

# Effective potential for Polyakov loops from a center symmetric effective theory in three dimensions

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# Outline

- Introduction / Motivation
- Center symmetric effective theory of Wilson Lines
- Lattice Simulation
- Summary

# Introduction

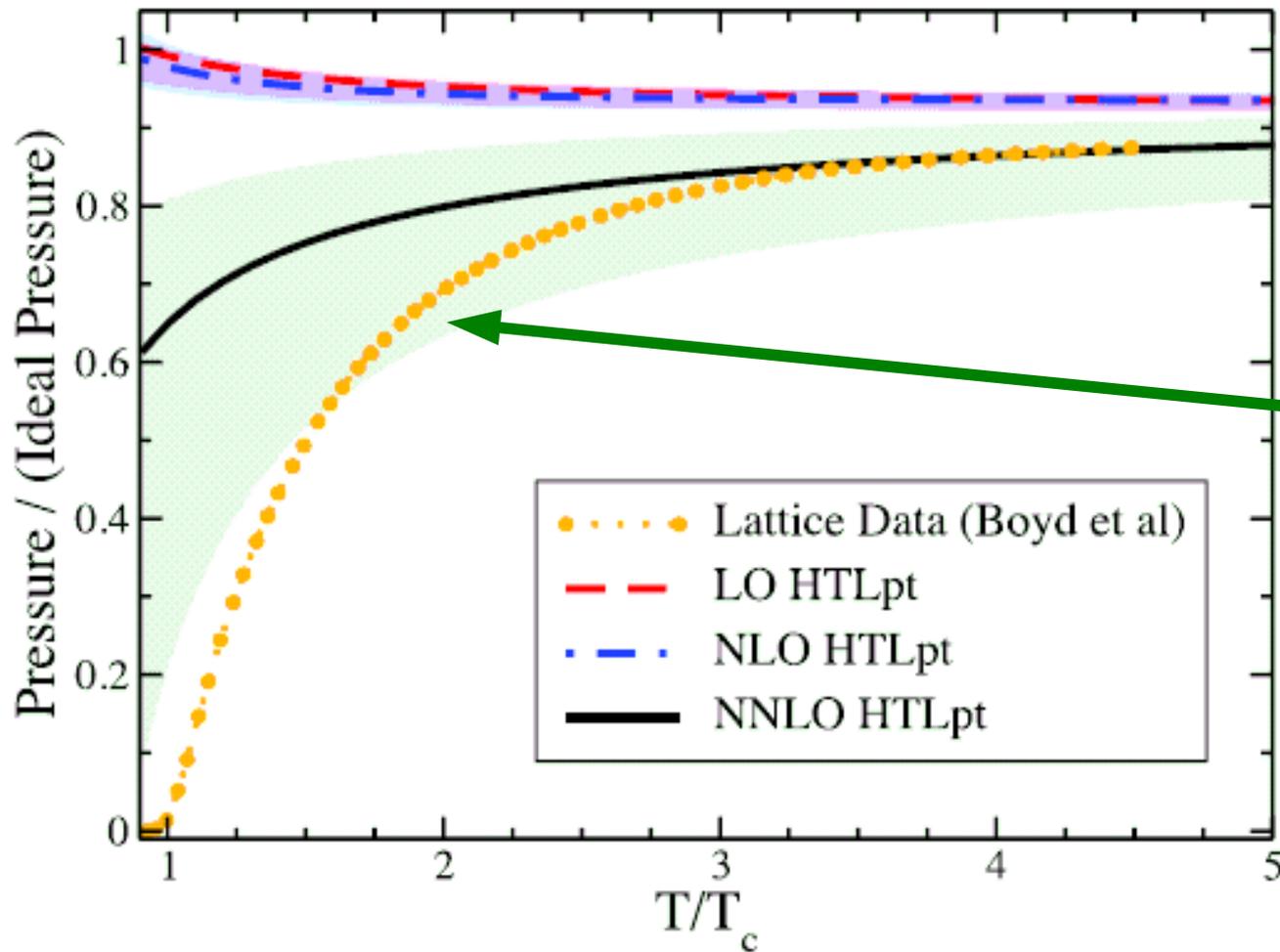
- Quark-Gluon Plasma is produced in heavy-ion collisions at high energies.
- Experiments and simulations suggest a non-perturbative phase right above phase transition ("sQGP").

M. Gyulassy, L. McLerran,  
Nucl. Phys. A750 (2005) 30-63



- Hard-thermal-loop perturbation theory and other approaches fail.

- Example: Pressure



Andersen,  
Strickland, Su  
arXiv: 0911.0676

4d SU(3) Lattice  
data

- HTL deviates from lattice below  $\sim 3T_c$  !
- Why ??

