Large-N Approximation for the Quark-Meson Effective Theory

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QCD in Extreme Conditions, Trondheim, February 23-26, 2010

Captatio Benevolentiae

Dyson-Schwinger approach is a competitive method for quantitative treatment of several relevant field theories, if solved with similar formal rigorousness

like Monte Carlo simulations of the Euclidean path integral.

Outline

- **Scale insensitivity** requirement for effective models of QCD
- From the $U(n) \times U(n)$ symmetric model to the N-vector model
- **QuarkNLO** Renormalisation of the Quark-Meson model
- **MesonNLO** Renormalisation of the $O(N)$ model
- Conclusions

Quark-Meson Effective theory

Thermodynamical degrees of freedom for both phases are present
Widely used for exploration of the thermodynamics of quark-hadron phase transformation:
Scavenius *et al.* (2001), Schaefer & Wambach (2005, 2007); Schaefer & Wagner (2009); Kovács & Szép (2009)

Perturbative renormalisability limited by triviality conveniently ensures scale insensitivity order by order in the meson-fermion coupling, though
is insufficient for (partially) resummed perturbative series

Resummation of perturbative series is incomplete without

COUNTERTERM RESUMMATION

Large N expansion

Devised for strongly coupled field theories

The old testing ground: $O(N)$-model LO
1974: Dolan & Jackiw; Coleman, Jackiw and Politzer; Schnitzer;
1983: Bardeen & Moshe

$O(N)$-model NLO: Root (1974); v. Smekal (1994); Cooper *et al.* (2004/05); Andersen *et al.* (2004, 2008); Fejős *et al.* (2009)

$U(n) \times U(n)$ (quark-meson model)
Interest and difficulty of large n solution: Paterson (1980)
Trick: thinning the meson fields down to an n^2-component vector interacting with quarks in the fundamental rep. of $SU(n)$
Large-n approach to phenomenology: Jakovác *et al.* (2004)
Markó & Szép (in preparation)

The linear (constituent) Quark – (light) Meson model

$$
\begin{aligned}
L_{QM} &= L_M + L_Y, \\
L_M &= \mathrm{Tr}[\partial_\mu M \partial^\mu M^\dagger - m^2 M M^\dagger] \\
&\quad -\frac{g_1}{n^2}(\mathrm{Tr}\, M M^\dagger)^2 - \frac{g_2}{n}\mathrm{Tr}(M M^\dagger)^2 + \sqrt{2n^2} h_0 s^0, \\
L_Y &= \bar\psi(x)(i\gamma_\mu\partial^\mu - m_0)\psi(x) - \frac{g}{\sqrt{n}}\bar\psi(x) M_5(x)\psi(x).
\end{aligned}
$$

$$
M = (s^a + i\pi^a)T^a, \quad M_5 = (s^a + i\gamma_5\pi^a)T^a,
$$
$$
\mathrm{Tr}\, T^a T^b = \tfrac{1}{2}\delta^{ab}, \qquad a = 0, 1, ..., n^2 - 1.
$$

The $n \times n$ meson matrix M transforms according to the $U(n) \times U(n)$ approximate global chiral symmetry

Leading order in n: the meson sector

$$
\begin{aligned}
L_M &= \frac{1}{2}[(\partial_\mu s^a)^2 + (\partial_\mu \pi^a)^2 - m^2((s^a)^2 + (\pi^a)^2)] + \sqrt{2n^2} h_0 s^0 \\
&- \frac{g_1}{4n^2}((s^a)^2 + (\pi^a)^2)^2 - \frac{g_2}{2n}(U^a)^2
\end{aligned}
$$

$$
U^a = \frac{1}{2} d^{abc}(s^b s^c + \pi^b \pi^c) - f^{abc} s^b \pi^c.
$$

Introduction of auxiliary composite fields:

$$
\Delta L_M = -\frac{1}{2}\left(X - i\sqrt{\frac{g_1}{2n^2}}((s^a)^2 + (\pi^a)^2)\right)^2 - \frac{1}{2}\left(Y^a - i\sqrt{\frac{g_2}{n}} U^a\right)^2.
$$

