Dissipative processes in superfluid quark matter

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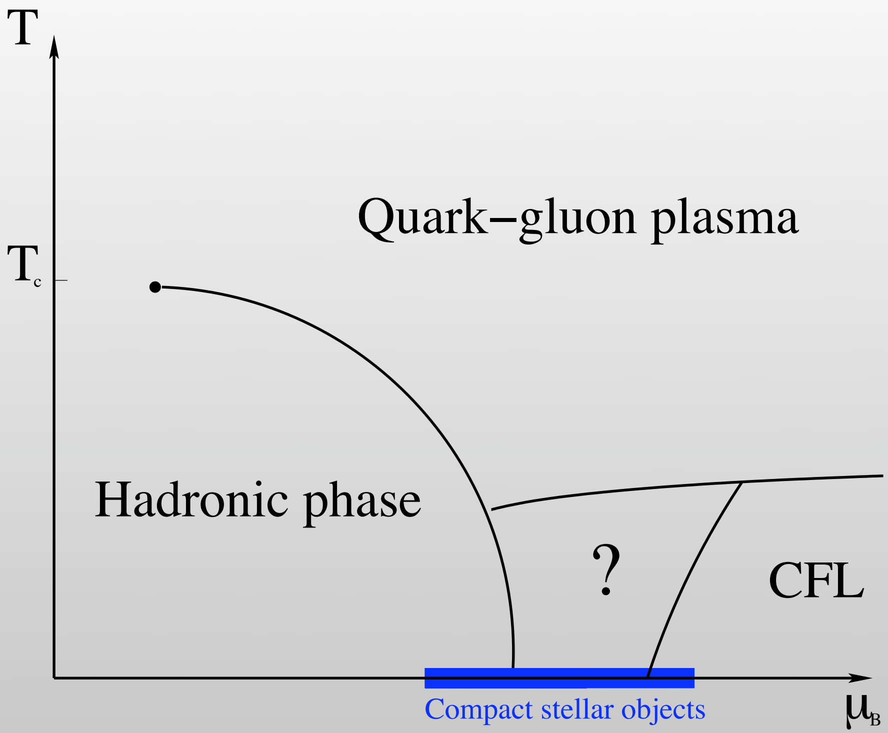
Outline

- ✦ QCD phase diagram
- ✦ Viscid and unviscid fluids
- ✦ Superfluids
- ✦ Dissipative processes
- ✦ Color Superconducting (CSC) phase
- ✦ R-mode oscillations

QCD PHASE DIAGRAM



so far no evidence of any phase transition

$\mu \gg m_s \quad < \psi_{\alpha i} C \gamma_5 \psi_{\beta j} > \sim \Delta\, \epsilon_{I\alpha\beta} \epsilon_{Iij}$

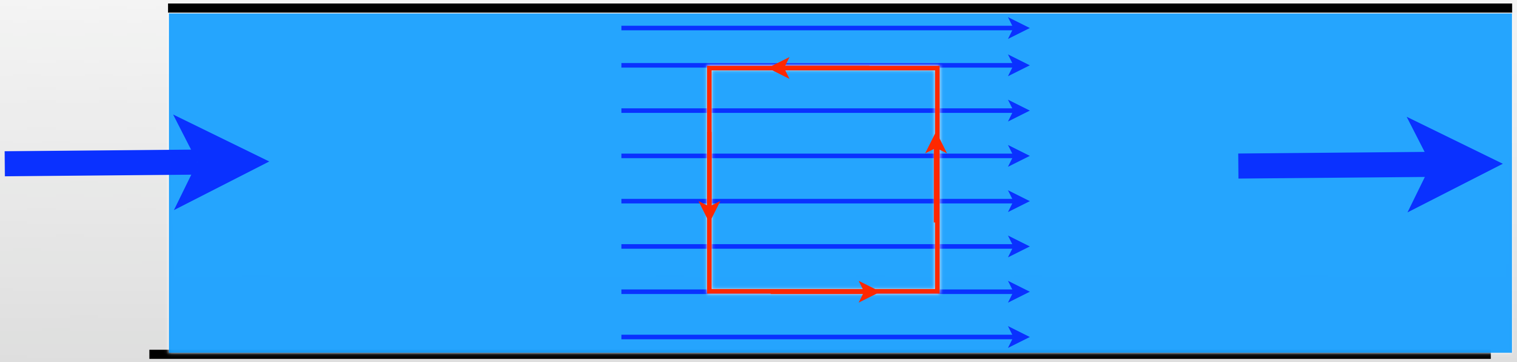
CFL phase

Alford, Rajagopal, Wilczek hep-ph/9804403

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2$$

- $U(1)_B$ **is spontaneously broken: CFL is a SUPERFLUID**

UNVISCID (DRY) FLUID



Description of the fluid

Continuity equation $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Euler equation $$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla \mathbf{p}}{\rho} - \nabla \phi$$