- Action of SU(N) Yang-Mills theory:  $S = - \int_0^\beta d\tau \int d^3x \mathcal{L}$

$$\mathcal{L} = \frac{1}{4} \text{tr} F_{\mu\nu}^a F_a^{\mu\nu} \quad F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$$

- Invariant under periodic gauge transformations:

$$A_\mu(\vec{x}, \tau) \rightarrow S(\vec{x}, \tau) (A_\mu(\vec{x}, \tau) + i \partial_\mu) S(\vec{x}, \tau)^\dagger$$

$$S(\vec{x}, \beta) = S(\vec{x}, 0) \quad S(\vec{x}, \tau) \in SU(N)$$

- Additionally: Transformations periodic up to "twist".

$$s(\vec{x}, \beta) = z s(\vec{x}, 0) \quad z \in Z(N) \quad \text{global factor !}$$

- Order parameter for this symmetry: Polyakov Loop

$$\ell = \frac{1}{N} \text{tr} L \quad L(\vec{x}) = P \exp \left( ig \int_0^{1/T} A_0(\vec{x}, \tau) d\tau \right) \quad L \in SU(N)$$

- Under gauge transformations:

$$L(\vec{x}) \rightarrow s(\vec{x}, \beta)^\dagger L(\vec{x}) s(\vec{x}, 0)$$

- For twisted transformations:  $\text{tr} L \rightarrow z \text{tr} L$

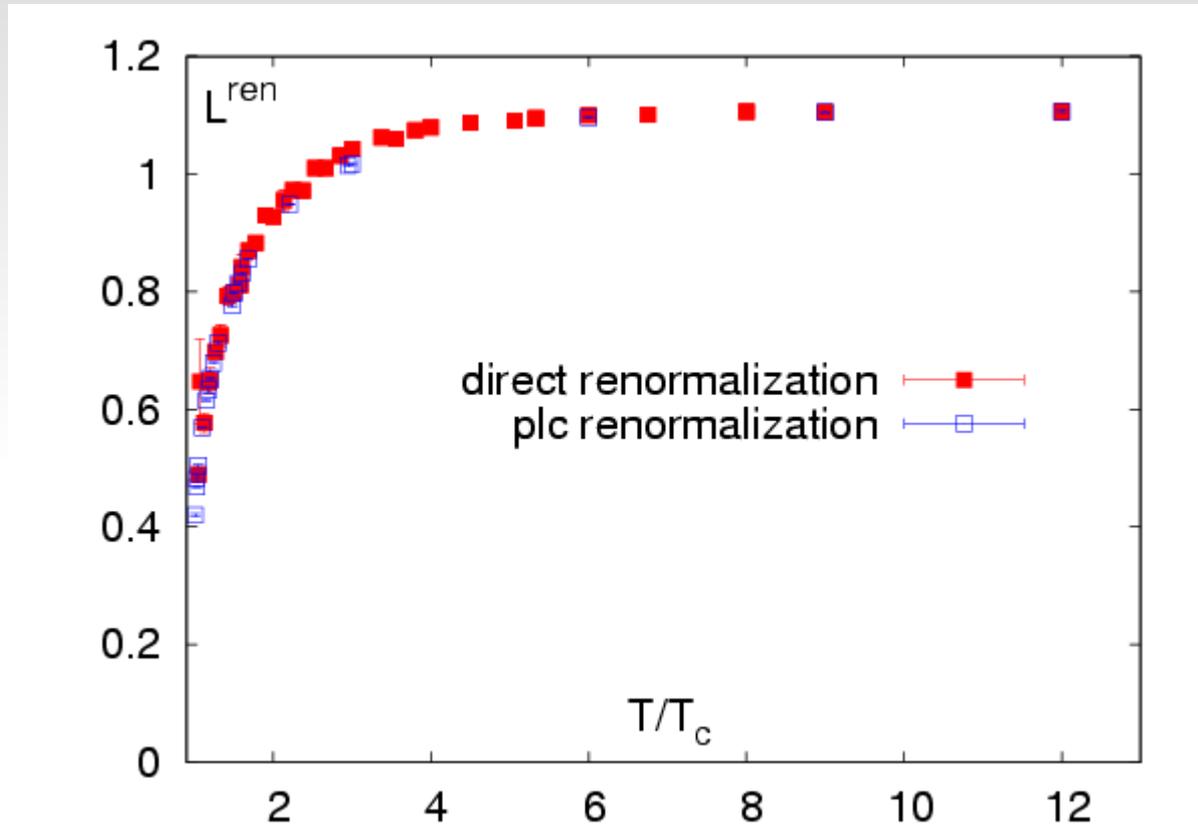
- Related to free energy of a static quark:

$$|\langle \ell \rangle| = e^{-\beta F_q}$$

- Non-zero expectation value signals deconfinement (spontaneous symmetry breaking).
- There are N different vacuum states related by Z(N) transformation of the Polyakov Loop !
 
$$\ell \rightarrow e^{i\frac{2\pi j}{N}} \ell$$
- HTL P.T.  Assumption of small  $A_0/T \ll 1$
- Chooses groundstate where L is close to unity !

$$L(\vec{x}) = P \exp\left(ig \int_0^{1/T} A_0(\vec{x}, \tau) d\tau\right) \approx 1 \quad \ell = \text{tr} L = 1$$

- At high temperatures this choice is fine:  
 $Z(N)$  is spontaneously broken.



Kaczmarek,  
Gupta,  
Hübner,  
hep-lat/0710.2277

(renormalized  
SU(3) loops)

- At non-asymptotic temperatures fluctuations between different vacuum states should become important.

