Cold Quark Matter

or Neutron stars to 3 loops

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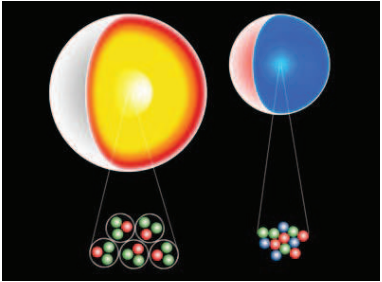
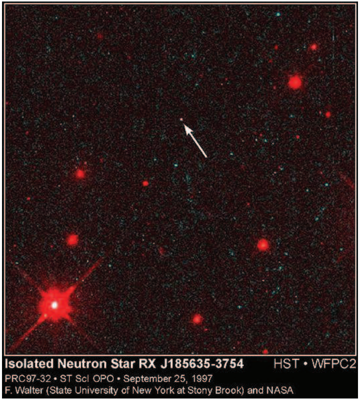
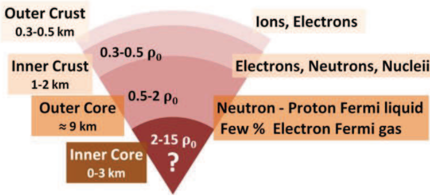
Compact stars

- Masses $\lesssim 2.0 M_\odot$
- Radii ~ 15km
- $T <$ KeV
- $n \lesssim 15\rho_0 \quad (\rho_0 = 0.16\mathrm{fm}^{-3})$
- $\mu_B \lesssim 1.6$GeV







Outline

How can we apply the perturbative $P(\mu_B, B, \bar{\Lambda}, m_s)$ to the compact stars?

1. Stability of hadrons against strange quark matter
   - How unstable am I?
2. Quark-Hadron phase transition
   - Hybrid equations of state with $P_{hadronic}(\mu_{pt}) = P_{quark}(\mu_{pt})$
3. Mass-Radius relationships from TOV equation
   - Connection to observations

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Strange quark matter hypothesis

If the energy per baryon in quark matter is less than

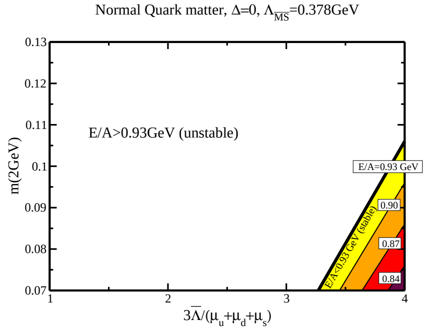
$$E/A = 3\mu_c = 0.93\mathrm{GeV} \qquad {}^{56}\mathrm{Fe}$$

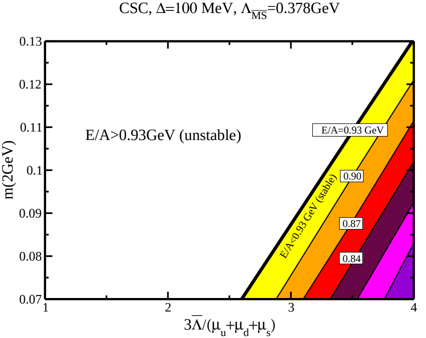
then quark matter is the true ground state $\rightarrow$ Nuclear matter metastable.

- Lifetime:
  - Nucleons $\rightarrow$ 2 Flavor quark matter:
    - Equilibration through Strong interaction $\longrightarrow t_{relax} \sim 1/\Lambda_{QCD}$
    - Short-lived. Ruled out by experiment!
  - Nucleons $\rightarrow$ 3 Flavor quark matter:
    - Equilibration through Weak Interactions (10^{60} years for $A > 6$)
    - Adding d.o.fs increases pressure $\rightarrow$ more likely to be stable
    - Experimentally plausible, let's find out what the theory says!

