Gauge / String Duality at Finite Temperature

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Norwegian Winter Workshop on QCD in Extreme Conditions
Trondheim, Norway, 24 - 26 February 2010

Issues

1. Motivation
2. Black Holes
3. Thermodynamics
4. Polyakov Loop
5. Heavy $\bar{q}q$ free energy at $T \leq T_c$

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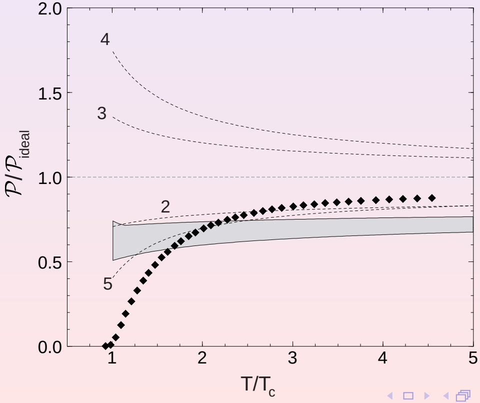
Motivation

Pressure of Gluodynamics

Weak Coupling Expansion and Resummed Perturbation Theory

E. Braaten and A. Nieto (1996), J.O. Andersen et al (1999).



Motivation

Interaction Measure in Gluodynamics

Weak Coupling Expansion and Resummed Perturbation Theory

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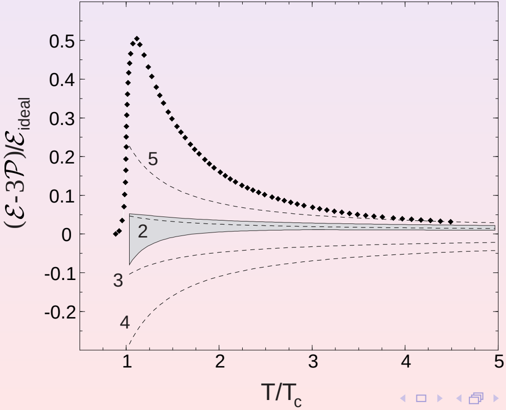
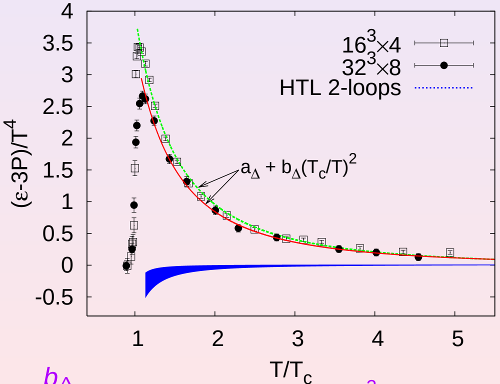


Motivation

Trace Anomaly $N_c = 3, N_f = 0$
G. Boyd et al., Nucl. Phys. B469, 419 (1996).



$$\frac{\epsilon - 3P}{T^4} = a_\Delta + \frac{b_\Delta}{T^2}\,, \quad b_\Delta = (3.46 \pm 0.13) T_c^2\,, \qquad 1.13 T_c < T < 4.5 T_c\,.$$

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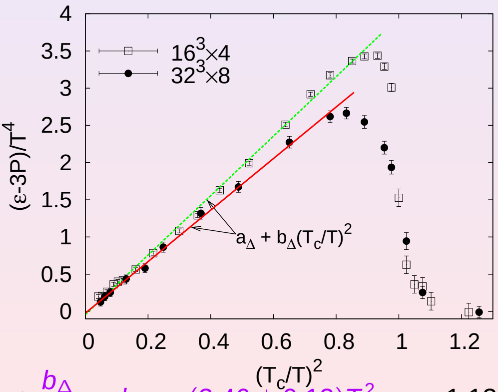