# Effective theory of Wilson lines

- Partition function

$$Z(T) = \int D\Phi(\vec{x}, \tau) \exp\left(-\int_0^\beta d\tau \int d^3x \mathcal{L}\right)$$

- Example, massless scalar:  $\mathcal{L} = \frac{1}{2} (\partial_\mu \Phi)^2 + \frac{g^2}{24} \Phi^4$
- Boundary conditions:  $\Phi(\vec{x}, \tau = \beta) = \Phi(\vec{x}, \tau = 0)$
- Expansion in terms of Matsubara frequencies:

$$\phi(\vec{x}, \tau) = \sqrt{1/T} \sum_n \int_p \phi_n(p) e^{i\vec{p}\vec{x} + i\omega_n \tau}$$

- Matsubara frequencies act like masses:

$$\Delta(\omega_n, \vec{p}) = \frac{1}{p^2 + \omega_n^2} \quad \omega_n = 2n\pi T \quad \text{(bosons)}$$

- Nonstatic modes decouple from long-distance dynamics!

$$x \gg 1/T$$

Appelquist,  
Carrazone,  
Phys. Rev. D11, 2856

- Effective theory in three dimensions. **”dimensional reduction”**

$$\mathcal{L} = \frac{1}{2} (\nabla \Phi)^2 + \frac{1}{2} m^2 \Phi^2 + \frac{\lambda}{24} \Phi^4 + \dots$$

- Applied to Yang-Mills theory:  $\mathcal{L} = \frac{1}{4} \text{tr} F_{\mu\nu}^a F_a^{\mu\nu}$

- Generates debye mass:

$$\mathcal{L} = \frac{1}{4} \text{tr} G_{ij} G_{ij} + \text{tr} |D_i A_0|^2 + m_D^2 \text{tr} A_0^2 + \dots$$

(“EQCD”)

- Infinite series in powers of  $A_0$  .
- Assumption of small  $A_0$  necessary for calculations.
- Again: Violates center symmetry !

- Alternative to explicit reduction procedure:
- Write down most general Lagrangian that respects the symmetries.
- Demand correspondence at large distances.  $x \gg 1/T$
- Fields need not be directly related !
- **Effective theory of Wilson Lines:**

R.D. Pisarski  
hep-ph/0608242  
hep-ph/0006205

Similar approach:

A. Vuorinen, L. Yaffe  
Phys.Rev. D74 (2006) 025011  
P. de Forcrand, A. Kurkela, A.Vuorinen  
Phys Rev. D77 (2007) 125014  
C. Korthals Altes  
arXiv: 0810.3325

- Lagrangian includes kinetic energy and 1-loop potential.

$$\mathcal{L}_0^{eff} = \frac{1}{2} tr G_{ij}^2 + \frac{T^2}{g^2} tr |L^+ D_i L|^2 - \frac{2}{\pi^2} T^4 \sum_{n \geq 1} \frac{1}{n^4} |tr L^n|^2$$

- Minimized by perturbative vacuum !

Gross, Pisarski,  
Yaffe,  
Rev. Mod. Phys. 53  
43 (1981)

- Idea: add "fuzzy bag" term for non-perturbative effects.

$$\mathcal{L}^{eff} = \mathcal{L}_0^{eff} + B_f T^2 |tr L|^2$$

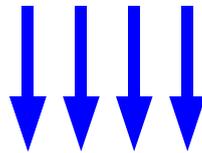
- **What is potential for Polyakov loops?**

# Lattice Simulation

- Focus on SU(2) Wilson Lines !

- Lattice action:

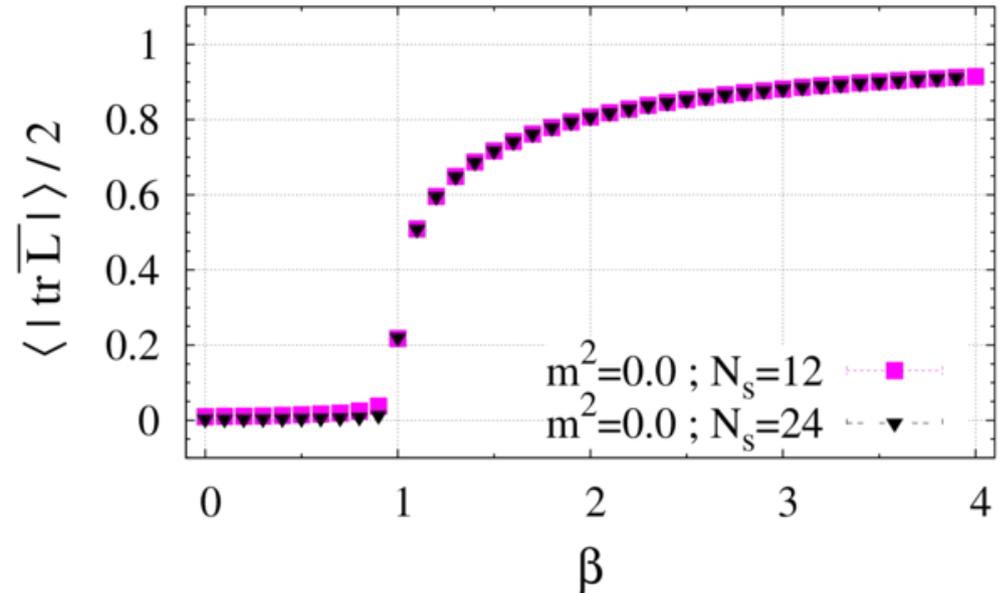
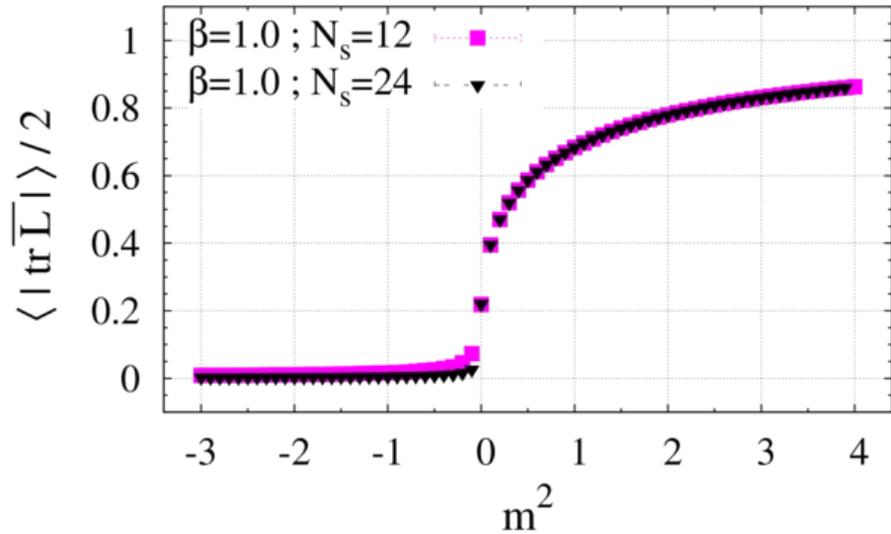
$$\mathcal{L}^{eff} = \frac{1}{2} tr G_{ij}^2 + \frac{T^2}{g^2} tr |L^+ D_i L|^2 - \frac{2}{\pi^2} T^4 \sum_{n \geq 1} \frac{1}{n^4} |tr L^n|^2 + B_f T^2 |tr L|^2$$



