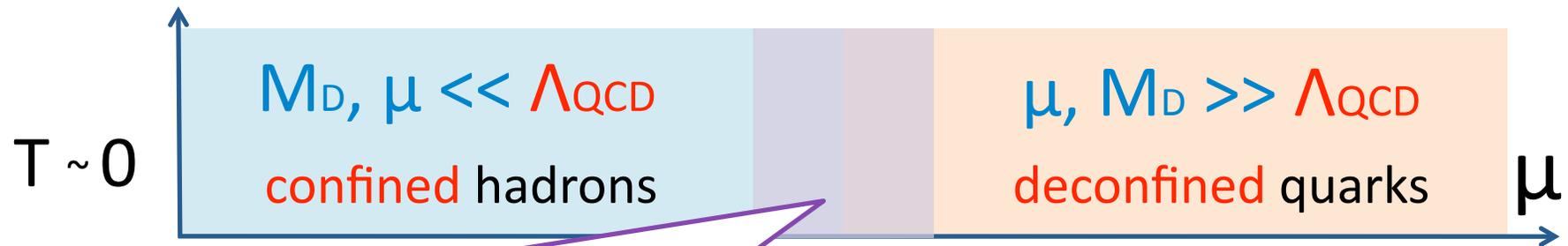


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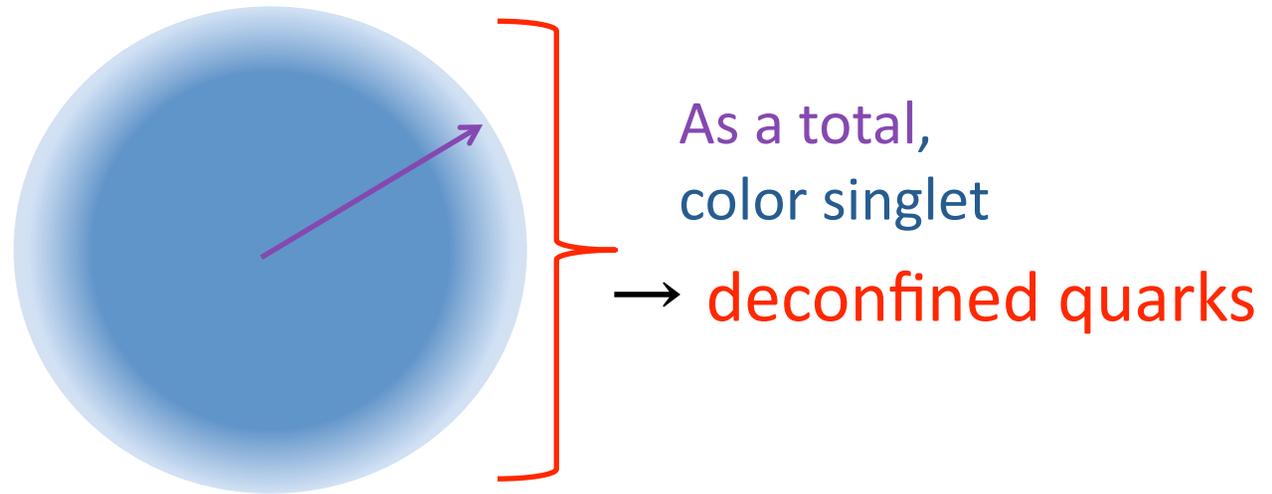
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$M_D \ll \Lambda_{\text{QCD}} \ll \mu$   
quark Fermi sea with confined excitations

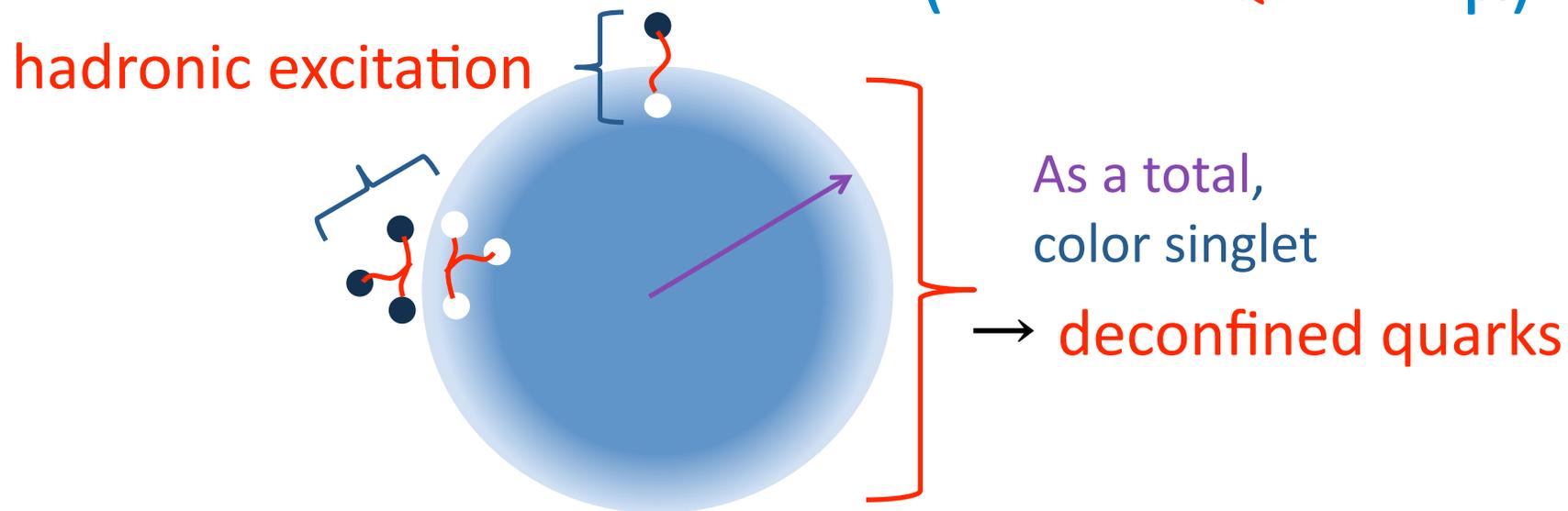
# Quarkyonic Matter

McLerran & Pisarski (2007) ( $M_D \ll \Lambda_{\text{QCD}} \ll \mu$ )



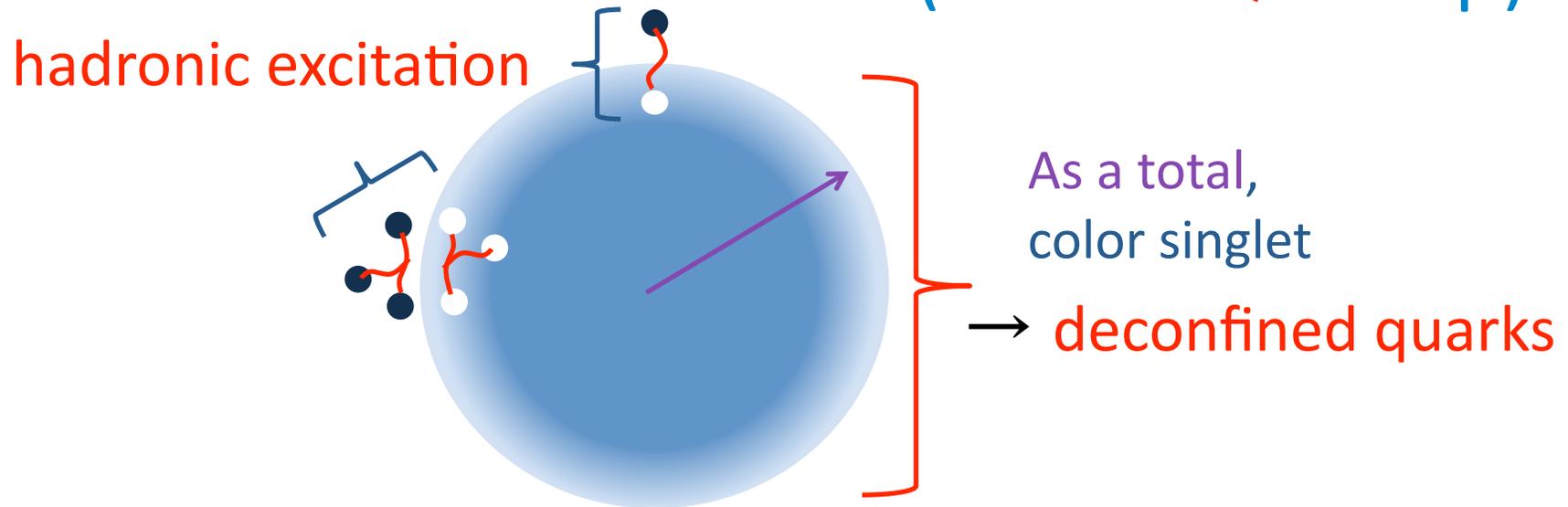
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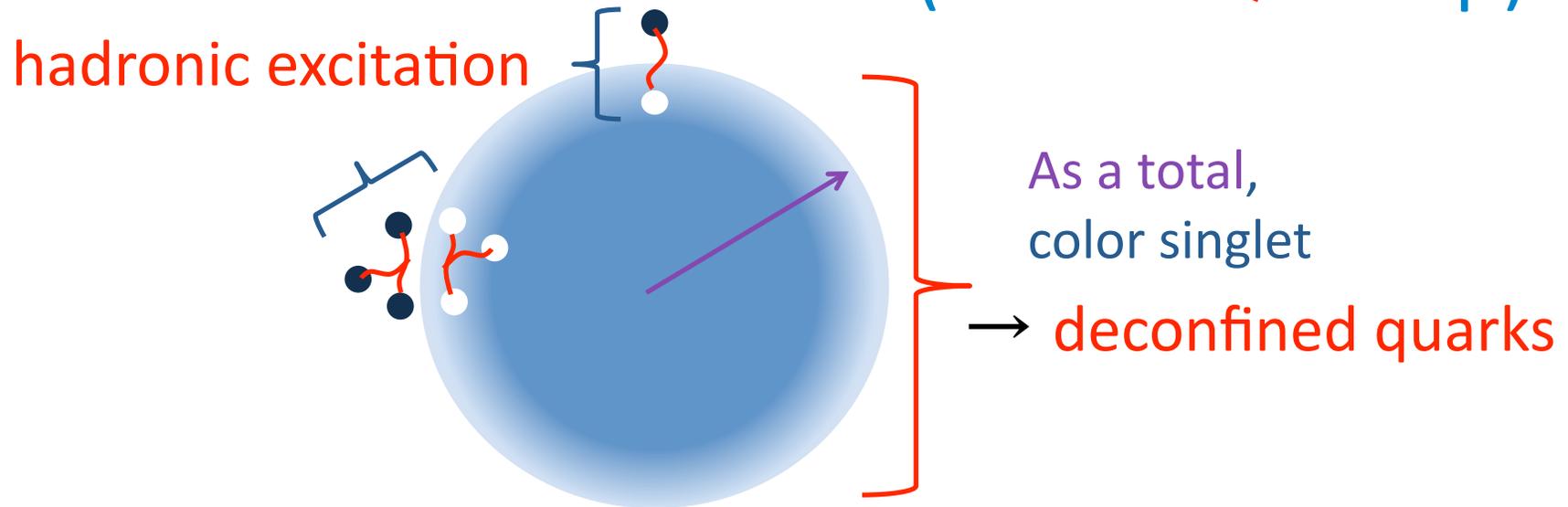
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Quark Fermi sea + baryonic Fermi surface → Quarkyonic