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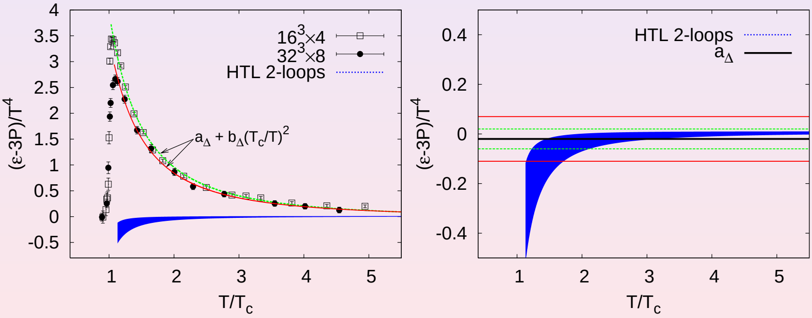
Motivation

$$\Delta \equiv \frac{\epsilon - 3P}{T^4} = a_{\Delta,P} + \frac{b_\Delta}{T^2}$$



Perturbation Theory and Hard Thermal Loops only yield a_Δ !!.

Schwarzschild black hole

General Relativity with no source $\Longrightarrow$ Einstein-Hilbert action

$$S_{EH} = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g}\, R \,, \qquad R = g^{\mu\nu} R_{\mu\nu}$$

Classical solution $\frac{\delta}{\delta g_{\mu\nu}} \Longrightarrow$ Einstein equations

$$E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \underset{\text{spherical}}{\Longrightarrow} R_{\mu\nu} = 0$$

Schwarzschild solution in spherical coordinates (1915):

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -f(r) dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\Omega_2^2 \,, \qquad f(r) = 1 - \frac{r_h}{r}$$

r_h is the horizon. Not physical singularity: $R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = 12 \frac{r_h^2}{r^6}$.
Large distance limit: $g_{tt}(r) \underset{r\to\infty}{\sim} -(1 + 2V_{Newton}(r)) \Longrightarrow r_h = 2G_4 M$.

Black hole thermodynamics

$$Z = \mathrm{Tr}\left(e^{-\beta H}\right), \qquad \beta = \frac{1}{T}$$

Periodicity in euclidean time ($\tau = it$): $\Phi(\tau + \beta) = \Phi(\tau)$

- Regularity: Expansion around the horizon $r = r_h(1 + \rho^2)$:

$$ds^2 = 4r_h^2\left(d\rho^2 + \rho^2 \underbrace{\left(\frac{d\tau}{2r_h}\right)^2}_{d\theta^2} + \frac{1}{4}d\Omega_2^2\right)$$

$\Longrightarrow$ Periodicity: $\frac{\tau}{2r_h} \to \frac{\tau}{2r_h} + 2\pi \Longrightarrow \tau \to \tau + 4\pi r_h =: \tau + \beta$

$$T = \frac{1}{8\pi M G_4}$$

Thermodynamics interpretation of black holes:

$$dM = TdS \Longrightarrow S = \int \frac{dM}{T} = 4\pi G_4 M^2$$

Black hole thermodynamics

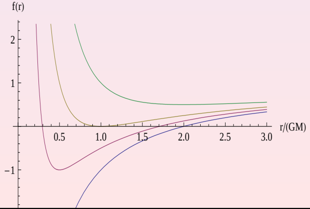
Area of the event horizon: $\mathcal{A} = 4\pi r_h^2 = 16\pi (G_4 M)^2$.
Bekenstein-Hawking entropy formula:

$$S = \frac{\mathcal{A}}{4G_D}$$

- Reissner-Nordström black hole:

$$S_{\rm Einstein-Maxwell} = \int d^D x \sqrt{-g} \left(\frac{1}{16\pi G_D} R - \frac{1}{4} F_{\mu\nu}^2 \right)$$

Solution: $f(r) = 1 - \frac{2G_4 M}{r} + \frac{G_4 Q^2}{r^2}$.