Vorticity $\mathbf{\Omega} = \nabla \times \mathbf{v}$ $$\frac{\partial \mathbf{\Omega}}{\partial t} + \nabla \times (\mathbf{\Omega} \times \mathbf{v}) = \mathbf{0}$$

If at a certain time $\Omega = 0$ it is still zero at any later time

The flow is permanently irrotational $\mathbf{v} = \nabla \varphi$

VISCOUS FLOW

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = -\nabla p - \rho\nabla\phi + \eta\nabla^2\mathbf{v} + \zeta\nabla(\nabla\cdot\mathbf{v})$$

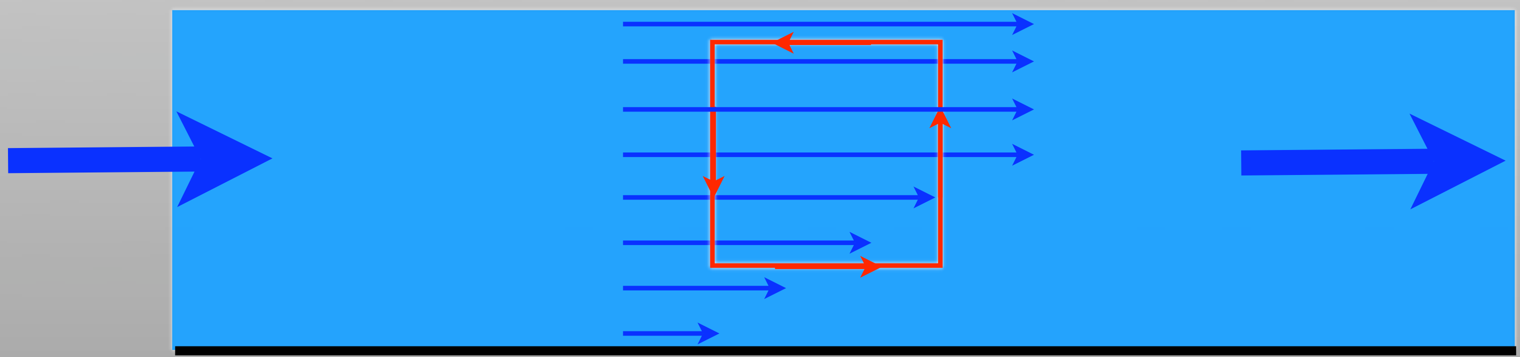
shear viscosity

bulk viscosity

Using the vorticity

$$\frac{\partial \mathbf{\Omega}}{\partial t} + \nabla\times(\mathbf{\Omega}\times\mathbf{v}) = \frac{\eta}{\rho}\nabla^2\mathbf{\Omega}$$

Diffusion equation for vorticity: vorticity is generated by the shear viscosity



SUPERFLUIDITY

Superfluidity: all the particles of the system are in the same quantum state

At T=0 shear and bulk viscosity are zero

At T $\neq$0 viscosity is due to the presence of low energy excitations

Landau's criterion

Superfluidity occurs when $v < v_{cr} = \text{Min}\frac{\epsilon(p)}{p}$ $\quad \text{Min}\frac{\epsilon(p)}{p} \neq 0$

Example

Consider a system with a global $U(1)$ symmetry

Spontaneous symmetry breaking: Goldstone boson $\epsilon(p) = c\,p$

NON-RELATIVISTIC SUPERFLUIDS

Landau two-fluid theory

$$\rho = \rho_n + \rho_s \qquad \mathbf{j} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s$$

Normal component
phonons, rotons...