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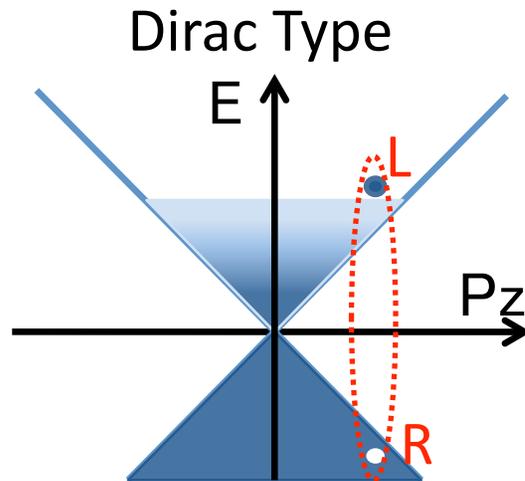
▪ Large  $N_c$ :  $M_D \sim N_c^{-1/2} \rightarrow 0$

Quarkyonic regime always holds.

(so we can use **vacuum** gluon propagator )

# Chiral Pairing Phenomena

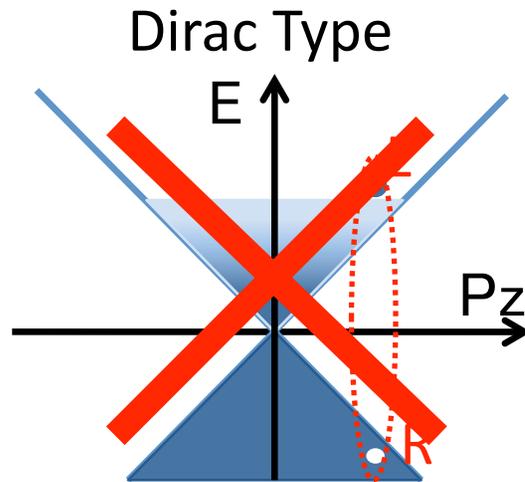
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$P_{\text{Tot}}=0$  (uniform)

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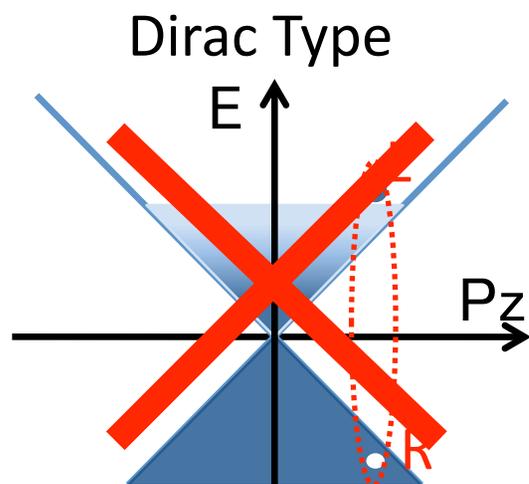


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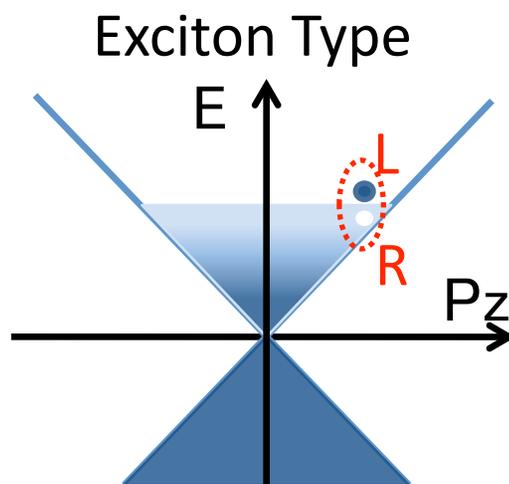
It costs large energy,  
so does not occur **spontaneously**.

# Chiral Pairing Phenomena

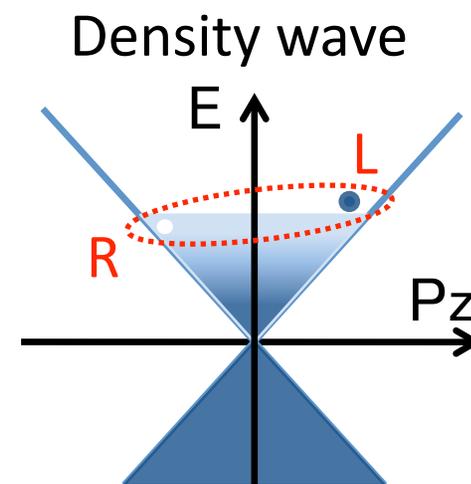
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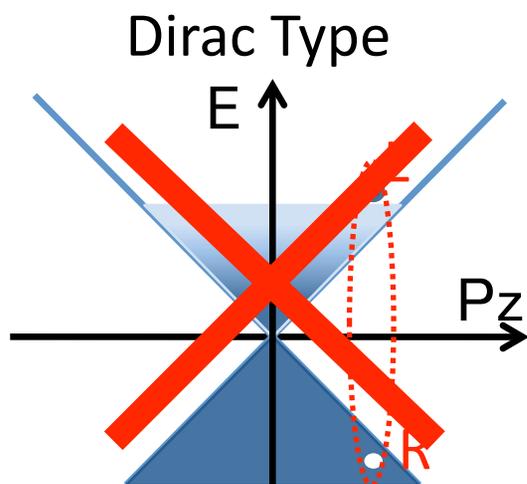
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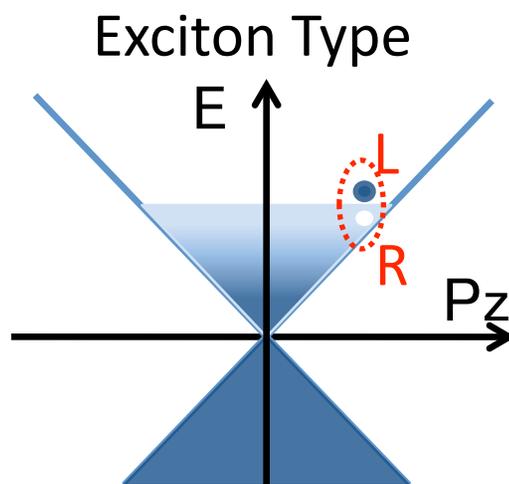
$P_{\text{Tot}}=2\mu$  (nonuniform)

# Chiral Pairing Phenomena

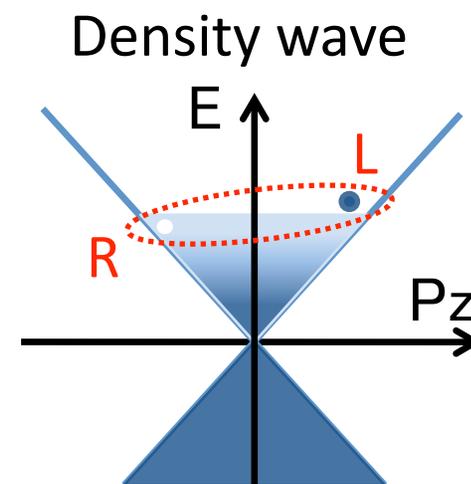
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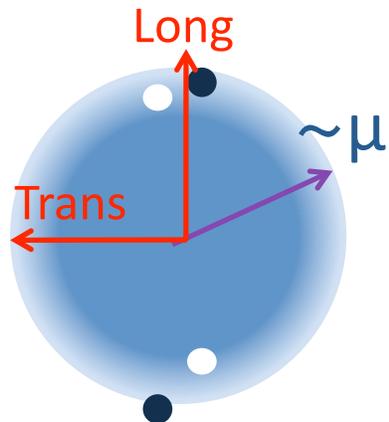
$P_{\text{Tot}}=0$  (uniform)



$P_{\text{Tot}}=0$  (uniform)



$P_{\text{Tot}}=2\mu$  (nonuniform)



We will identify the most relevant pairing:  
**Exciton & Density wave** solutions  
 will be treated and compared simultaneously.

# Set up of the problem

- Confining propagator for quark-antiquark:

$$D_{\mu\nu} = C_F \times g_{\mu 0} g_{\nu 0} \times \frac{\sigma}{(\vec{p}^2)^2} \quad (\text{linear rising type})$$

cf) leading part of **Coulomb** gauge propagator (ref: Gribov, Zwanziger)

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$1/N_c \rightarrow 0$  : Vacuum propagator is not modified

(ref: Glozman, Wagenbrunn, PRD77:054027, 2008;  
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$\Lambda_{\text{QCD}}/\mu \rightarrow 0$  : Factorization approximation

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We will perform the dimensional reduction of nonperturbative self-consistent equations, **Schwinger-Dyson** & **Bethe-Salpeter** eqs.