$Q^2 > G_4 M^2$ no singularity
$Q^2 = G_4 M^2$ extremal, T=0
$Q^2 < G_4 M^2$ two singularities
$Q^2 = 0$ one singularity

The 5D Einstein-dilaton model

$$S_{5D} = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{4}{3}\partial_\mu \phi \partial^\mu \phi - V(\phi) \right) - \frac{1}{8\pi G_5} \int_{\partial M} d^4x \sqrt{-h}\, K \, .$$

Finite temperature solutions (E. Kiritsis et al. JHEP (2009) 033):

- Thermal gas solution (confined phase):

$$ds^2 = b_0^2(z)(-dt^2 + d\vec{x}^2 + dz^2) \, , \qquad t \sim t + i\beta$$

- Black hole solution (deconfined phase):

$$ds^2 = b^2(z) \left[-f(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right]$$

In the UV ($z \simeq 0$): flat metric $b(z) \simeq L/z$ and $f(0) = 1$.
There exists an horizon $f(z_h) = 0$.
Regularity at the horizon $\Longrightarrow T = \frac{|\dot{f}(z_h)|}{4\pi}$.

The 5D Einstein-dilaton model

Einstein equations $\frac{\delta}{\delta g_{\mu\nu}}$:

$$\underbrace{\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right)}_{E_{\mu\nu}} - \underbrace{\left(\frac{4}{3}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g_{\mu\nu}\left(\frac{4}{3}(\partial\phi)^2 + V(\phi)\right)\right)}_{T_{\mu\nu}} = 0$$

$$(a) \qquad \frac{\ddot{f}}{\dot{f}} + 3\frac{\dot{b}}{b} = 0\,, \Longrightarrow f(z) = 1 - \frac{\int_0^z \frac{d\bar{z}}{b(\bar{z})^3}}{\int_0^{z_h} \frac{d\bar{z}}{b(\bar{z})^3}}$$

$$(b) \qquad 6\frac{\dot{b}^2}{b^2} - 3\frac{\ddot{b}}{b} = \frac{4}{3}\dot{\phi}^2\,,$$

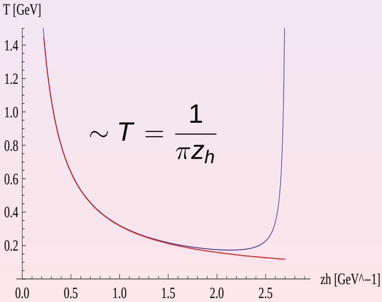
$$(c) \qquad 6\frac{\dot{b}^2}{b^2} + 3\frac{\ddot{b}}{b} + 3\frac{\dot{b}}{b}\frac{\dot{f}}{f} = \frac{b^2}{f} V$$

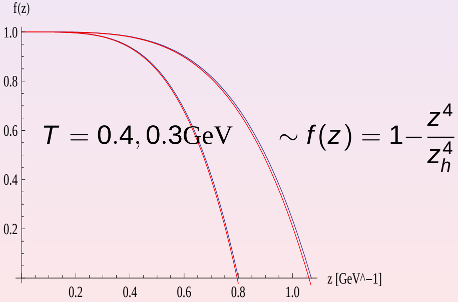
Conformal solution:

$$V(\phi) = \frac{12}{L^2}\,, \quad \dot{\phi} = 0 \Longrightarrow b(z) = \frac{L}{z}\,, \quad f(z) = 1 - \left(\frac{z}{z_h}\right)^4\,, \quad T = \frac{1}{\pi z_h}$$

The 5D Einstein-dilaton model

$$b^2(z) = e^{-\frac{4}{3}\phi(z)} \frac{L^2}{z^2} h(z)\,, \quad \text{Input}: \quad h(z) = \frac{\log(\epsilon)}{\log(\epsilon + (\Lambda z)^2)} \quad \text{Pirner\&Galow'09.}$$





Thermodynamics

Postulate: Entropy of gauge theories is equal to the Bekenstein-Hawking entropy of their string duals.