Strategy: Find out if SQM stable in the parameter space $(m_s, B, \bar{\Lambda})$ with

- $n_s > 0$ (quark mass extremely important!)
- $E/A < 0.31\text{GeV}$





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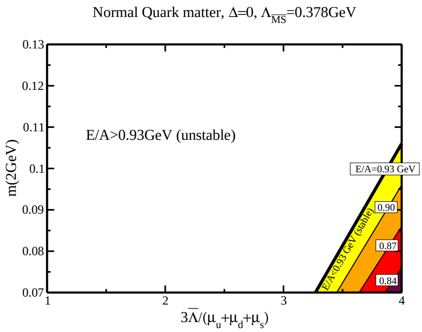

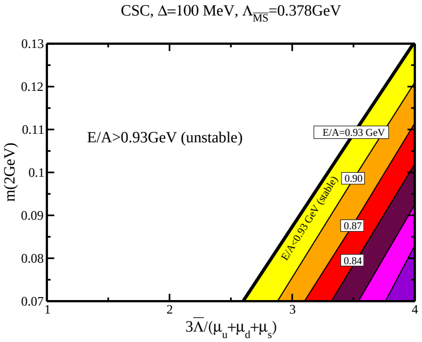

- Parameter space very hostile for $(\Delta = 0)$. Including CSC makes SQM more plausible
  $\longrightarrow$ Stable SQM disfavored but not ruled out.

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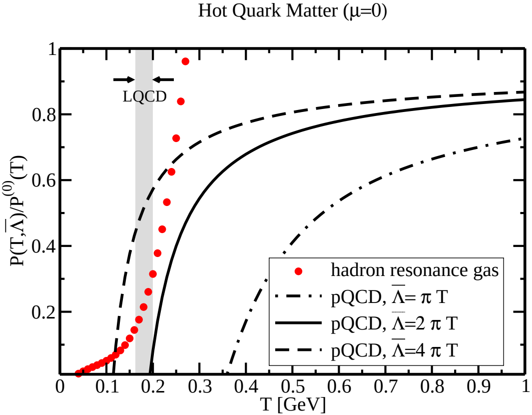
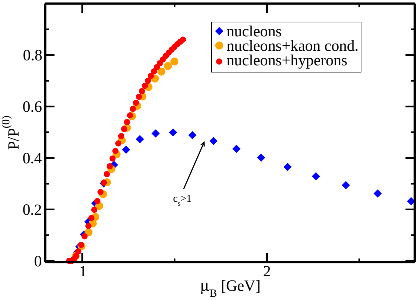
Quark-Hadron transition

Inspiration from finite T-case:



- Low-T: Hadron resonance gas
- High-T: Pert. theory

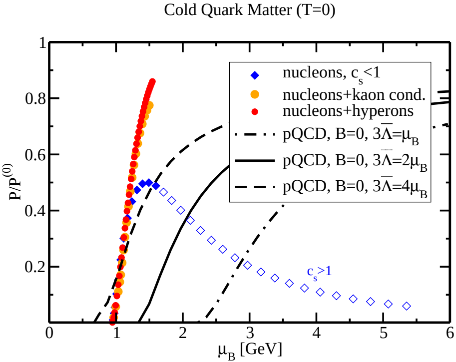
At low densities ($n_B \sim 0.16\mathrm{fm}^{-3}, \mu_B = 3\mu_q \sim 1\mathrm{GeV}$):



- Write down 2- and 3-body Hamiltonian for nuclei
- Match them to experimentally known potentials:
  - 2-body: Argonne v_{18}
  - 3-body: Urbana IX...
  - Properties of nuclei and hypernuclei ...
- Solve many-body Schrödinger equation for the EoS
- Hope for the best, fear for the worst...

If there are no exotic phases, there will be a phase transition between the hadronic and quark matter phases at some μ_{pt}.