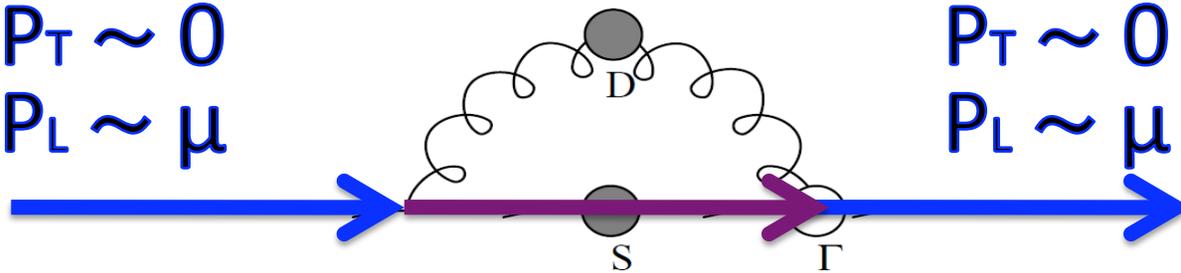
e.g.) Dim. reduction of Schwinger-Dyson eq. <sup>7/14</sup>

quark self-energy

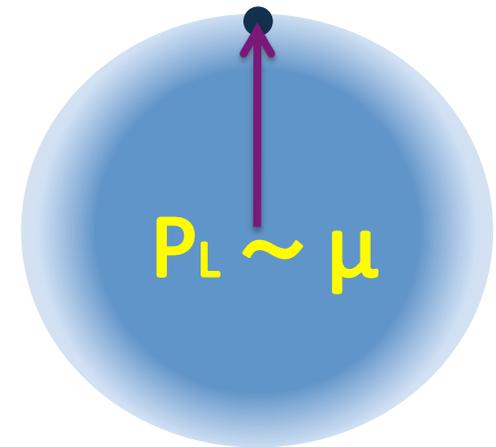
$$\not{Z}(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2 \vec{k}_T}{(2\pi)^4} \gamma_4 S(\vec{k}) \gamma_4 \frac{\sigma}{|\vec{p} - \vec{k}|^4}$$

including  $\Sigma$

$P_T \sim 0$   
 $P_L \sim \mu$



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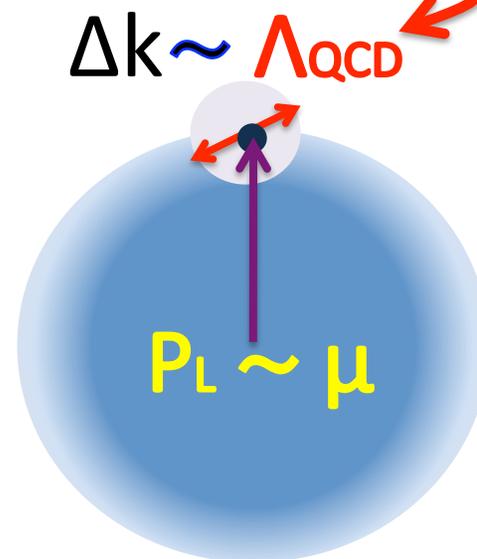
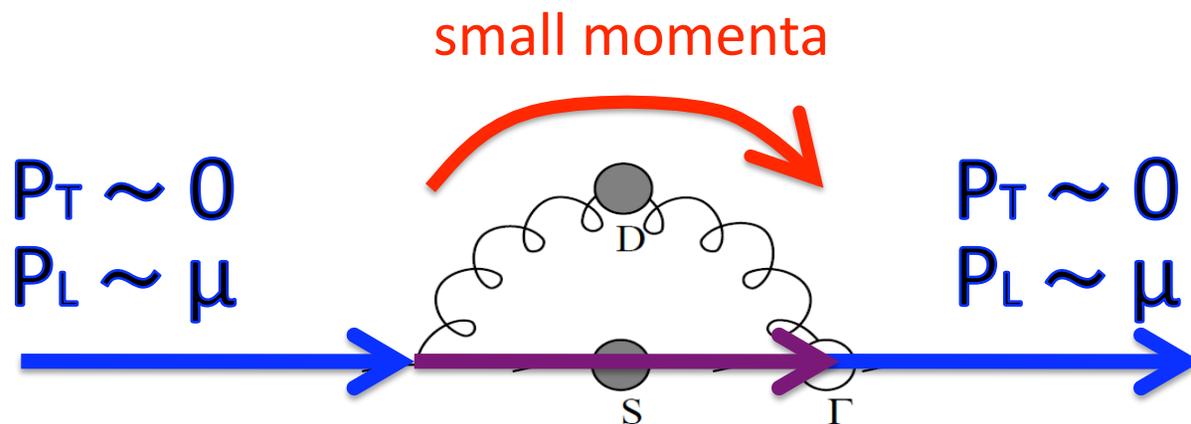
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quark self-energy

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- **Note1:** Mom. restriction from **confining** interaction.







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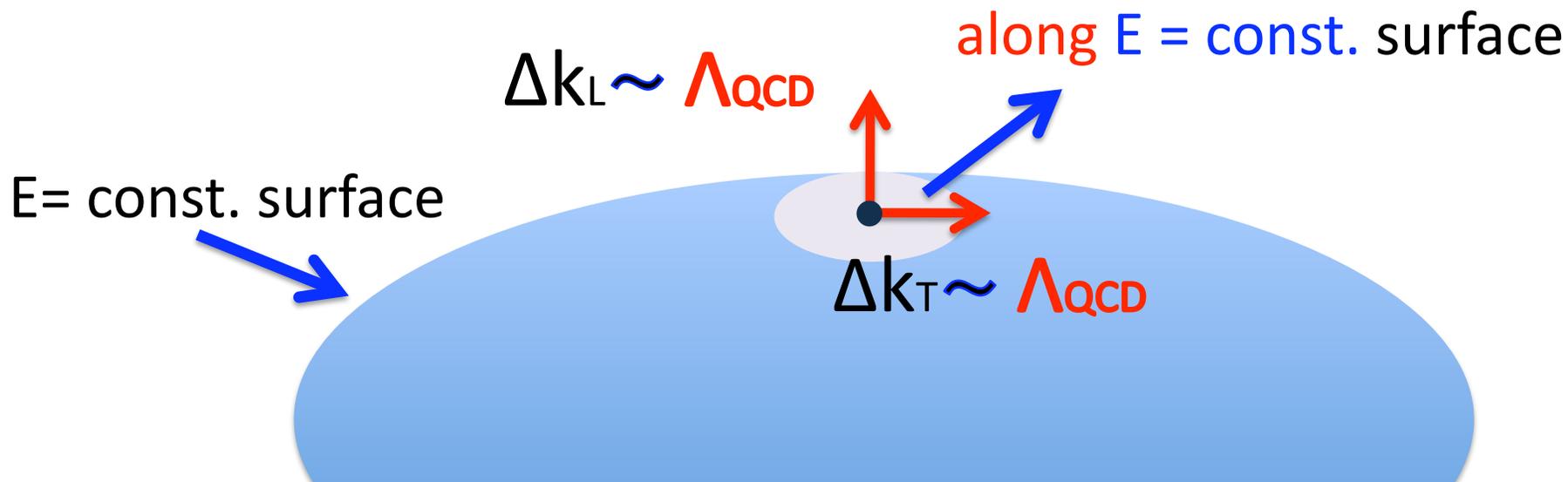
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- **Note2:** Suppression of **transverse** and **mass** parts:

$$S(k) = \gamma_0 S_0 - \gamma_z S_z - \vec{\gamma}_T \vec{S}_T + S_m$$

$\sim \mu$                        $\sim \Lambda_{\text{QCD}}$

- **Note3:** **quark energy** is **insensitive** to small change of  $k_T$ :



e.g.) Dim. reduction of Schwinger-Dyson eq. <sup>8/14</sup>

insensitive to kT

$$\not{Z}(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2 \vec{k}_T}{(2\pi)^4} \gamma_4 S(\vec{k}) \gamma_4 \frac{\sigma}{|\vec{p} - \vec{k}|^4}$$

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factorization

$$\gamma_4 \Sigma_4 + \gamma_z \Sigma_z = \int \frac{dk_4 dk_z}{(2\pi)^2} \gamma_4 S(k_4, k_z, \underline{\vec{0}_T}) \gamma_4 \otimes \int \frac{d\vec{k}_T}{(2\pi)^2} \frac{\sigma}{|\vec{p} - \vec{k}|^4}$$

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smearing

confining propagator in (1+1)D:

$$\frac{\sigma}{2\pi} \frac{1}{|p_z - q_z|^2}$$

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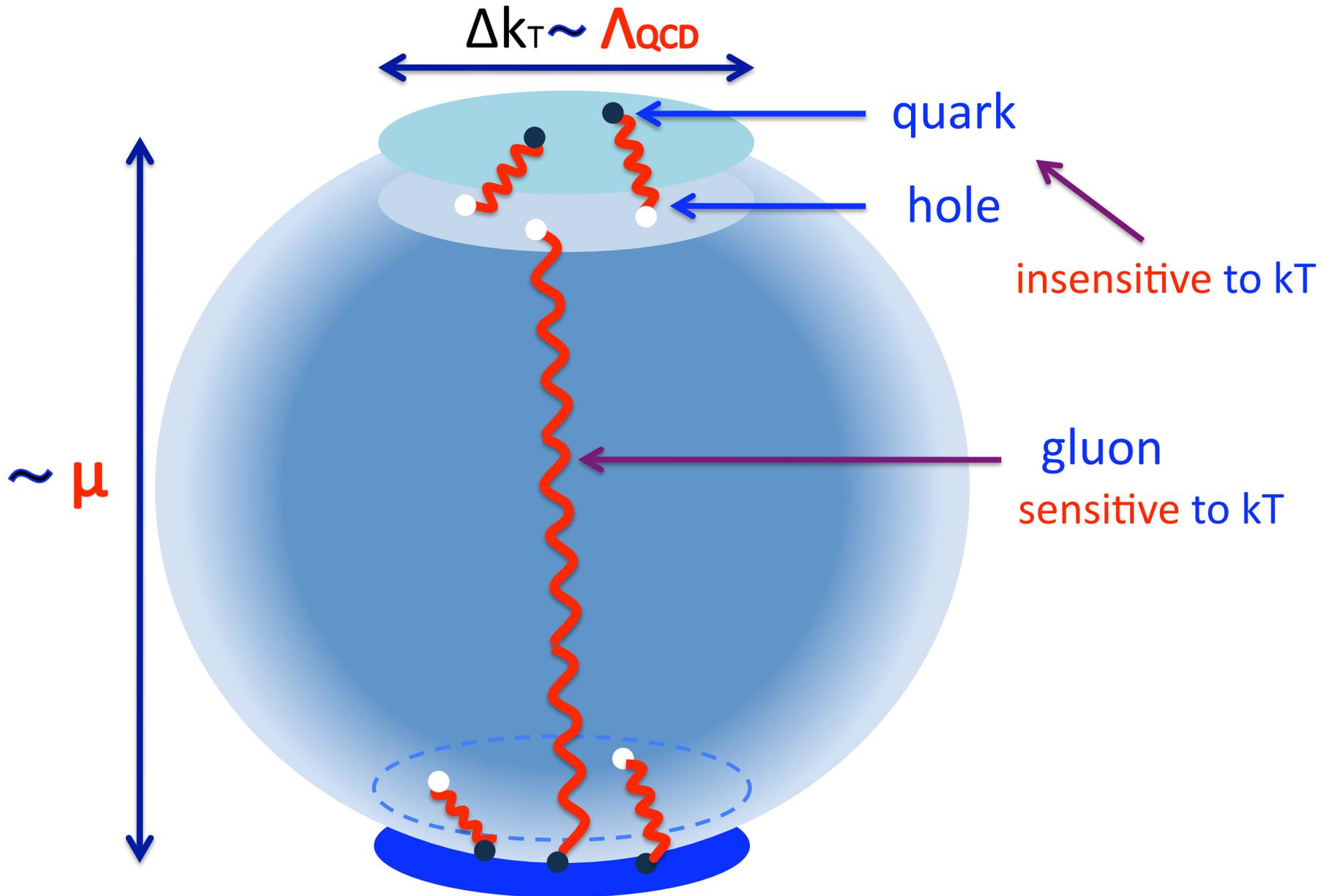
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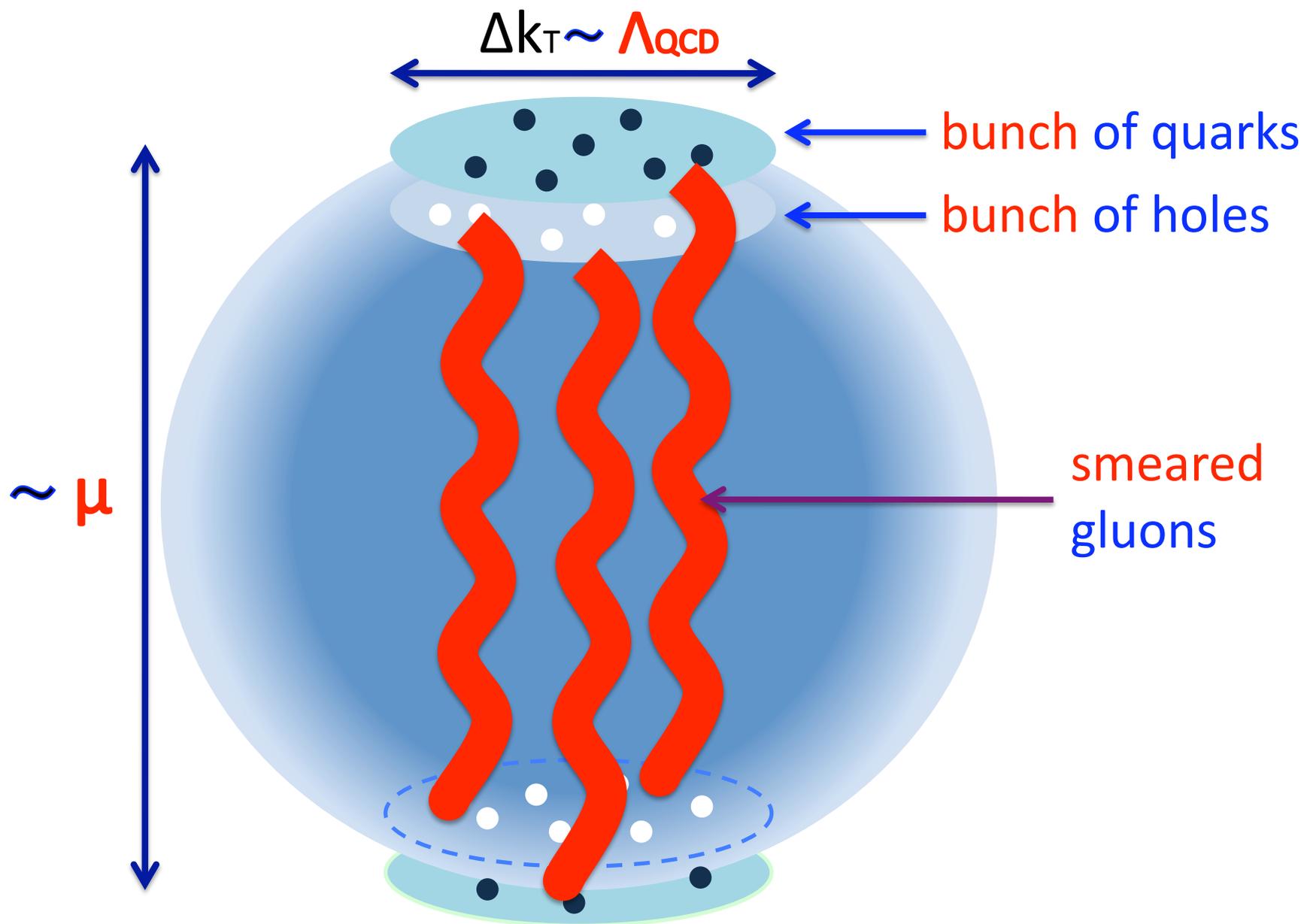
Schwinger-Dyson eq. in (1+1) D QCD in  $A_1=0$  gauge

Bethe-Salpeter eq. can be also converted to (1+1)D

# Catoon for Pairing dynamics before reduction



## 1+1 D dynamics of patches after reduction



# Flavor Doubling

- At leading order of  $1/N_c$  &  $\Lambda_{\text{QCD}}/\mu$

Dimensional reduction of Non-pert. self-consistent eqs:  
4D “QCD” in Coulomb gauge  $\longleftrightarrow$  2D QCD in  $A_1=0$  gauge  
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suppression of spin mixing

no angular d.o.f in (1+1) D

spin  $SU(2) \times SU(N_f)$   
 (3+1)-D side

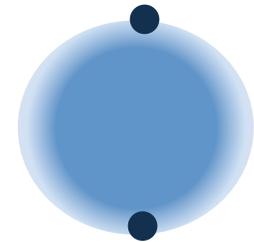


$SU(2N_f)$   
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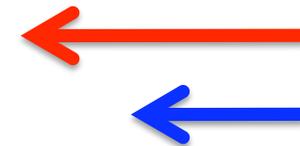
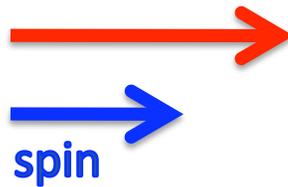
cf) Shuster & Son, NPB573, 434 (2000)

# Flavor Multiplet

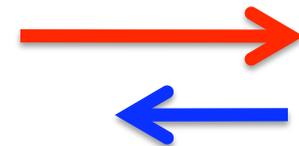
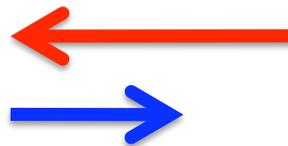
particle near north & south pole



R-handed

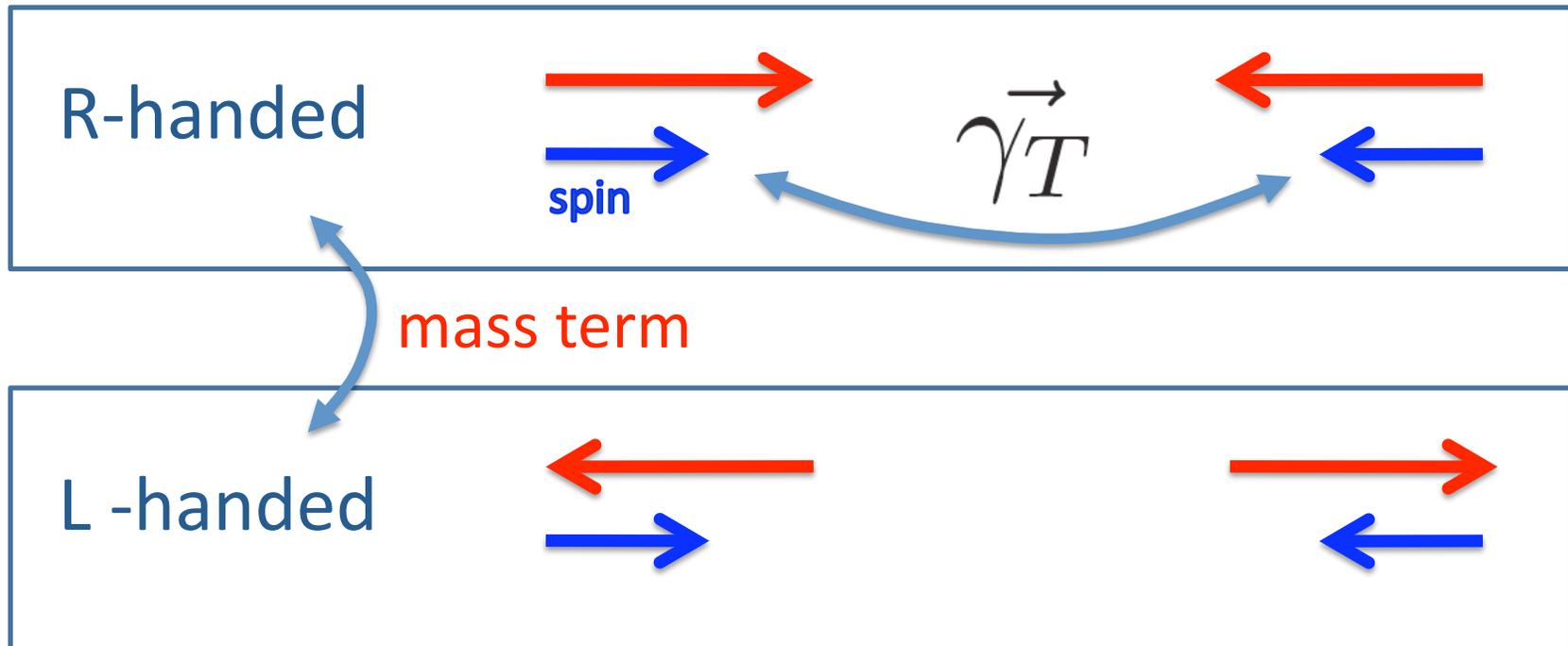
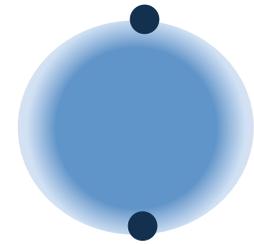