$$S(T) = \frac{\mathcal{A}(z_h)}{4G_5} = \frac{V_3 b^3(z_h)}{4G_5} = V_3 s_0 \frac{h^{\frac{3}{2}}(z_h)}{z_h^3}\,, \qquad z_h = \frac{1}{\pi T}$$

High temperature limit: $s(T) \underset{T\to\infty}{\sim} s_0 \pi^3 T^3 = \frac{32}{45}\pi^2 T^3 =: s_{\mathrm{ideal}}(T)$

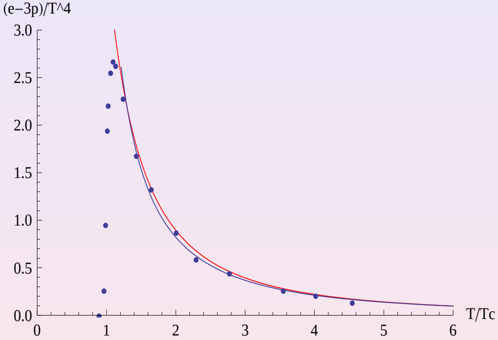
One can compute all the thermodynamics quantities:

$$s(T) = \frac{d}{dT} p(T)\,, \qquad \Delta(T) \equiv \frac{\epsilon - 3p}{T^4} = \frac{s}{T^3} - \frac{4p}{T^4}\,.$$

One can choose several warp factors:

- Andreev & Zakharov '07: $h_A(z) = e^{1/2cz^2}$.
- Pirner & Galow '09: $h_P(z) = \frac{\log(\epsilon)}{\log(\epsilon + (\Lambda z)^2)}\,, \qquad \epsilon = \frac{l_s^2}{L^2}$.

Thermodynamics



$$1.13 \leq T/T_c \leq 4.5$$

$$\Lambda = 264\mathrm{MeV}\,, \;\; T_c = 270\mathrm{MeV} \Longrightarrow \epsilon = 0.77\,, \;\; \chi^2/\mathrm{dof} = 0.56\,.$$

Polyakov Loop

$$L(T) := \langle P \rangle = \int DXe^{-S_w} \underset{\text{semiclassically}}{\Longrightarrow} \langle P \rangle = \sum_i w_i e^{-S_i} \simeq w_0 e^{-S_0}$$

Nambu-Goto Action:

$$S_{\text{NG}} = \frac{1}{2\pi l_s^2} \int d\sigma d\tau \sqrt{\det h_{ab}} = \frac{1}{2\pi l_s^2} \int d\tau d\sigma \sqrt{\det g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu} ,$$

$$\mu, \nu = t, \vec{x}, z \qquad a, b = \sigma, \tau$$

Modified AdS_5-metric at finite temperature:

$$ds_E^2 = \frac{L^2}{z^2} h(z) \left(f(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right) , \qquad f(z) = 1 - \left(\frac{z}{z_h} \right)^4$$

Static configurations: $\tau = t \,, \quad \sigma = z$

$$h_{ab} = \frac{L^2}{z^2} h(z) \begin{pmatrix} f & 0 \\ 0 & \frac{1}{f} + \dot{x}^2 \end{pmatrix} .$$

Polyakov Loop

The NG action writes:

$$S_{\mathrm{NG}} = \frac{L^2}{2\pi l_s^2} \int_0^{1/T} d\tau \int_0^{z_h} dz \, \frac{h(z)}{z^2} \sqrt{1 + \dot{x}^2 f(z)} \,,$$

Equation of motion for x:

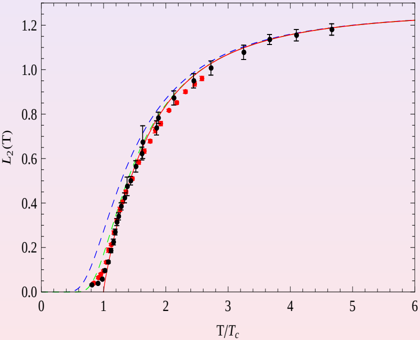
$$\frac{\partial}{\partial z} \left[\frac{h(z)}{z^2} \frac{\dot{x} f}{\sqrt{1 + \dot{x}^2 f}} \right] = 0 \,, \qquad \text{Boundary cond.:} \;\; x(0) = x(z_h) \equiv x_0$$