$$S = \beta \sum_{Pl.} \left(1 - \frac{1}{2} Re Tr U_{Pl.}\right) - \frac{1}{2} \beta \sum_{\langle ij \rangle} Tr(L_i U_{ij} L_j^+ U_{ij}^+ + h.c.) - m^2 \sum_i |Tr L_i|^2$$

- Updates with metropolis algorithm, heatbath and overrelaxation.

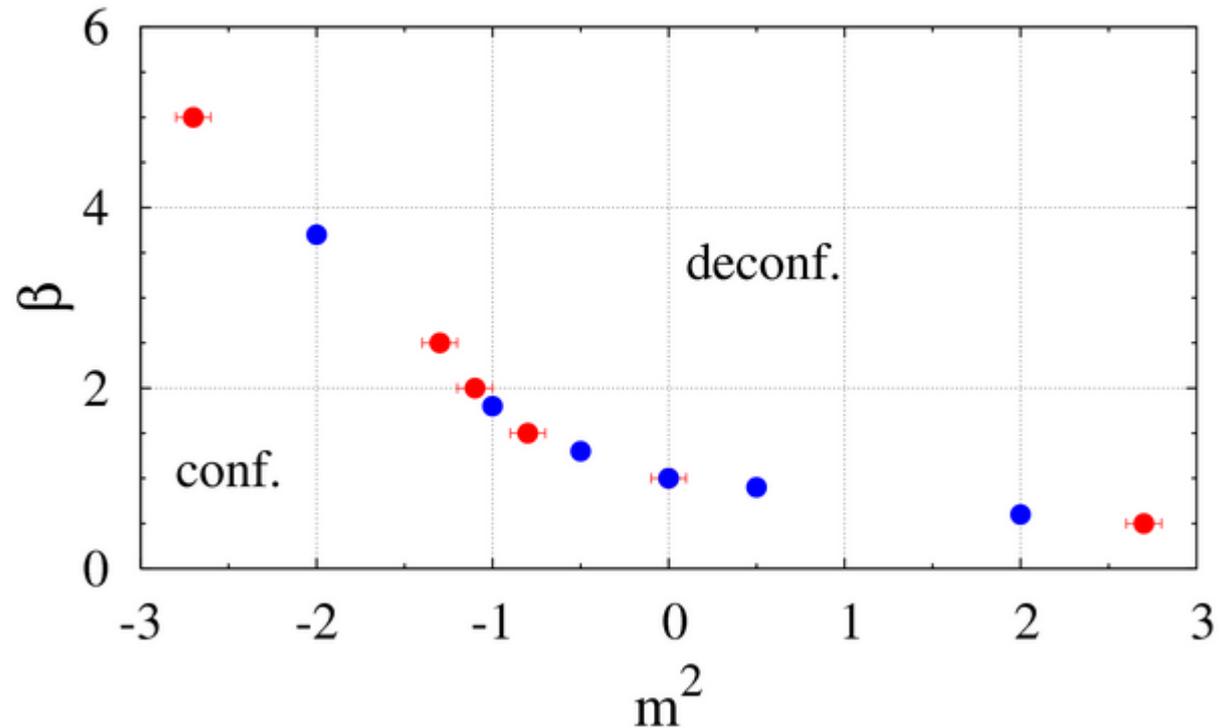
- Mapping of the phase diagram



- Model deconfines for large values of  $\beta$  or  $m^2$ .
- Transition second order for not too large  $\beta$ .

$$S = \beta \sum_{Pl.} \left(1 - \frac{1}{2} \text{ReTr} U_{Pl.}\right) - \frac{1}{2} \beta \sum_{\langle ij \rangle} \text{Tr}(L_i U_{ij} L_j^+ U_{ij}^+ + h.c.) - m^2 \sum_i |\text{Tr} L_i|^2$$

- Phase diagram:



- Polyakov loop potential can be studied with distribution of eigenvalues.