L-handed



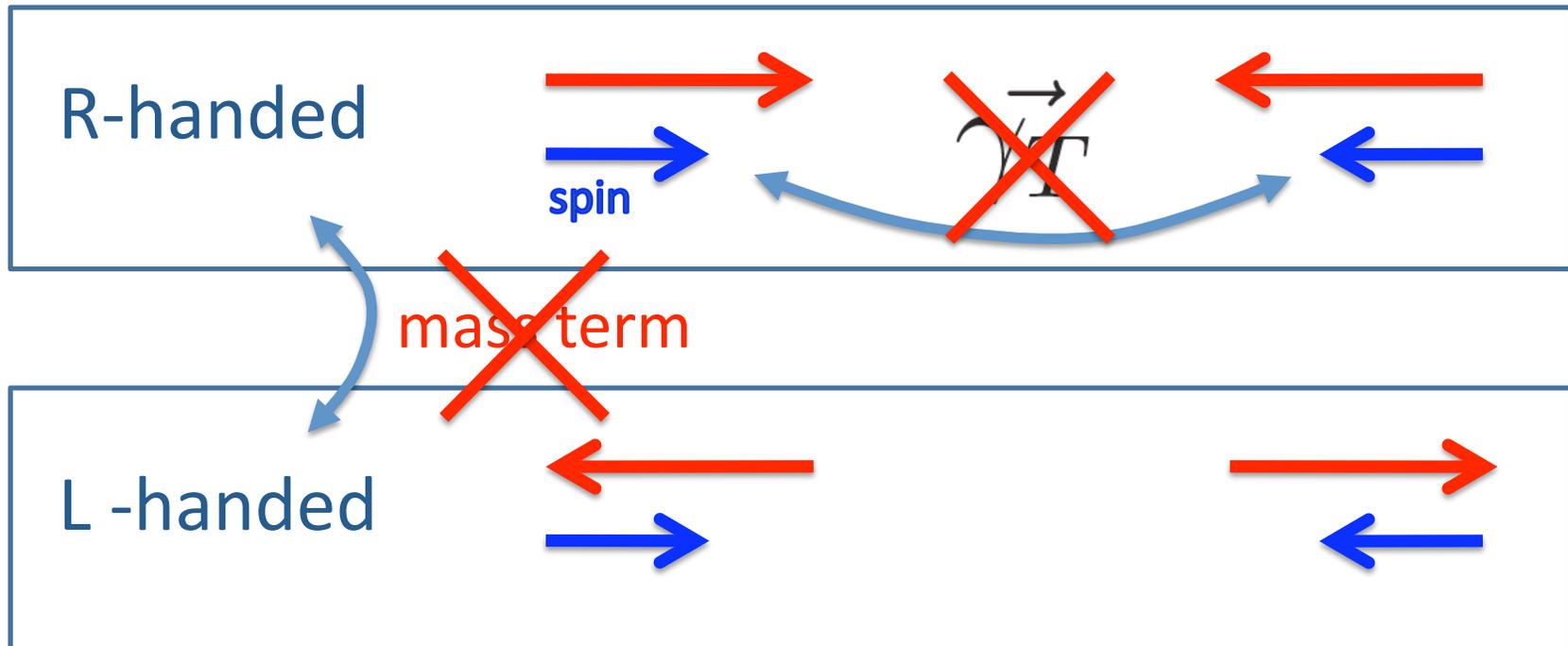
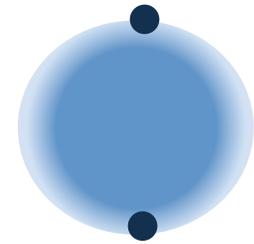
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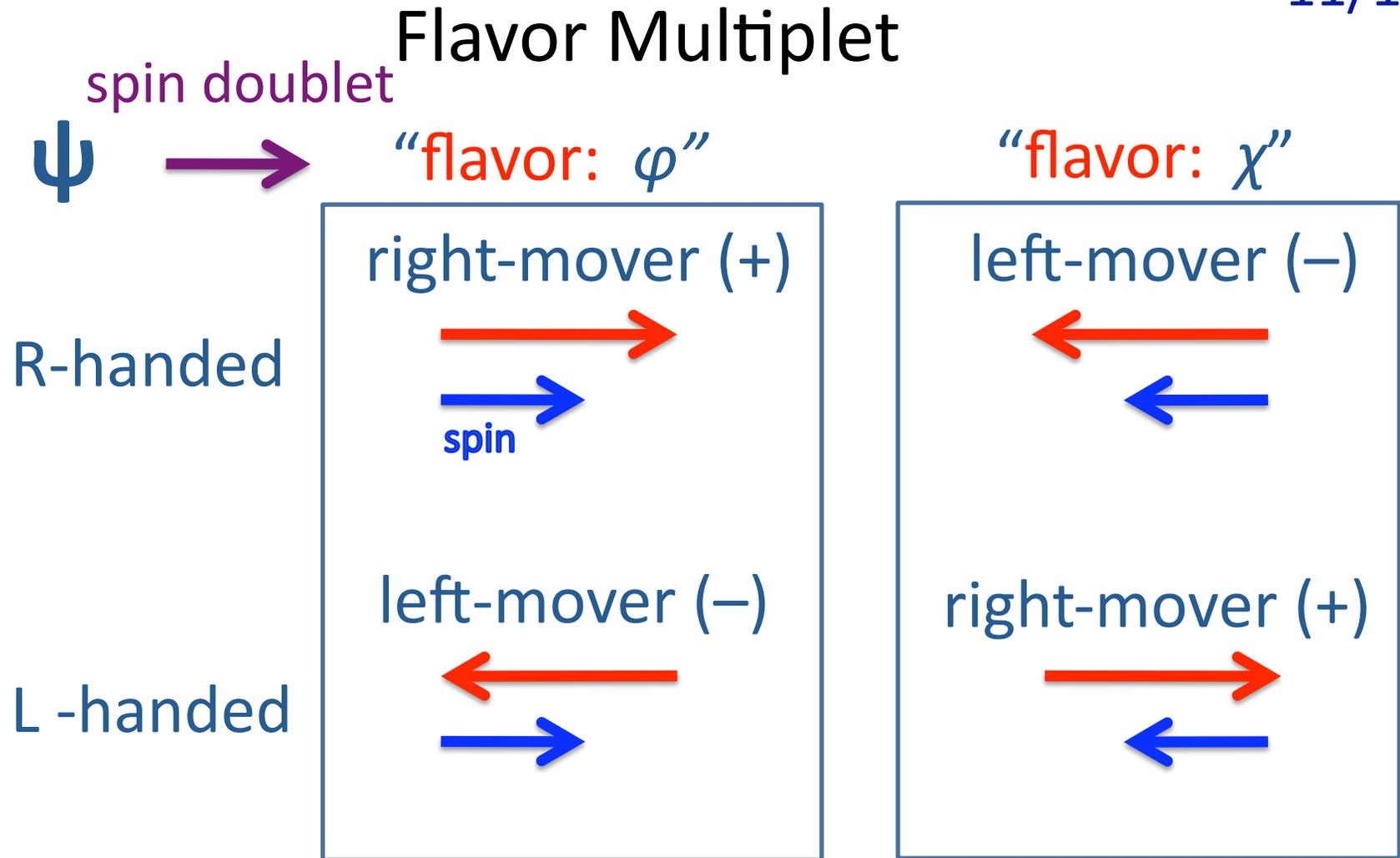
particle near north & south pole



# Flavor Multiplet

particle near north & south pole





Moving direction: (1+1)D “chirality”

(3+1)D – CPT sym. directly convert to (1+1)D ones

# Relations between composite operators

- 1-flavor (3+1)D operators without spin mixing:

$$\bar{\psi}\psi$$



$$\bar{\Psi}\Psi$$

$$\bar{\psi}\gamma^0\psi$$



$$\bar{\Psi}\Gamma^0\Psi$$

$$\bar{\psi}\gamma^z\psi$$



$$\bar{\Psi}\Gamma^z\Psi$$

$$\bar{\psi}\gamma^0\gamma^z\psi$$



$$\bar{\Psi}\Gamma^5\Psi$$

Flavor singlet in (1+1)D

# Relations between composite operators

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$$\begin{array}{cccc}
 \bar{\psi}\psi & \bar{\psi}\gamma^0\psi & \bar{\psi}\gamma^z\psi & \bar{\psi}\gamma^0\gamma^z\psi \\
 \downarrow & \downarrow & \downarrow & \downarrow \\
 \bar{\Psi}\Psi & \bar{\Psi}\Gamma^0\Psi & \bar{\Psi}\Gamma^z\Psi & \bar{\Psi}\Gamma^5\Psi
 \end{array}$$

Flavor singlet in (1+1)D

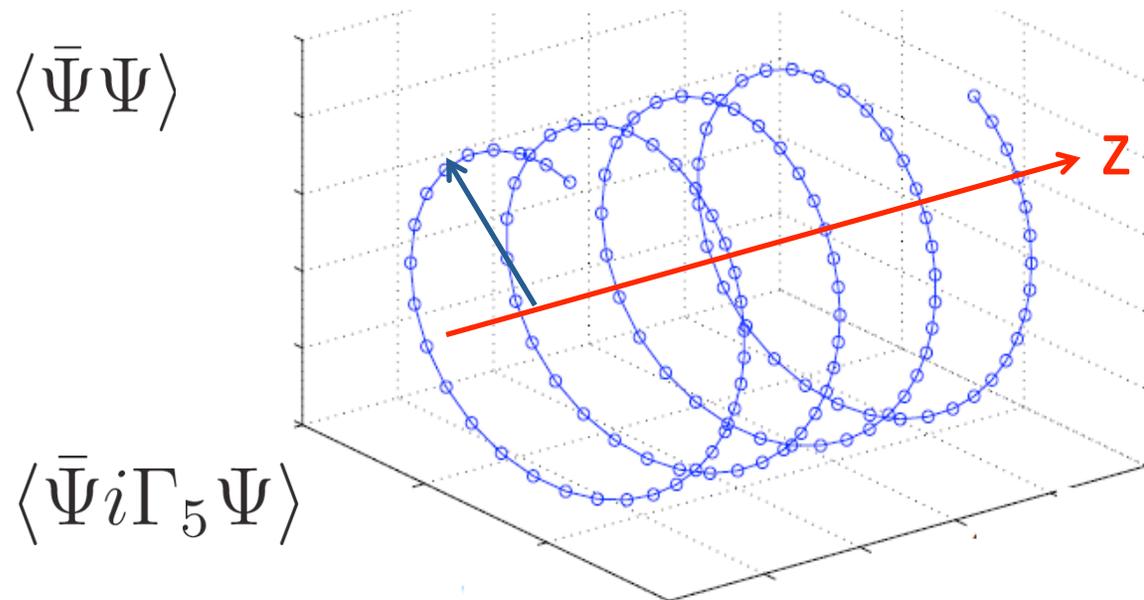
- All others have spin mixing:

$$\begin{array}{l}
 \text{ex) } \bar{\psi}\gamma^5\psi \longrightarrow \bar{\Psi}\tau_3\Psi \\
 \bar{\psi}\gamma^1\psi \longrightarrow \bar{\Psi}\tau_2\Psi
 \end{array}
 \quad \text{(They will show no flavored condensation)}$$

Flavor non-singlet in (1+1)D

# Chiral Spirals in (1+1)D

- At  $\mu \neq 0$ : periodic structure (**crystal**) which **oscillates in space**.