Superfluid component

$$\partial_t \rho + \mathrm{div}\mathbf{j} = 0$$

The two "components" correspond to two different motions of the fluid

Normal component: viscous fluid

Superfluid component: unviscid fluid

Not completely correct: neglects interactions

NON-RELATIVISTIC HYDRODYAMICS

Non dissipative equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$ **Mass conservation**

$$\frac{\partial j_i}{\partial t} + \partial_j \Pi_{ij} = 0$$ **Momentum conservation**

$$\frac{\partial \mathbf{v_s}}{\partial t} + \nabla \left(\mu + \frac{\mathbf{v_s}^2}{2} \right) = 0$$ **Josephson equation**

Adding the most generic dissipative terms

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial j_i}{\partial t} + \partial_j (\Pi_{ij} + \tau_{ij}) = 0$$

$$\frac{\partial \mathbf{v_s}}{\partial t} + \nabla \left(\mu + \frac{\mathbf{v_s}^2}{2} + h \right) = 0$$

dissipative terms

Dissipative energy flux

$$\mathbf{Q}' = \mathbf{q} + h(\mathbf{j} - \rho \mathbf{v}_n) + \tau \cdot \mathbf{v}_n$$

Entropy production rate

$$R = -h\nabla \cdot (\rho_s(\mathbf{v_n} - \mathbf{v_s})) - \tau_{ik}\partial_k v_{ni} - \frac{1}{T}\mathbf{q} \cdot \nabla T$$

Close to equilibrium

$$\begin{aligned}
\tau_{ij} &= -\eta\big(\partial_j v_{ni} + \partial_i v_{nj} - \frac{2}{3}\delta_{ij}\nabla \cdot \mathbf{v}_n\big) - \delta_{ij}\big(\zeta_1 \nabla \cdot (\rho_s(\mathbf{v}_s - \mathbf{v}_n)) + \zeta_2 \nabla \cdot \mathbf{v}_n\big) \\
h &= -\zeta_3 \nabla \cdot (\rho_s(\mathbf{v}_s - \mathbf{v}_n)) - \zeta_4 \nabla \cdot \mathbf{v_n} \\
\mathbf{q} &= -\kappa \nabla T
\end{aligned}$$

Onsager symmetry principle: $\zeta_1 = \zeta_4$

Non-negative entropy production: $\zeta_1^2 \leq \zeta_2 \zeta_3$ **and** $\kappa, \eta, \zeta_2, \zeta_3$ **positive**

Physical interpretation

$$\begin{aligned}
P &= P_{\mathrm{eq}} - \zeta_1 \mathrm{div}(\rho_s(\mathbf{v_n} - \mathbf{v_s})) - \zeta_2 \mathrm{div}\, \mathbf{v_n} \\
\mu &= \mu_{\mathrm{eq}} - \zeta_3\, \mathrm{div}(\rho_s(\mathbf{v_n} - \mathbf{v_s})) - \zeta_4 \mathrm{div}\, \mathbf{v_n}
\end{aligned}$$

ROTATING SUPERFLUID

$$\mathbf{V} = \frac{\rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n}{\rho} \qquad \mathbf{w} = \mathbf{v}_s - \mathbf{v}_n$$

c.m. **relative motion**

Small fluctuations in a rotating superfluid (corotating frame)

$$\partial_t \delta V_i + 2\epsilon_{ijk}\Omega_j \delta V_k = -\frac{1}{\rho}\partial_i \delta p - \partial_i \delta\phi - \frac{1}{\rho}\partial_k \delta\tau_{ik} \, ,$$

$$\partial_t \delta w_i + 2\epsilon_{ijk}\Omega_j \delta w_k = -\frac{\rho}{\rho_s \rho_n}\delta F_i^{SN} + \frac{\rho}{\rho_n}(\delta(s\partial_i T) - \partial_i \delta h) + \frac{1}{\rho_n}\partial_k \delta\tau_{ik}$$