$$\rho = \frac{1}{2} |\lambda_1 + \lambda_2| = \sqrt{\ell^2}$$

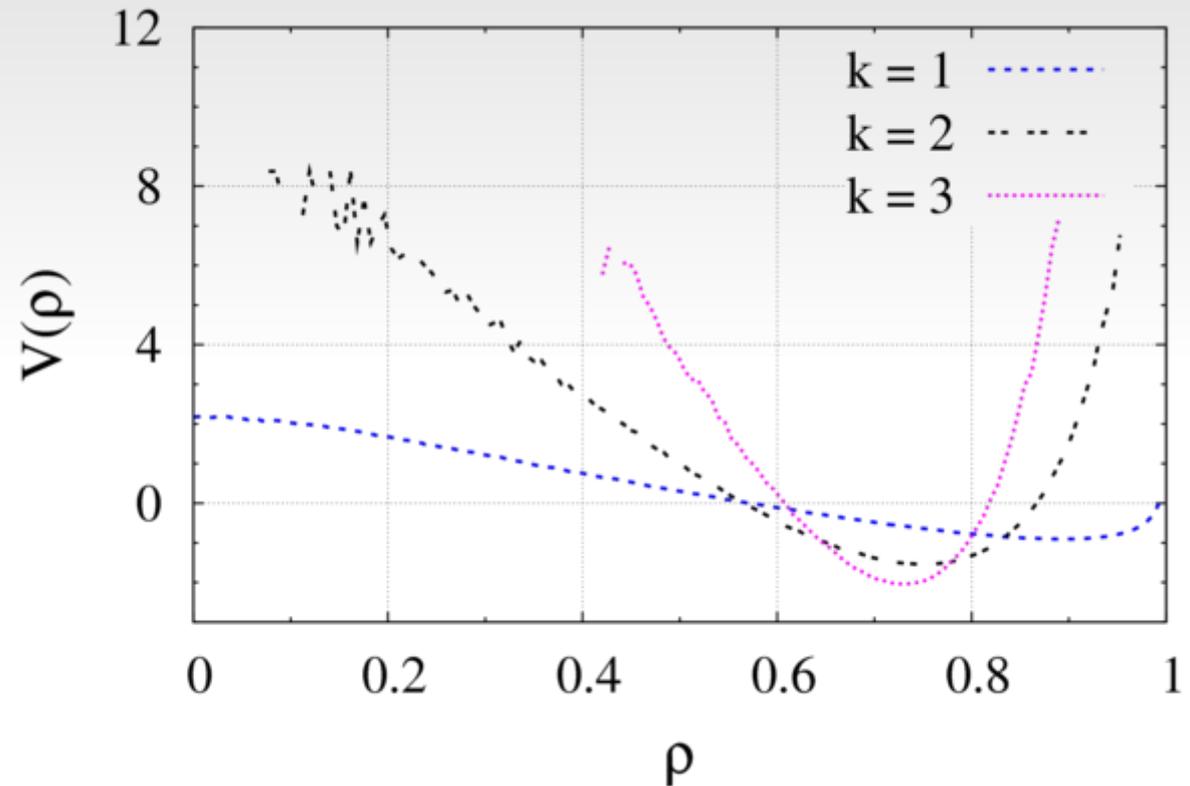
$$V_{eff}(\rho) = -\ln P(\rho) \quad Z(T) = \int d\rho P(\rho) = \int d\rho e^{-V(\rho)}$$

- To compare with mean field models: "Block spins"

$$\bar{\ell}_i^{(k)} = \frac{1}{k^3} \sum_{\vec{n}} \frac{1}{2} \text{tr} L(\vec{i} + \vec{n}) \quad \vec{n} = (0,0,0) \dots (k, k, k)$$

- Example:  $\beta = 1.5$   
 $m^2 = 0.0$

"cooling"



- We take  $k = 3$  as good description of long distance physics ! (self consistency)

- Certainly present: Contribution from integration measure
- Measure from integral over group elements.

$$Z(T) = \int dU e^{-S} \quad U \in SU(N)$$

- Appears when transforming to integral over parameters:

$$\int dU f(U) = \int d\alpha_1 \dots d\alpha_n J(\alpha) f(U(\alpha))$$

- Acts as a potential term in the action !