- cf) **Chiral Gross Neveu model (with continuous chiral symmetry)**

Schon & Thies, hep-ph/0003195; 0008175; Thies, 06010243

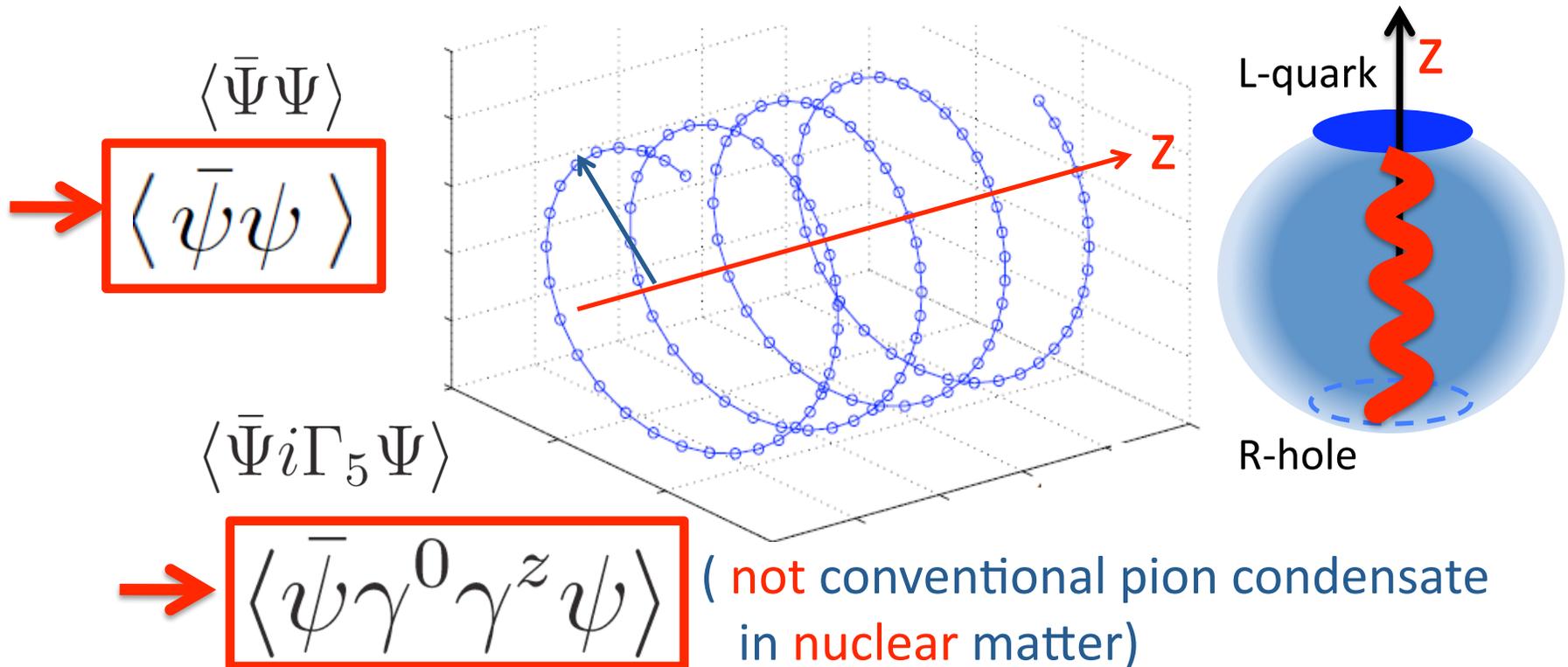
Basar & Dunne, 0806.2659; Basar, Dunne & Thies, 0903.1868

- 'tHooft model, massive quark (1-flavor)**

B. Bringoltz, 0901.4035

# Quarkyonic Chiral Spirals in (3+1)D

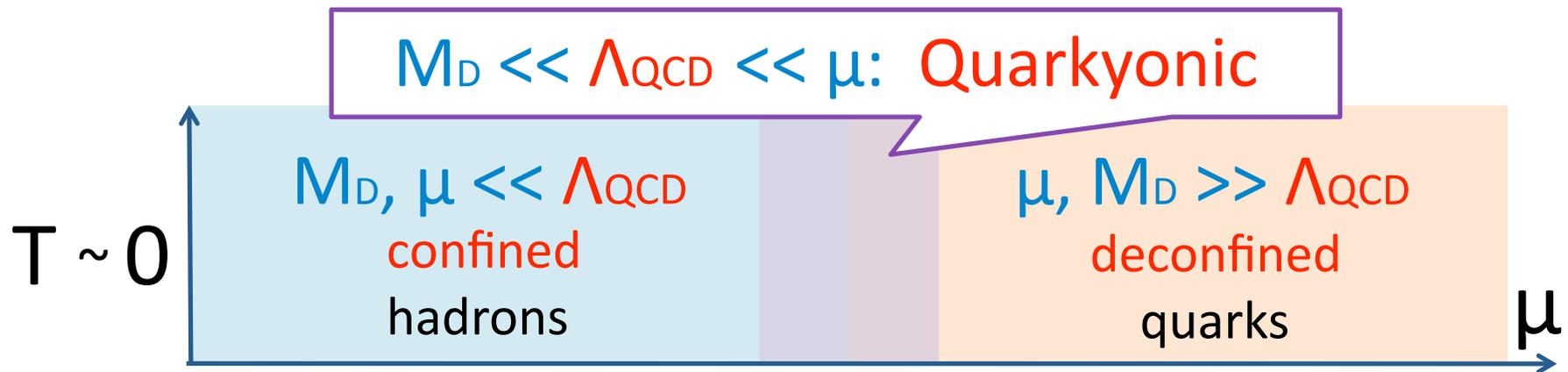
- Chiral rotation evolves in the longitudinal direction:



- Baryon number is spatially constant.
- No other condensates appear in quarkyonic limit.

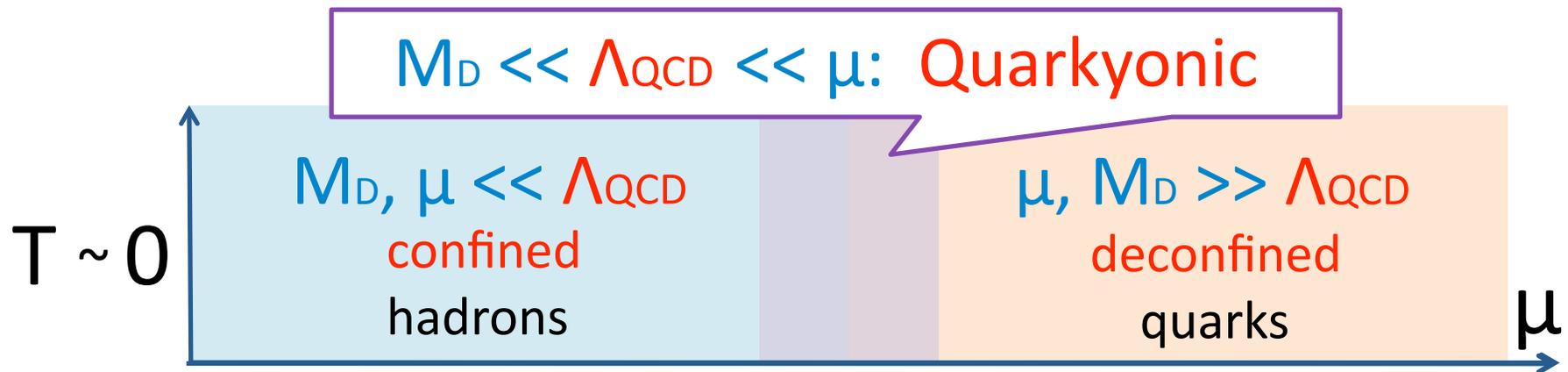
# Summary of large $N_c$ , low $T$ phase diagram