mutual friction force

thermomechanical force

shear and bulk viscosities

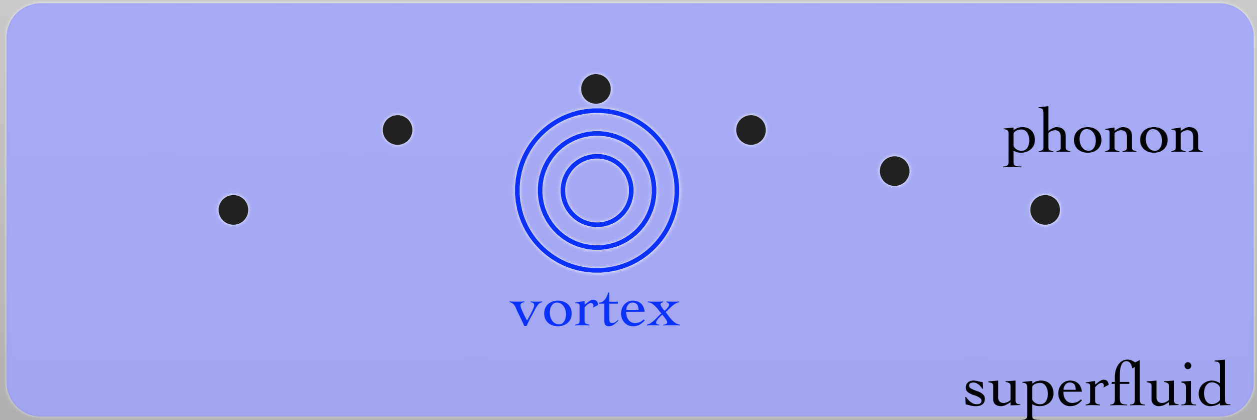
MUTUAL FRICTION

- **Mutual friction**

Force between the superfluid component and the normal component mediated by phonon-vortex interaction

$$\rho_s \frac{d\mathbf{v}_s}{dt} = -\frac{\rho_s}{\rho}\nabla p - \rho_s \nabla\phi - \mathbf{F}^N$$

$$\rho_n \frac{d\mathbf{v}_n}{dt} = -\frac{\rho_n}{\rho}\nabla p - \rho_n \nabla\phi + \mathbf{F}^N + \eta\nabla^2\mathbf{v}_n$$



Forces acting on a Vortex

- **Magnus force** — Standard hydrodynamic force

$$\mathbf{F}^M = \kappa\rho_s(\mathbf{v}_s - \mathbf{v}_L) \times \hat{\mathbf{z}}$$

- **Friction force** — Scattering of phonons off vortices

$$\mathbf{F}^N = D(\mathbf{v}_n - \mathbf{v}_L) + D'\hat{\mathbf{z}} \times (\mathbf{v}_n - \mathbf{v}_L)$$

- **Mutual friction**
  - elastic scattering off vortices: $$\frac{d\sigma}{d\theta} = \frac{c_s}{2\pi E}\frac{\cos^2\theta}{\tan^2\frac{\theta}{2}}\sin^2\frac{\pi E}{\Lambda}$$
  - inelastic scattering on vortices: work in progress

i) At low temperatures most of the system is superfluid, the normal component is dominated by phonons

$$\mathcal{N}_{\rm ph} \sim T^3$$

ii) Need to understand how the friction can be transmitted from the normal component to the superfluid component

iii) The friction will depend on the particular process considered

Examples:
The movement of the superfluid on a surface will be frictionless
The movement of a body in the superfluid will not be frictionless

Phonon contribution

phonon dispersion law $\epsilon_p = c_s p + B p^3 + \mathcal{O}(p^5)$

$$\partial_t \mathcal{N}_{\rm ph} + {\rm div}(\mathcal{N}_{\rm ph}\mathbf{v_n}) = -\frac{\Gamma_{\rm ph}}{T}\mu_{\rm ph}$$

$$\begin{array}{ll} B > 0 & \phi \to \phi\phi \\ B < 0 & \phi\phi \to \phi\phi\phi \end{array}$$

$$\begin{aligned}
\zeta_1 &= -\frac{T}{\Gamma_{\rm ph}}\frac{\partial \mathcal{N}_{\rm ph}}{\partial \rho}\left(\mathcal{N}_{\rm ph} - S\frac{\partial \mathcal{N}_{\rm ph}}{\partial S} - \rho\frac{\partial \mathcal{N}_{\rm ph}}{\partial \rho}\right) = -\frac{T}{\Gamma_{\rm ph}} I_1 I_2 \\
\zeta_2 &= \frac{T}{\Gamma_{\rm ph}}\left(\mathcal{N}_{\rm ph} - S\frac{\partial \mathcal{N}_{\rm ph}}{\partial S} - \rho\frac{\partial \mathcal{N}_{\rm ph}}{\partial \rho}\right)^2 = \frac{T}{\Gamma_{\rm ph}} I_2^2 \\
\zeta_3 &= \frac{T}{\Gamma_{\rm ph}}\left(\frac{\partial \mathcal{N}_{\rm ph}}{\partial \rho}\right)^2 = \frac{T}{\Gamma_{\rm ph}} I_1^2
\end{aligned}$$