- For SU(2):  $V = -\frac{1}{2} \ln(1 - \rho^2)$

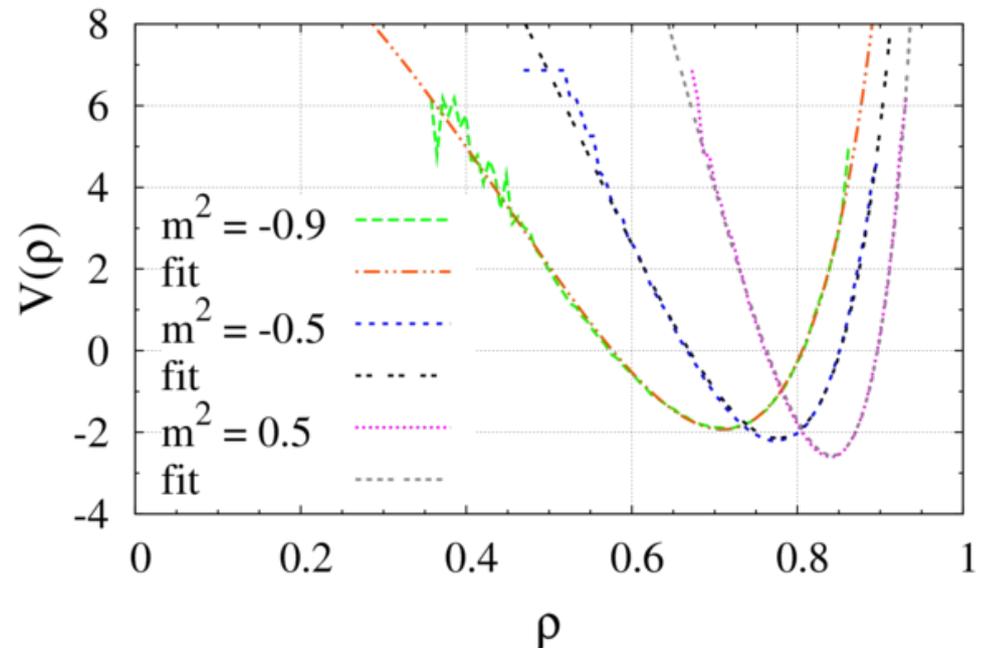
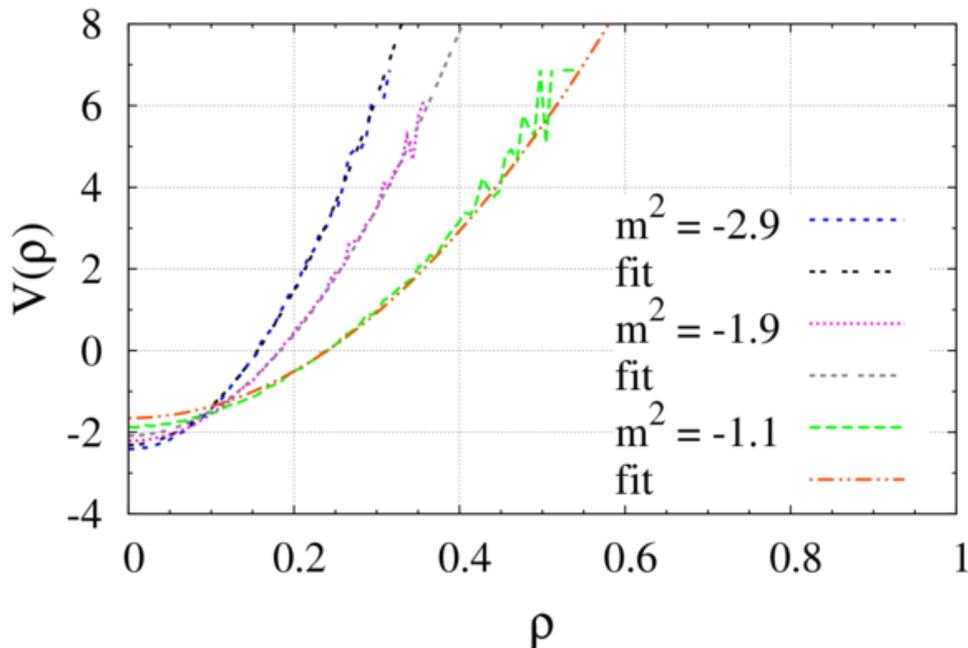
**(Vandermonde potential)**

- Other terms: Analogous to Landau-Ginzburg.
- For large range of  $\beta$  and  $m^2$ :

$$V = -\frac{1}{2}d_0 \ln(1 - \rho^2) + d_1 + d_2 \rho^2 + d_4 \rho^4$$

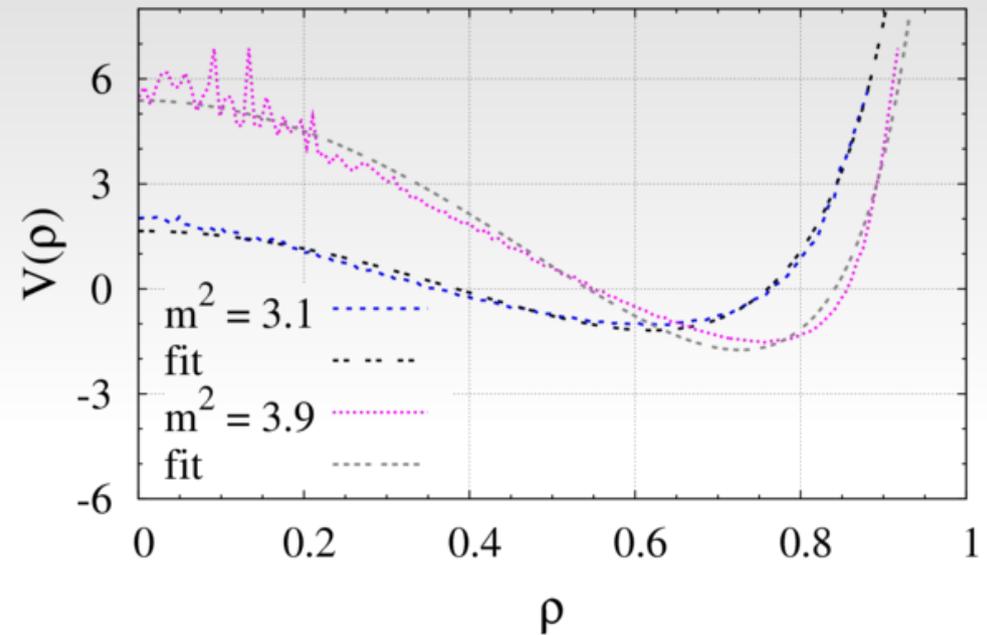
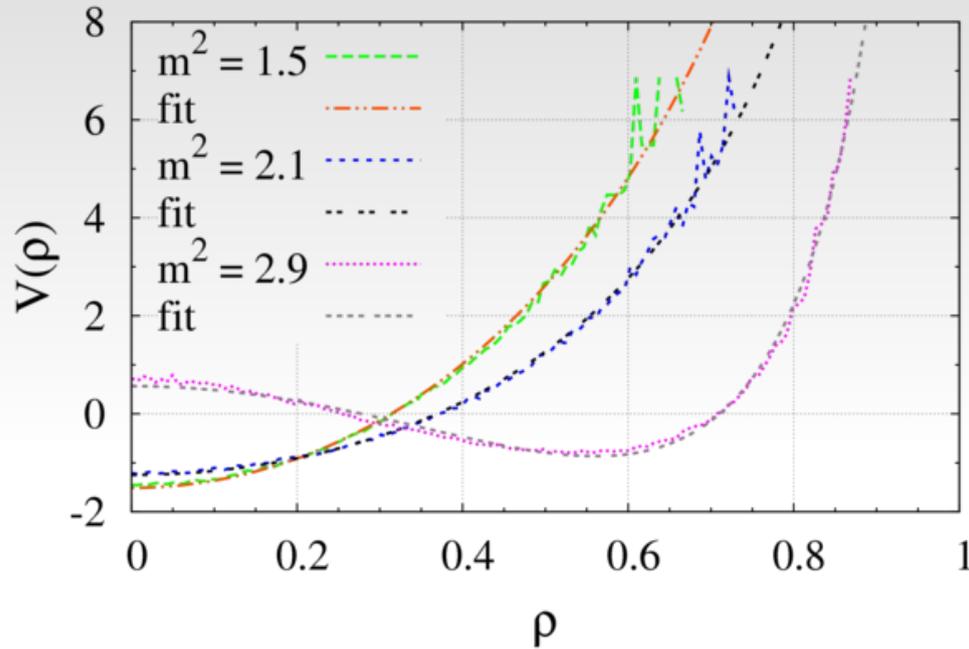
$$k = 3$$