## Confining aspects

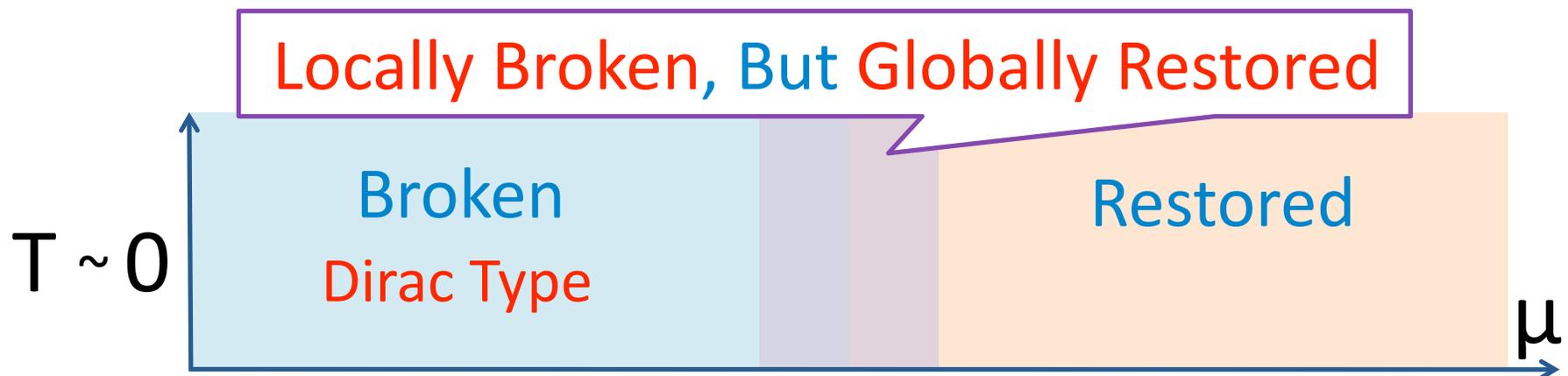


# Summary of large $N_c$ , low $T$ phase diagram

## Confining aspects



## Chiral aspects

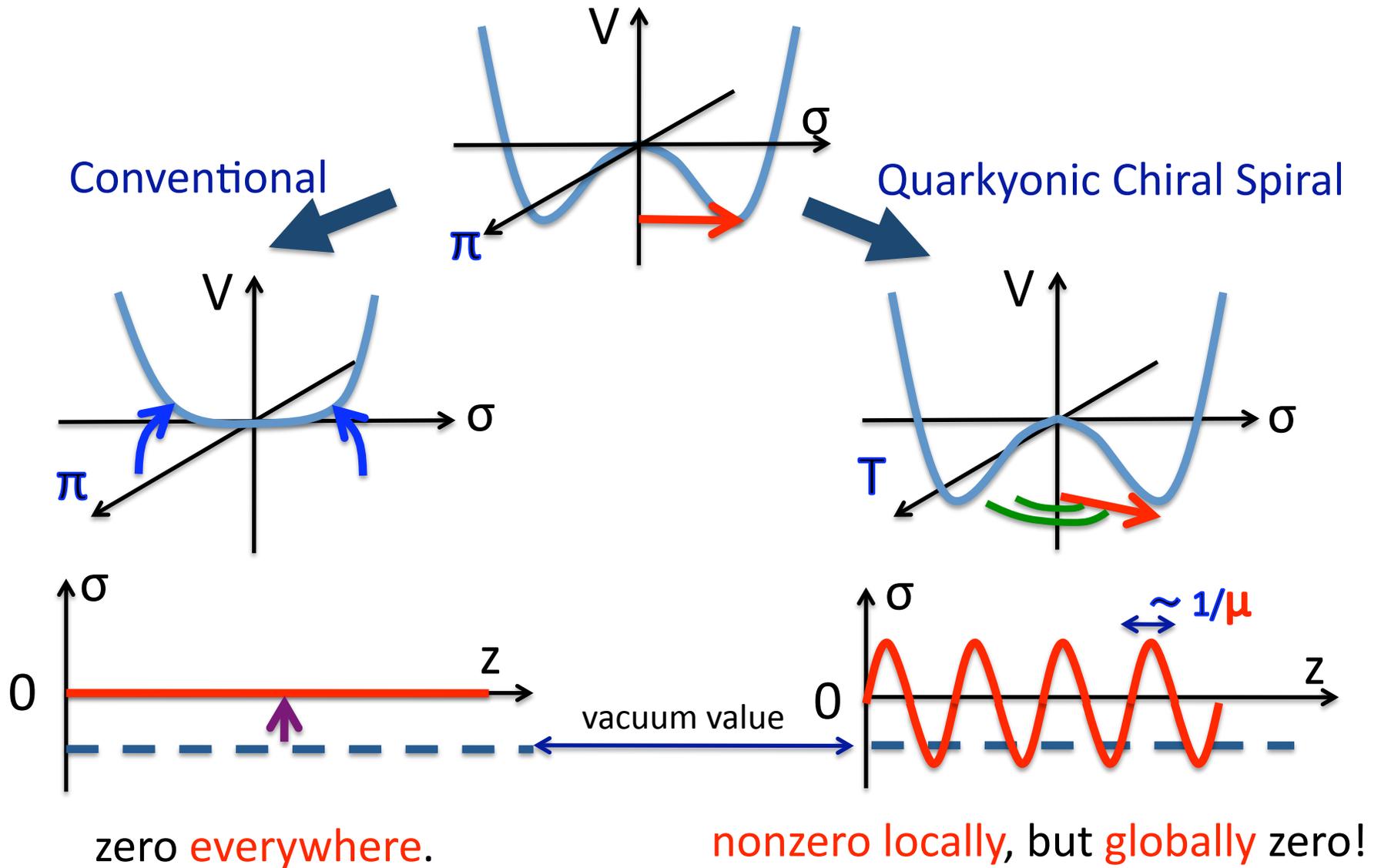


# Topics not discussed in this talk

(Please ask in discussion time or personally during workshop)

- Origin of self-energy divergences and how to avoid it.
  - Quark pole need not to disappear in linear confinement model.
- Explicit example of quarkyonic matter:
  - 1+1 D large  $N_c$  QCD has a quark Fermi sea but confined spectra.
- Beyond single patch pairing.
  - Issues on rotational invariance, working in progress.

# Summary: Chiral sym. breaking/restoration



# Why Chiral Rotation in (1+1)D ?

- Key observation: Moving direction = (1+1)D Chirality

$\varphi_+$	$P \sim \mu$	$P \sim -\mu$	$\varphi_-$
			
$\bar{\varphi}_-$			$\bar{\varphi}_+$
$\langle \bar{\varphi}_- \varphi_+ \rangle = \Delta e^{\frac{2i\mu z}{}}$		$\langle \bar{\varphi}_+ \varphi_- \rangle = \Delta e^{-\frac{2i\mu z}{}}$	

Opposite phase

$$\rightarrow \langle \bar{\varphi} \Gamma_5 \varphi \rangle = \langle \bar{\varphi}_- \varphi_+ \rangle - \langle \bar{\varphi}_+ \varphi_- \rangle = \Delta i \sin 2\mu z \neq 0$$

Density wave of  $\bar{\Psi} \Psi$  inevitably accompanies  $\bar{\Psi} \Gamma^5 \Psi$

(1+1)D: Chiral Density wave  $\rightarrow$  Chiral Spiral

## 2<sup>nd</sup> Dictionary: $\mu = 0$ & $\mu \neq 0$ in (1+1)D

- $\mu \neq 0$  2D QCD can be mapped onto  $\mu = 0$  2D QCD

$$\Psi' = e^{i\mu z \Gamma_5} \Psi \quad : \text{chiral rotation (or boost)}$$

$$\bar{\Psi} \left[ i\Gamma^\mu \partial_\mu + \mu \Gamma^0 \right] \Psi \rightarrow \bar{\Psi}' i\Gamma^\mu \partial_\mu \Psi'$$

$(\mu \neq 0)$ 
 $(\mu = 0)$

(due to special geometric property of 2D Fermi sea)

- Dictionary between  $\mu = 0$  &  $\mu \neq 0$  condensates:

$$\mu = 0$$

$$\mu \neq 0$$

$$\langle \bar{\Psi}' \Psi' \rangle \rightarrow \langle \bar{\Psi} \Psi \rangle \cos(2\mu z) + \langle \bar{\Psi} i\Gamma_5 \Psi \rangle \sin(2\mu z)$$

$$\langle \bar{\Psi}' \Gamma^0 \Psi' \rangle \rightarrow \langle \bar{\Psi}' \Gamma^0 \Psi' \rangle + \frac{\mu}{2\pi}$$

$(= 0)$ 
 $(= 0)$

induced by anomaly  
 “correct baryon number”

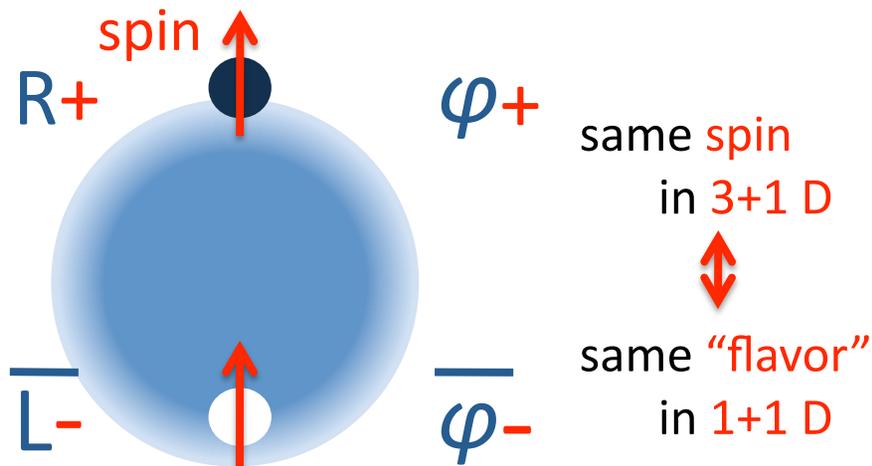
# Chiral Density Wave VS Chiral Exciton

IF dimensionally reduced models respect “flavor” symmetry

→ there is **no** chiral exciton condensates:

## Chiral Density Wave

→  $R_+$   $\bar{L}_-$  pairing

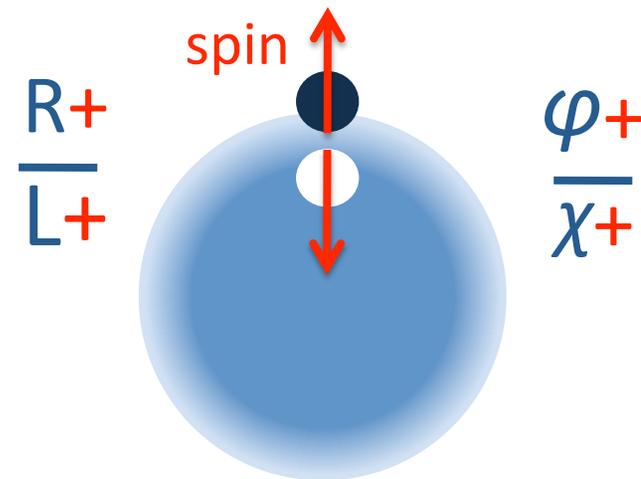


Possible (1+1)D flavor singlet

$$\bar{\Psi}\Psi \quad \bar{\Psi}\Gamma^5\Psi$$

## Chiral Exciton

→  $R_+$   $\bar{L}_+$  pairing



No flavor singlet pairing

→ No chiral exciton!

# Preceding works on the chiral density waves

(Many works, so incomplete list)

- **Nuclear matter or Skyrme matter:**

Migdal '71, Sawyer & Scalapino '72.. : effective lagrangian for nucleons and pions

- **Quark matter:**

- Perturbative regime with Coulomb type gluon propagator:**

- Deryagin, Grigoriev, & Rubakov '92: Schwinger-Dyson eq. in large  $N_c$

- Shuster & Son, hep-ph/9905448: Dimensional reduction of Bethe-Salpeter eq.

- Rapp, Shuryak, and Zahed, hep-ph/0008207: Schwinger-Dyson eq.