Notice that $\zeta_1^2 = \zeta_2 \zeta_3$ the system tends toward the state where bulk viscosity does not lead to dissipation

RELATIVISTIC HYDRODYAMICS

Non dissipative equations

$$\partial_\mu n^\mu = 0 \quad \text{current conservation}$$

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{energy-momentum conservation}$$

$$u^\mu \partial_\mu \phi + \mu = 0 \quad \text{Josephson equation}$$

Adding the most generic dissipative terms

$$\partial_\mu n^\mu = 0$$

$$\partial_\mu (T^{\mu\nu} + T_d^{\mu\nu}) = 0$$

$$u^\mu \partial_\mu \phi + \mu + \chi = 0$$

dissipative terms

Close to equilibrium

$$\chi = -\zeta_3 \partial_\mu (V^2 w^\mu) - \zeta_4 \partial_\mu u^\mu$$

$$\begin{aligned} T_d^{\mu\nu} &= \kappa \left(\Delta^{\mu\gamma} u^\nu + \Delta^{\nu\gamma} u^\mu\right) \left(\partial_\gamma T + T u^\delta \partial_\delta u_\gamma\right) \\ &+ \eta \Delta^{\mu\gamma} \Delta^{\nu\delta} \left(\partial_\delta u_\gamma + \partial_\gamma u_\delta + \frac{2}{3} g_{\gamma\delta} \partial_\alpha u^\alpha\right) \\ &+ \Delta^{\mu\nu} \left(\zeta_1 \partial_\gamma (V^2 w^\gamma) + \zeta_2 \partial_\gamma u^\gamma\right) \end{aligned}$$

where

$$w^\mu = -\left(\partial^\mu \varphi + \mu u^\mu\right)$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

for a rotating superfluid one has to include the mutual friction force

CFL EFFECTIVE ACTION

Fluid of quasiparticles (phonons) moving on the top of the superfluid background

Effective Lagrangian (low energy, asymptotic densities) Son hep-ph/0204199

$$\mathcal{L}_{\rm eff} = \frac{3}{4\pi^2}\left[(\partial_0\varphi - \mu_q)^2 - (\partial_i\varphi)^2\right]^2$$

Scale separation

$$\varphi(x) = \bar{\varphi}(x) + \phi(x)$$

superfluid → $\bar{\varphi}$; phonon → ϕ

bulk → $\bar{\varphi}(x)$; long-wavelength fluctuations → $\phi(x)$

$$S[\varphi] = S[\bar{\varphi}] + \frac{1}{2}\int d^4x \left.\frac{\partial \mathcal{L}_{\rm eff}}{\partial(\partial_\mu\varphi)\partial(\partial_\nu\varphi)}\right|_{\bar{\varphi}} \partial_\mu\phi\partial_\nu\phi + \cdots$$

Phonon's action

$$S[\phi] = \frac{1}{2}\int d^4x\sqrt{-g}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