- Example:  $\beta = 2.0$



- Example:  $\beta = 0.5$

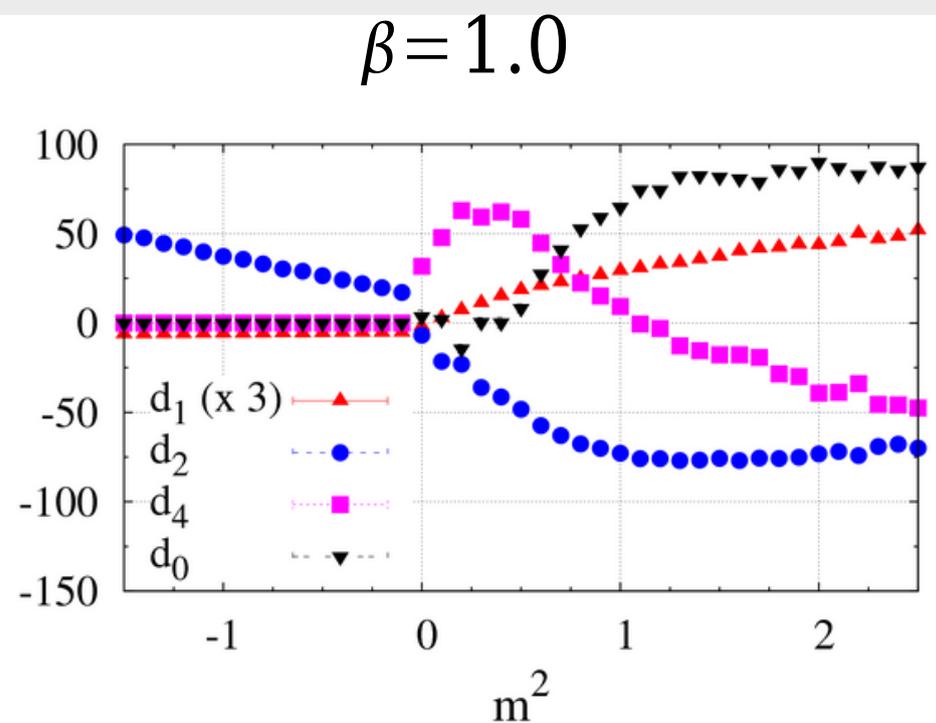
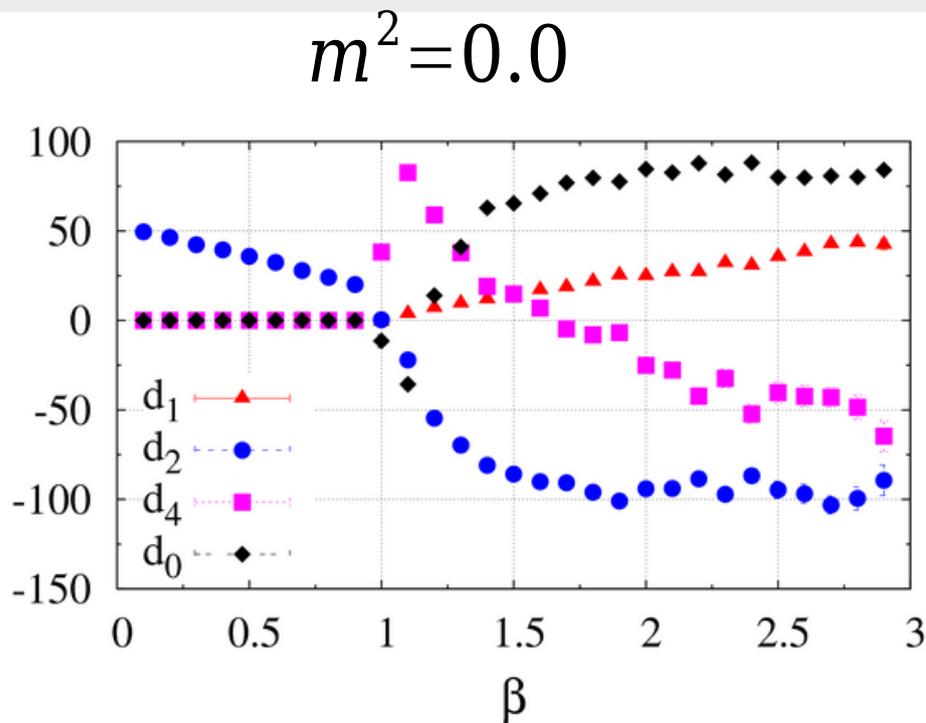
$$V = -\frac{1}{2}d_0 \ln(1 - \rho^2) + d_1 + d_2 \rho^2 + d_4 \rho^4$$