- Catalog all possible EoS:s $(B, \bar{\Lambda})$ fulfilling:
  - Equal pressure for both phases at the phase transition
  - Monotonically increasing energy density



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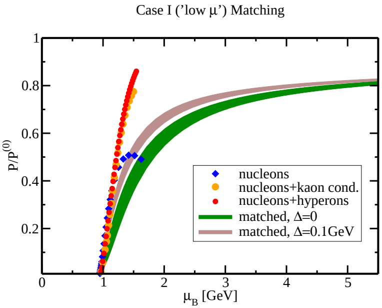
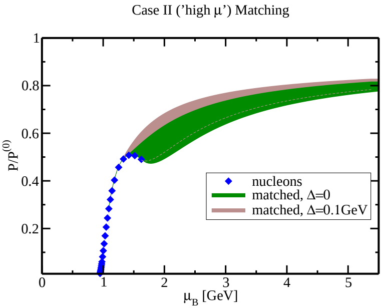
- Matching possible in two disjoint regimes:





$0.16\mathrm{fm}^{-3} < n_B \lesssim 0.32\mathrm{fm}^{-3}$ $\qquad$ $n_B > 0.64\mathrm{fm}^{-3}$

$\longrightarrow$ Represents the best educated guess available for the true EoS

- Matching reduces significantly the perturbative uncertainties
- The location of μ_{pt} *cannot* be further studied in PT

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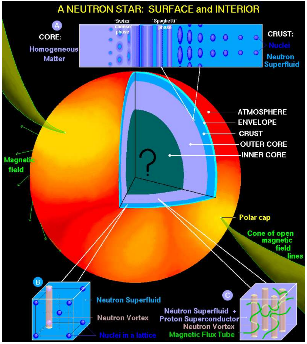
Compact stars

- The mass-radius relationship is very sensitive to the EoS
  - M-R relation given by TOV-equation:

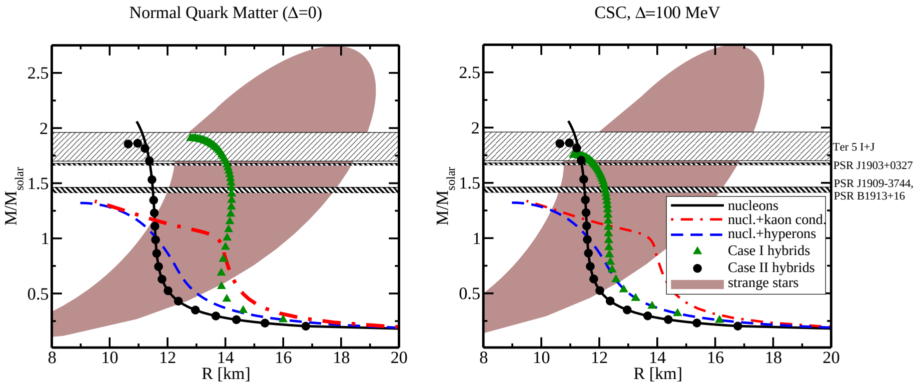
$$
\begin{aligned}
dM(r) &= 4\pi r^2 \varepsilon(r) dr, \\
dP(r) &= -\frac{G\left(P(r)+\varepsilon(r)\right)\left(M(r)+4\pi r^3 P(r)\right)}{r\left(r-2GM(r)\right)} dr,
\end{aligned}
$$

  - Takes $\varepsilon(p)$ as an input.

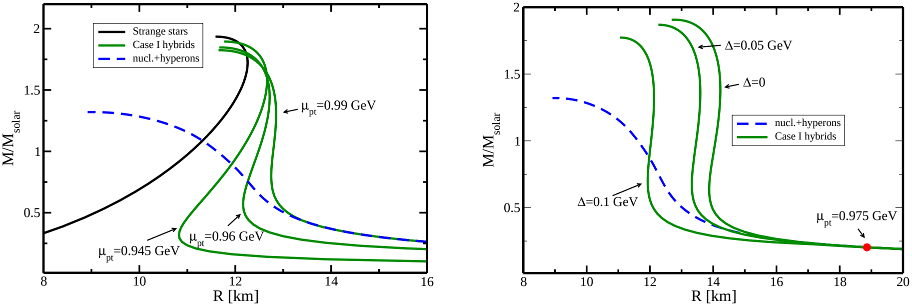


Let's consider compact stars:

- Made of nucleonic matter
- Made of pure quark matter
- Hybrid stars with
  - Mainly quark matter with thin nucleonic crust (Case I)
  - Mainly nucleonic matter with small quark core (Case II)

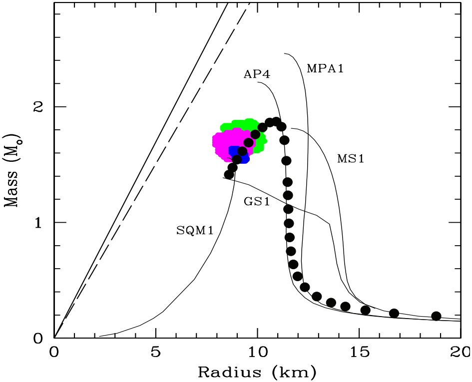


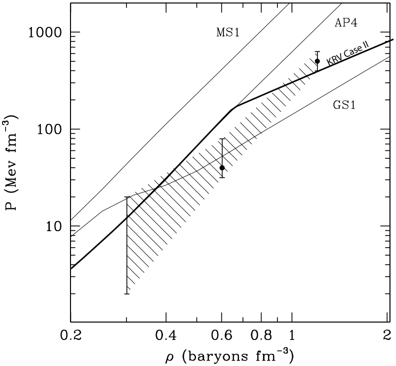
- Effect of CSC very small.
- Hyperonic/Kaonic EoS ruled out.
- For neutron and hybrid stars, $M \lesssim 2M_{\mathrm{solar}}$.
- Cannot exclude very large stars in case of stable strange quark matter
    - Dense quark stars ruled out!



- Maximal mass independent of the maching point
- CSC increases pressure $\longrightarrow$ smaller stars

Experimental data from last thursday!





- Case II agrees with the data better than any of the standard EoS:s.
- Overestimates the radius
  $\longrightarrow$ Accounting for the (possible) 2-component admixture in the transition:
  - smoothens the EoS
  - Reduces radius, doesn't affect the maximum mass.
- Superconductivity reduces radius $\longrightarrow$ Improve the treatment of CFL

Conclusions

- The grand potential of QCD at finite density with finite m_s computed to α_s^2.
- Modelled the EoS in the full range of μ (three logical possibilities):
  1. Hardon / quark matter transition
     - A realistic description of thermodynamics on all values of μ
  2. Absolutely stable strange quark matter
     - is disfavored, but not ruled out
     - ...but the observation of stars $M > 2M_{\mathrm{solar}}$ might be a strong evidence in the opposite direction.
  3. Exotic (non-CSC) phases between hadrons and quark matter
     - Well, at least me improved the perturbative side...

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  3. Exotic (non-CSC) phases between hadrons and quark matter
     - Well, at least me improved the perturbative side...
- *Our Case II matching seems to perform better than any of the standard EoS to model the recent experimental data.*

Outlook

- Improve the modelling of CSC:
  - By computing the mismatch of the fermi spheres
  - Assessing the different possibilities for CSC: CFL, 2CS...
- Improve the perturbative calculation:
  - $\alpha_s^3 \log(\alpha_s)$: only ring diagrams involved.
  - α_s^3: Major undertaking.
- Improve the compact star calculations:
  - *Two-component mixtures of hadronic and quark matter*
  - Moment of inertia, glichtes
  - Neutron star oscillations
  - Rotating stars, r-modes
  - Cooling rates and transport effects
  - ...

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  - ...
- ...and of course:
  the observations are advancing very fast,
  new data expected to come anytime!!