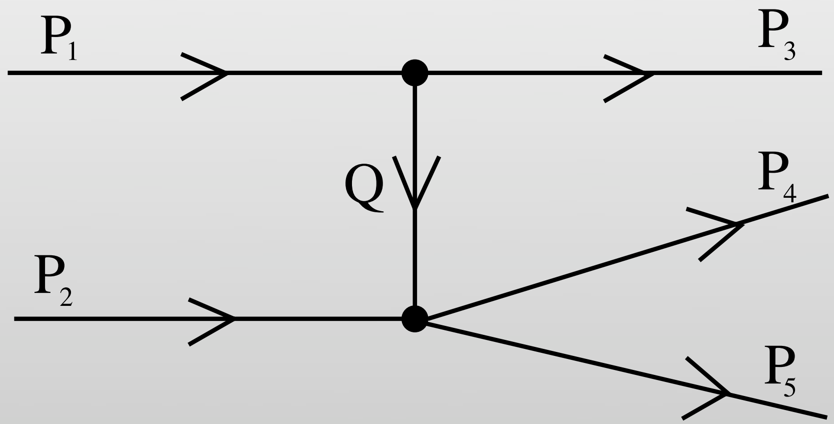
Acoustic metric

$$g_{\mu\nu} = \eta_{\mu\nu} + (c_s^2 - 1)v_\mu v_\nu$$

PHONONS IN CFL

Low temperatures $T \lesssim 0.01$ MeV

- In CFL B<0

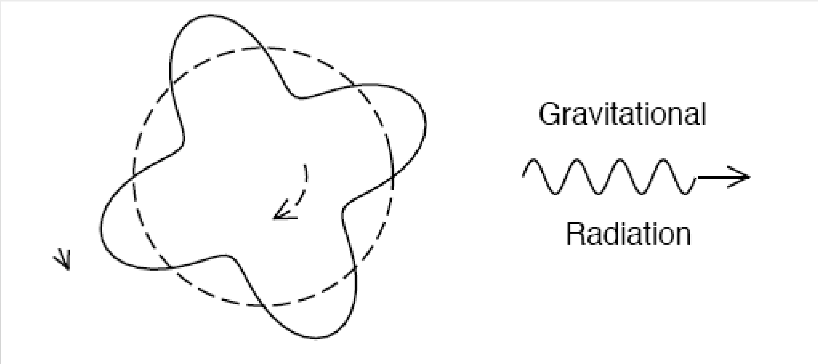


Conformal limit $\zeta_1 = \zeta_2 = 0$ $\zeta_3 \sim \frac{\mu^6}{T\Delta^8}$

Conformal breaking due to m_s $\zeta_1 \sim \frac{m_s^2 \mu^7}{T\Delta^8}$ $\zeta_2 \sim \frac{m_s^4 \mu^8}{T\Delta^8}$

Importance of dissipative processes

R-mode instability



Lindblom, astro-ph/0101136

Modes retrograde in the corotating frame (but prograde in the inertial frame) are unstable.
See also Andersson, Kokkotas gr-qc/0010102

R-mode oscillation difficult to damp in CFL stars
Madsen, Phys. Rev. Lett. 85, 10 (2000)

↓

Emitting gravitational radiation the star quickly spins down

dissipative processes damp these oscillations when $\nu \lesssim 1\,\mathrm{Hz}$

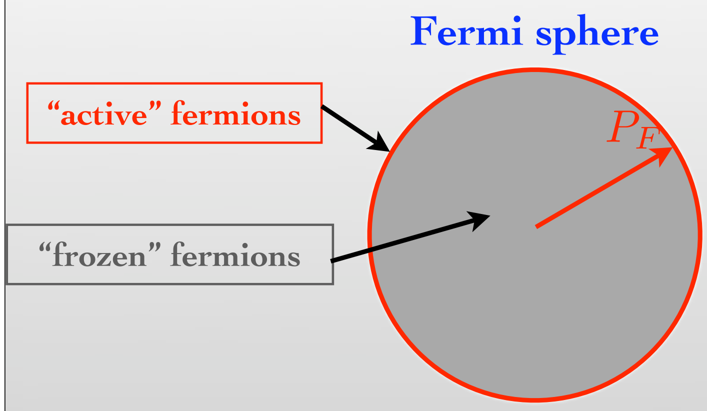
Summary

- CFL is a superfluid
- Contribution of phonons to the bulk viscosity coefficients of CFL
- For rotating superfluids one has to include the mutual friction force
- Damping of star oscillations, especially gravitationally unstable r-modes

Addenda

Degenerate Fermions

- System of degenerate Fermions at high density and low temperature



- ❖ Fermions fill energy levels up to the Fermi energy
- ❖ Exciting fermions deep in the Fermi sphere has an energy cost
- ❖ Fermions close to the Fermi surface can easily scatter

- Cooper theorem: Any arbitrarily attractive interaction ⟶ Cooper pairing

The difermion condensate induces some symmetry breaking

✱ Breaking of global symmetry, the system is superfluid

✱ Breaking of gauge symmetry (chromo) magnetic field is expelled: Meissner effect

Goldstone boson with a linear dispersion law ⟶ Landau criterion for superfluidity is satisfied

COLOR SUPERCONDUCTOR

Cold quark matter at extreme densities

- **Degenerate system of quarks**
- **Attractive interaction between quarks**

Instantons (intermediate density)

One-gluon excange (high density)

$$\mu \gg m_s$$

CFL phase

Alford, Rajagopal, Wilczek hep-ph/9804403

$$< \psi_{\alpha i} C \gamma_5 \psi_{\beta j} > \sim \Delta\, \epsilon_{I\alpha\beta} \epsilon_{Iij}$$

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \to SU(3)_{c+L+R} \times Z_2$$