- Effective model:**

- Nakano & Tatsumi, hep-ph/0411350, D. Nickel, 0906.5295

- Nonperturbative regime with linear rising gluon propagator:**

- Present work

## 4, A closer look at QCS

Coleman's theorem & Strong chiral phase fluctuations

# Excitation modes in quarkyonic limit

- **Fermionic** action for (1+1)D QCD:

$$S = \int d^2x [\Psi_+ i\partial_- \Psi_+ + \Psi_- i\partial_+ \Psi_-] + \text{gauge int.}$$

- **Bosonized** version:

U(1) free bosons & Wess-Zumino-Novikov-Witten action :

(Non-linear  $\sigma$  model + Wess-Zumino term)

“Charge – Flavor – Color Separation”

$$S = \underbrace{S_{U(1)}[\phi] + S_{k=N_c}^{flavor}[g]}_{\text{conformal inv.}} + \underbrace{S_{k=N_f}^{color}[h]}_{\text{dimensionful}} + \text{gauge int.}$$

conformal inv.

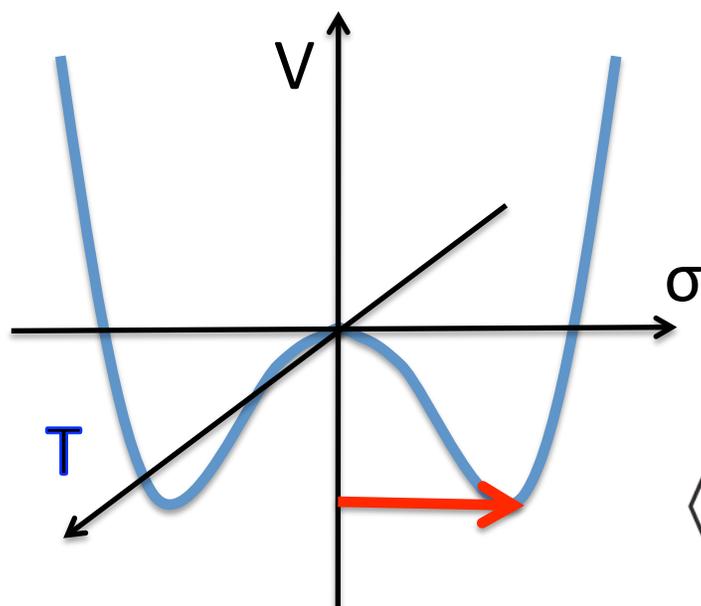
dimensionful

$4N_f^2$  gapless phase modes

gapped phase modes

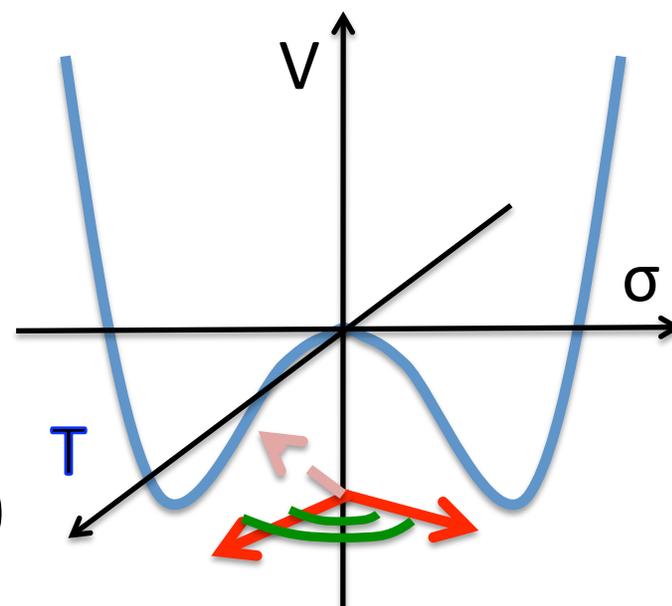
# Coleman's theorem ?

- Coleman's theorem: No **Spontaneous** sym. breaking in 2D



$$\langle e^{i\theta} \rangle \neq 0 \text{ (SSB)}$$

$$\langle \rho \rangle \neq 0$$



$$\langle e^{i\theta} \rangle = 0 \text{ (No SSB)}$$

IR divergence in (1+1)D  
phase dynamics

- Phase fluctuations belong to:

Excitations  
(physical pion spectra)

ground state properties  
(No pion spectra)

## 2, How to solve

Dimensional reduction from  $(3+1)D$  to  $(1+1)D$

# Quasi-long range order & large $N_c$

- Local order parameters:

$$\langle \bar{\Psi}_+ \Psi_- \rangle \sim \langle e^{i\sqrt{4\pi/N_c N_f} \phi} \rangle \otimes \langle \text{tr} g \rangle \otimes \langle \text{tr} h \rangle$$

gapless modes
gapped modes

0
0
finite

due to IR divergent phase dynamics

But this does **not** mean the system is in the usual **symmetric** phase!

- Non-Local order parameters:

$$\langle \bar{\Psi}_+ \Psi_-(x) \bar{\Psi}_- \Psi_+(0) \rangle \sim$$

(including **disconnected** pieces)

$$\left\{ \begin{array}{l} \cancel{e^{-m|x|}} : \text{symmetric phase} \\ \cancel{\langle \bar{\Psi}_+ \Psi_- \rangle^2} : \text{long range order} \\ |x|^{-\underline{C/N_c}} : \text{quasi-long range order} \\ \text{(power law)} \end{array} \right.$$

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# How neglected contributions affect the results?

- Neglected contributions in the dimensional reduction:

$$S(k) = \gamma_0 S_0 - \gamma_z S_z - \vec{\gamma}_T \vec{S}_T + S_m$$

$\underbrace{\quad}_{(3+1)\text{D}}$ 
 $\underbrace{\quad}_{\sim \mu}$ 
 $\underbrace{\quad}_{(1+1)\text{D}}$ 
 $\underbrace{\quad}_{\sim \Lambda_{\text{QCD}}}$

spin mixing → breaks the flavor symmetry **explicitly**

mass term → acts as mass term

## Expectations:

- Explicit breaking regulate the IR divergent phase fluctuations, so that quasi-long range order becomes long range order.
- Perturbation effects get smaller as  $\mu$  increases, but still introduce **arbitrary small explicit breaking**, which stabilizes quasi-long range order to long range order.

(As for **mass** term, this is confirmed by Bringoltz analyses for **massive** 'tHooft model.)

**Final results should be closer to our large  $N_c$  results!**

## 3, Two Dictionaries

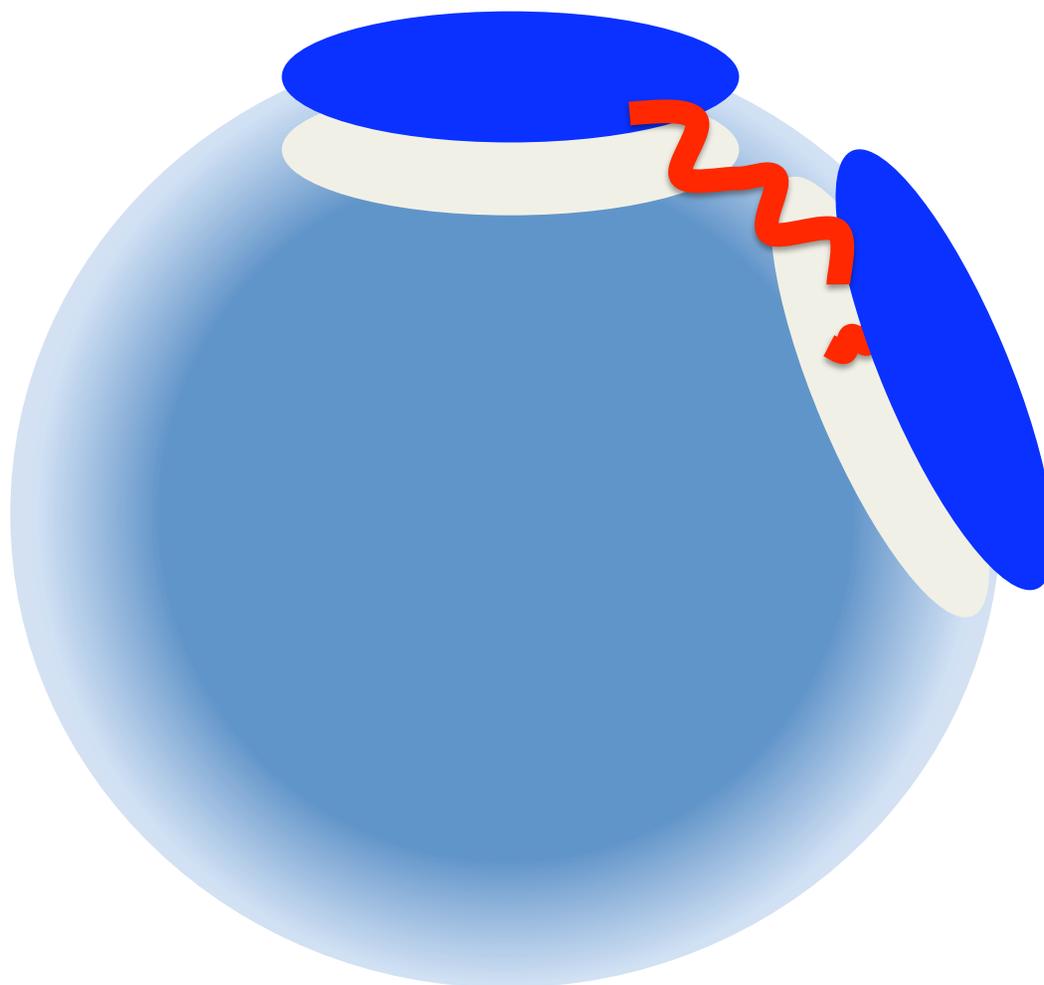
- between (3+1)D & (1+1)D quantum numbers
- between  $\mu = 0$  &  $\mu \neq 0$  condensates

# 1, Introduction

Quarkyonic matter, chiral pairing phenomena

In this work, we will **not** discuss the interaction between **different patches** **except** those at the **north** or **south** poles.

(Since we did not find satisfactory treatments)



## Confining aspects

$M_D \ll \Lambda_{\text{QCD}} \ll \mu$ : Quarkyonic



## Chiral aspects

Locally Broken, But Globally Restored



# Quarkyonic Matter near $T=0$

- **Bulk** properties: deconfined quarks in Fermi sea  
(All quarks contribute to Free energy, pressure, etc. )
- **Phase structure:** degrees of freedom near the Fermi surface  
cf) Superconducting phase is determined by dynamics near the Fermi surface.

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Is chiral symmetry broken in a Quarkyonic phase ?  
If so, how ?