- All constants  $d_0, d_1, d_2, d_4$  depend on  $\beta$  and  $m^2$  !
- Temperature dependent Vandermonde term !

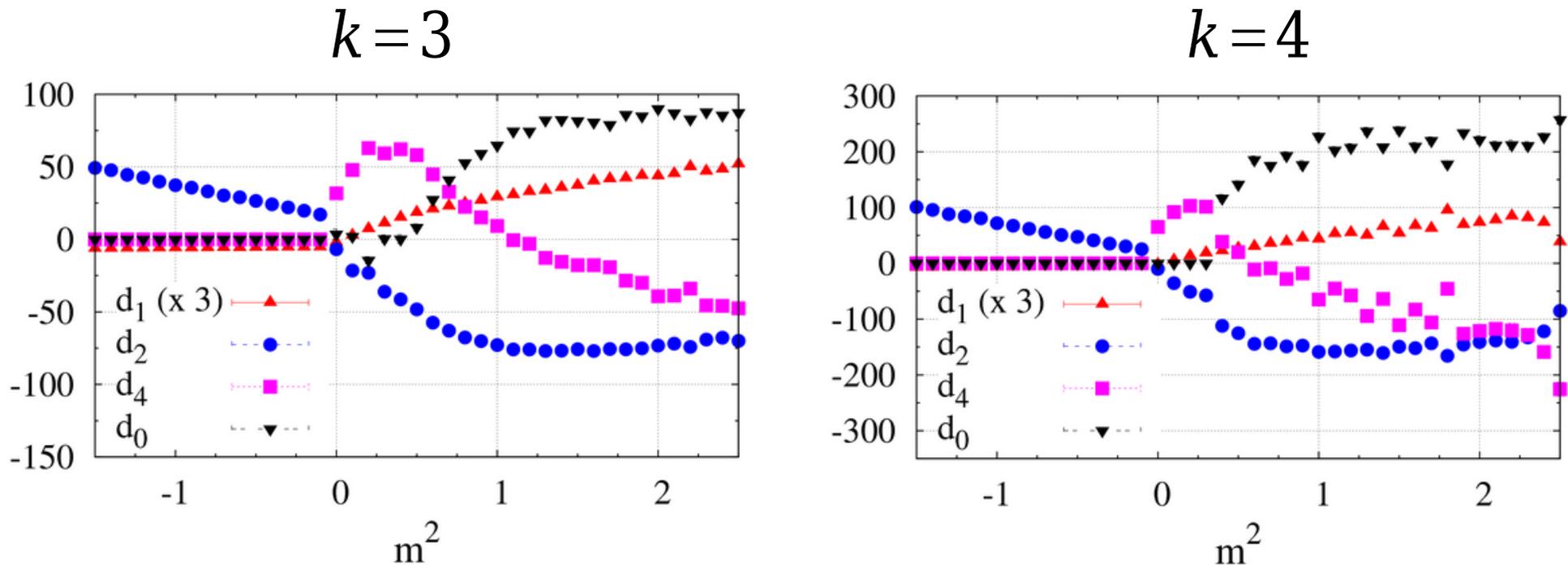
- Above phase transition: Small region with vanishing Vandermonde !

$$V = -\frac{1}{2}d_0 \ln(1 - \rho^2) + d_1 + d_2 \rho^2 + d_4 \rho^4$$



- Consistent with setting  $d_0$  to zero by hand in this area!

- Result is stable when going to higher  $k$  !  $\beta = 1.0$



- Overall scaling factor expected from definition:

$$Z = \exp\left(\int d^3x V\right) \rightarrow Z = \exp\left(\sum V_{eff}\right)$$

- Vanishing Vandermonde is not artificial.

# Summary

- Studied a center symmetric theory of Wilson lines
- Mapped the phase diagram
- Extrapolated effective potential for Polyakov loop
- "block-spinning" / cooling yields mean-field type potential with  $T$  dependent Vandermonde term.

## What hasn't been shown here:

- Correlation functions of several operators
- Detailed investigation of the magnetic sector (string tensions, plaquettes et. al.)
- Similar observables in 4D pure gauge theory

## Possible future:

- Precise non-perturbative matching to 4D theory
- Higher powers of  $L$
- Extend to  $SU(3)$

## References:

A.Dumitru, D.Smith

*Phys. Rev. D* 77, 094022 (2008), arXiv:0711.0868

D.Smith

*Nucl. Phys.* A820:227-230 (2009), arXiv: 0810.1129

D.Smith

arXiv: 0911.4037