- $U(1)_B$ **is spontaneously broken: CFL is a superfluid**

GRAVITY ANALOGS

Observation by Unruh

The effective action of a phonon in a fluid is equivalent to the action of a scalar field in a curved background

That is, one can describe the propagation of sound in a fluid as the propagation of a (scalar) particle in a non-flat metric

Use of this analogy

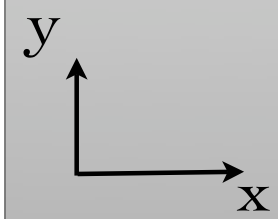
To study some aspects of general relativity, e.g. black hole evaporation

To study some systems using results of general relativity

I will only present few examples of gravity analogs

Gravity analogy
(Child view)

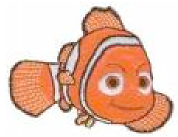
Consider a stream of a fluid

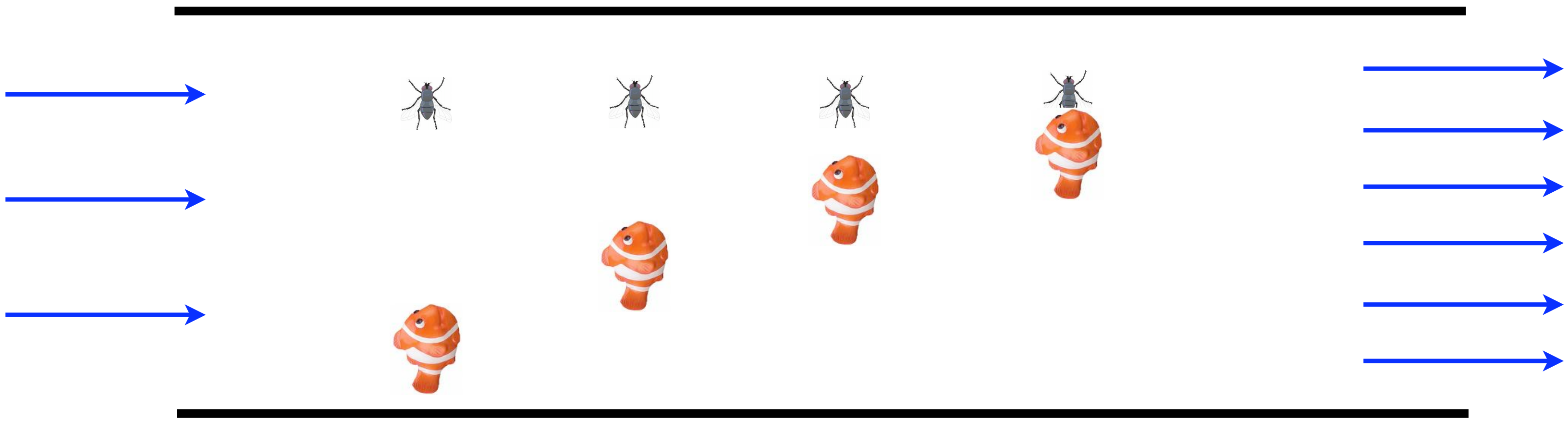


Fluid velocity: $\mathbf{v} = v(x)\hat{\mathbf{x}}$ and $\dfrac{\partial v}{\partial x} > 0$

We want to study the propagation of "particles" in the fluid

GRAVITY ANALOGY
(CHILD VIEW)