Solution: $x = x_0 =$ **constant.**

$$S_{\mathrm{reg}} = -\frac{1}{2\epsilon} + \frac{1}{2\pi\epsilon T} \int_0^{z_h} dz \, \frac{h(z) - 1}{z^2} \,.$$

Polyakov Loop

$$L(T) \simeq w_0 e^{-S_{\text{reg}} - c_R}$$



	$h_A(z)$	$h_P(z)$
ϵ	0.859	0.48
Λ	–	264 MeV
c	1.79 GeV^2	–
T_c	270 MeV	120 MeV

Lattice data ($N_f = 3$): P. Petreczky and K. Petrov, PRD70 (2004), M. Cheng et al., PRD77(2008).

Why are warp factors so good?

$h_A(z)$ and $h_P(z)$ both describe very well lattice results. This is because they include power corrections:

$$h_P(z) = \frac{\log(\epsilon)}{\log\left(\epsilon + (\Lambda z)^2\right)} = 1 + \frac{\Lambda^2}{\epsilon|\log\epsilon|} z^2 + \frac{(2+\log\epsilon)\Lambda^4}{2\epsilon^2\log^2\epsilon} z^4 + \mathcal{O}(z^6)\,.$$

They are related to condensates, starting from dimension 2.

- Dim 2 condensate: $\mathcal{C}_2 \equiv g^2\langle A_{0,a}^2\rangle = \frac{2N_c}{\pi^2}\frac{\Lambda^2}{\epsilon^2|\log\epsilon|} = (0.50\,\mathrm{GeV})^2$,
- Polyakov loop:

$$L \simeq e^{-S_0} = \exp\left(c_0 - \frac{\mathcal{C}_2}{4N_c T^2} - \frac{\mathcal{C}_4}{16N_c^2 T^4} + \cdots\right)\,,$$

- Trace anomaly:

$$\frac{\epsilon - 3p}{T^4} = \frac{33}{4\pi}\alpha_s\frac{\mathcal{C}_2}{T^2} + \mathcal{O}\left(\frac{\mathcal{C}_4}{T^4}\right)$$

Perturbation theory

But they don't fulfill UV behaviour given by perturbation theory.

$$L = 1 + \underbrace{\frac{4}{3}\sqrt{\pi}\alpha_s^{3/2}}_{\sim 1/\log^{3/2}(T/\Lambda)} + \underbrace{(2\log\alpha_s + 3 + 2\log\pi)\,\alpha_s^2}_{\sim 1/\log^2(T/\Lambda)} + \mathcal{O}(\alpha_s^{5/2}) \quad \text{Gava '81}$$

$$\frac{\epsilon - 3p}{T^4} = \underbrace{\frac{11}{3}\alpha_s^2}_{\sim 1/\log^2(T/\Lambda)} + \mathcal{O}(\alpha_s^{5/2})$$

One can try to construct the corresponding $h(z)$:

$$h(z) \sim \log^{-n}(\Lambda z)\,, \qquad n > 0$$

For the Polyakov loop:

$$h(z) = 1 + \epsilon\frac{16\sqrt{2}}{33\sqrt{11}}\pi^2\left(\frac{1}{\log^{3/2}(1/(\Lambda z))} - \frac{3}{2}\frac{1}{\log^{5/2}(1/(\Lambda z))}\right) \underset{z\to 0}{\longrightarrow} 1$$

A different approximation

Different approximation: choose the form of the dilaton potential (Kiritsis '09, Kajantie '09, Megias '09).

$$\beta(\alpha) = -\beta_0\alpha^2 - \beta_1\alpha^3 + \cdots \Longrightarrow$$

$$\Longrightarrow V(\alpha = e^\phi) = \frac{12}{L^2}\left(1 + \frac{8}{9}\beta_0\alpha + \left(\frac{23}{81} + \frac{4\beta_1^2}{9\beta_0^2}\right)(\beta_0\alpha)^2 + \cdots\right)$$

- Kajantie et al. arXiv:0905.2032 (2009), spatial string tension:

$$V(r) = \sigma_s r\,, \quad \sigma_s(T) = \frac{1}{2\pi\alpha'} b^2(z_h)\alpha^{4/3}(z_h)$$

  This is in contradiction with QCD: $\sigma_{\rm QCD} \sim T^2\alpha_s^2(T)$.

- Megías et al. (2010) compute the Polyakov loop:

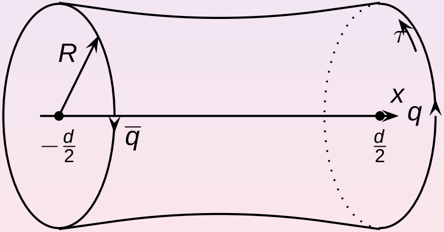
$$L(T) = \exp\left(c_0 + \frac{C_*}{2\epsilon}\alpha^{\frac{4}{3}}(z_h) + \mathcal{O}(\alpha^{\frac{7}{3}})\right)$$

  In contradiction with PT: $L_{\rm PT}(T) = \exp\left(\frac{4}{3}\sqrt{\pi}\alpha^{\frac{3}{2}} + \mathcal{O}(\alpha^2)\right)$.

Heavy $\bar{q}q$ free energy at $T \leq T_c$

K.Veschgini, E.Megías, J.Nian & H.J.Pirner, arXiv:0911.1680 ('09).

$$e^{-\beta F_{q\bar{q}}(\vec{d},T)} = \frac{1}{N_c^2}\left\langle \mathrm{tr}_c \Omega(\frac{\vec{d}}{2})\, \mathrm{tr}_c \Omega^\dagger(-\frac{\vec{d}}{2})\right\rangle \approx e^{-S_{\mathrm{NG}}} \,.$$



$$ds^2 = \frac{h(z)L^2}{z^2}(r^2 d\phi^2 + dr^2 + dx^2 + dz^2)\,, \qquad R = \frac{1}{2\pi T}$$

Heavy $\bar{q}q$ free energy at $T \leq T_c$

$$S_{\mathrm{NG}} = \frac{1}{2\pi l_s^2} \int_0^{2\pi} d\phi \int_{-d/2}^{d/2} dx \, \frac{L^2 h(z)}{z^2} r \sqrt{1 + (z')^2 + (r')^2}$$

- Euler-Lagrange equations:

$$\text{(a)} \qquad k = \frac{h(z) \cdot r}{z^2} \frac{1}{\sqrt{1 + (z')^2 + (r')^2}}$$

$$\text{(b)} \qquad r'' - \frac{h^2(z) \cdot r}{k^2 z^4} = 0$$

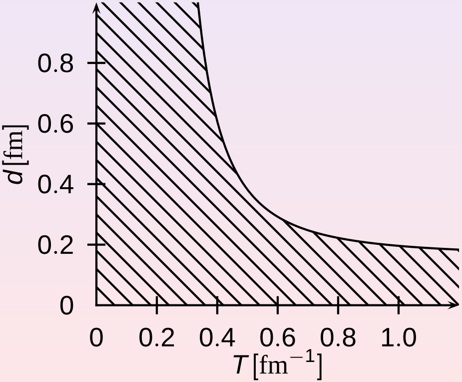
$$\text{(c)} \qquad z'' - \frac{h(z) \cdot r^2 \cdot (z \partial_z h(z) - 2h(z))}{k^2 z^5} = 0$$

- Boundary conditions:

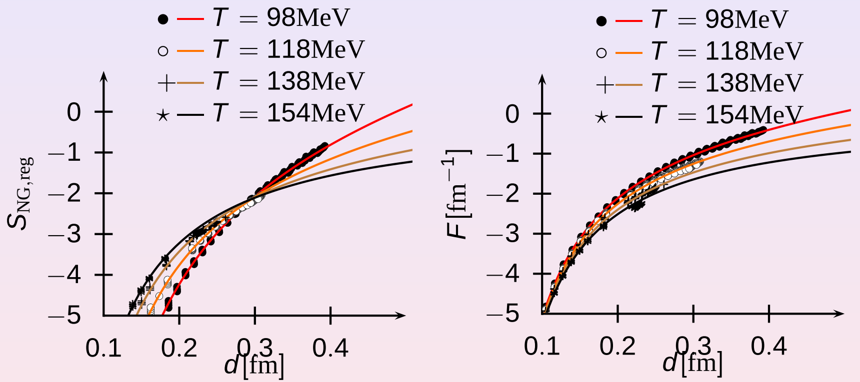
$$r(\pm d/2) = R = \frac{\beta}{2\pi} = \frac{1}{2\pi T} , \qquad z(\pm d/2) = 0 .$$

Heavy $\bar{q}q$ free energy at $T \leq T_c$

Numerical solutions only in a limited range.
No minimal surface $\Longrightarrow$ classical approximation not valid.



Heavy $\bar{q}q$ free energy at $T \leq T_c$



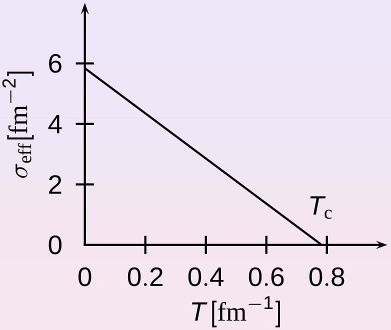
$$F_{\bar{q}q} = T \cdot S^{\rm fit}_{\rm NG,reg} = \frac{-0.48}{d} + d \underbrace{\left(\frac{-7.46}{\rm fm} T + \frac{5.84}{\rm fm^2} \right)}_{\equiv \sigma_{\rm eff}(T)} .$$

Heavy $\bar{q}q$ free energy at $T \leq T_c$

- Effective string tension:

$$\sigma_{\text{eff}}(T_c) = 0 \Longrightarrow T_c = 154\,\text{MeV}\,.$$

$$T_c^{\text{lattice},N_c=3,N_f=3} \overset{\text{Yagi'05}}{=} 155(10)\,\text{MeV}$$



Other thermodynamics quantities:

- Entropy:

$$S_{\bar{q}q} = -\frac{\partial F_{\bar{q}q}}{\partial T} = \frac{7.46}{\text{fm}} d\,,$$

- Inner energy:

$$E_{\bar{q}q} = F_{\bar{q}q} + T \cdot S_{\bar{q}q} = \frac{-0.48}{d} + \frac{5.84}{\text{fm}^2} d\,,$$

Conclusions:

- The non-perturbative behaviour of QCD near and above T_c is characterized by power corrections in T. These power corrections are high energy trace of non-perturbative low energy effects.
- AdS-QCD serves as a powerful tool to study the non perturbative regime of thermal QCD. Within our approximation we consider conformal breaking warp factors that naturally describe these power corrections.
- Good and unified description for:
  - Trace anomaly (pressure, entropy, ...)
  - Polyakov loop.
  - Heavy $Q\bar{Q}$ free energy.
- We are working in other observables:
  - Correlation of spatial Wilson loops $\Longrightarrow$ spatial string tension.
  - Glueball spectrum.
  - Mesons spectrum.
  - ...

We would like to kindly invite you to our Workshop:

STRING THEORY AND EXTREME MATTER
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Topics:

- AdS/CFT and AdS/QCD.
- What about quarks?
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