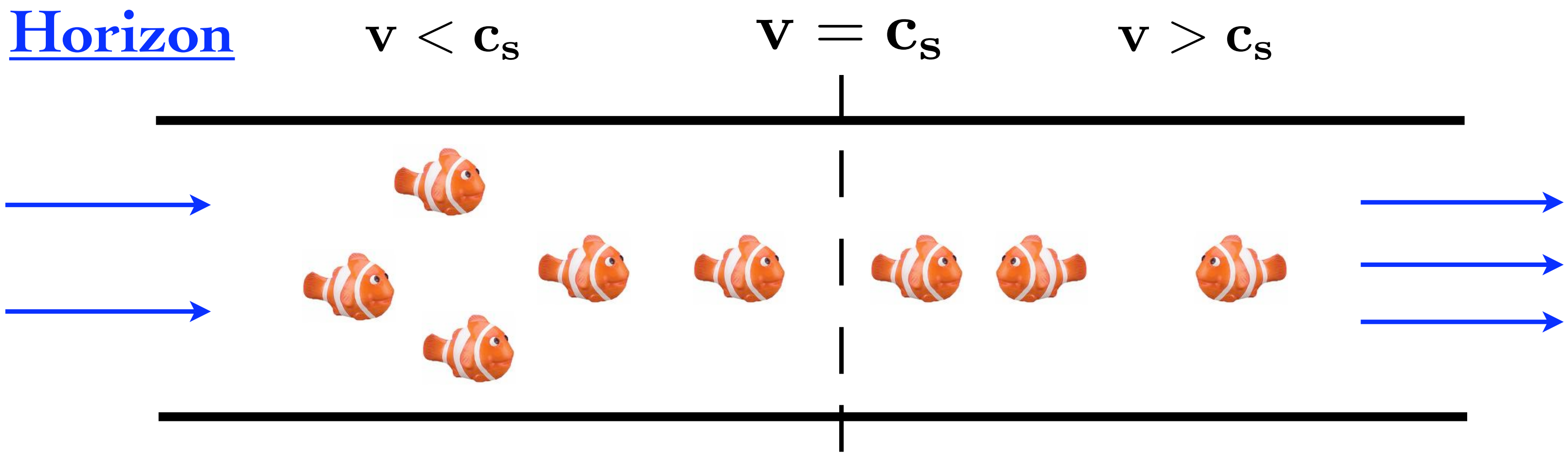
The probe:  moves at constant velocity c_s



The trajectory we observe is bended: $x(y) = \frac{1}{2}\frac{\partial v}{\partial x}\frac{y^2}{c_s^2}$

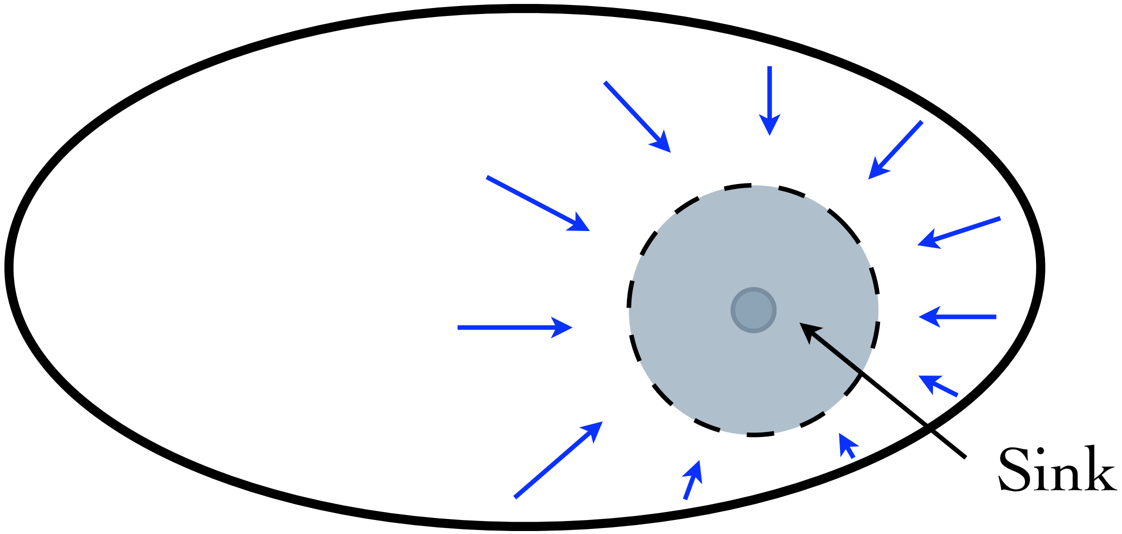
This looks like the bending of light in a free-falling lift

Gravity analogy
(Child view)



Gravity analogy

Trapped surface



Geometrical view

Velocity of sound in the lab frame

$$\frac{d\mathbf{x}}{dt} = c_s \hat{\mathbf{n}} + \mathbf{v}$$

with $\quad |\hat{\mathbf{n}}|^2 = 1$

$$c_s^2 dt^2 - (d\mathbf{x} - \mathbf{v}dt)^2 = 0$$

$$\Updownarrow$$

$$g_{\mu\nu} dx^\mu dx^\nu = 0$$

acoustic metric $\quad g_{\mu\nu} = \left(\begin{array}{c|c} c_s^2 - v^2 & \mathbf{v}^t \\ \hline \mathbf{v} & -I \end{array} \right)$