Symmetry breaking pattern:

$$
s^a \to s^a + \sqrt{2n^2} v \delta^{a0}, \qquad U^a \to U^a + 2\sqrt{n} v s^a + n\sqrt{2n}\delta^{a0} v^2
$$

Large-n Dyson-Schwinger equations:

$$X = i\sqrt{\frac{g_1}{2}}n(2v^2 + G_s^{(s)} + G_s^{(\pi)}),\quad Y^0 = i\sqrt{\frac{g_2}{2}}n(2v^2 + G_s^{(s)} + G_s^{(\pi)}),$$
$$M^2 \equiv m^2 - \frac{i}{n}(\sqrt{2g_1}X + \sqrt{2g_2}Y^0),\qquad M^2 v = i\sqrt{g_2}G_{sY} + h_0.$$

Coupled propagator sectors:

$$[G_{s^0,X},\quad G_{s^0,Y^0},\quad G_{X,Y^0},\quad G_{s^0,s^0},\quad G_{Y^0,Y^0},\quad G_{X,X}],$$
$$[G_{s^u,s^v},\quad G_{s^u,Y^v},\quad G_{Y^u,Y^v},\quad u=v\neq 0],\qquad [G_{\pi^u,\pi^v},\quad u=v].$$

Shorthand notations:

$$G_{s^0s^0}\equiv G_0^{(s)},\qquad G_{\pi^0\pi^0}\equiv G_0^{(\pi)},$$
$$G_{\pi^u\pi^u}\equiv G_s^{(\pi)},\quad G_{s^us^u}\equiv G_s^{(s)},\quad G_{s^uY^u}\equiv G_{sY},\quad G_{Y^uY^u}\equiv G_s^{(Y)},\quad u\neq 0.$$

The pion-propagators:

$$\begin{aligned} iG_0^{(\pi)-1} &= iD_0^{-1} - ig_2 G_s^{(Y)} G_s^{(\pi)}, \quad iD_0^{-1}(x,y) = -(\Box + M^2)\delta(x-y) \\ iG_s^{(\pi)-1} &= iD_0^{-1} - ig_2[G_s^{(Y)}(G_s^{(\pi)} + G_s^{(s)}) - G_{sY}G_{Ys}]. \end{aligned}$$

The scalar-"doublets": $\mathcal{G}_{(s,Y)}^{-1} = \begin{pmatrix} G_{ss}^{-1} & G_{sY}^{-1} \\ G_{Ys}^{-1} & G_{YY}^{-1} \end{pmatrix}$

$$\begin{aligned} G_{ss}^{-1} &= iD_0^{-1} - ig_2[G_s^{(Y)}(G_s^{(s)} + G_s^{(\pi)}) + G_{sY}G_{Ys}] \\ G_{sY}^{-1} = G_{Ys}^{-1} &= -2i\sqrt{g_2}v - ig_2 G_{Ys}(G_s^{(s)} - G_s^{(\pi)}) \\ G_{YY}^{-1} &= -I - i\frac{g_2}{2}[\mathcal{A} + 2G_s^{(s)}G_s^{(\pi)}] \end{aligned}$$

$\mathcal{A} = (G_s^{(s)})^2 + (G_s^{(\pi)})^2$

The scalar "triplet": $\mathcal{G}^{-1}_{(X,Y^0,s^0)} = \begin{pmatrix} G^{-1}_{XX} & G^{-1}_{XY^0} & G^{-1}_{Xs^0} \\ G^{-1}_{XY^0} & G^{-1}_{Y^0Y^0} & G^{-1}_{Y^0s^0} \\ G^{-1}_{s^0X} & G^{-1}_{s^0Y^0} & G^{-1}_{s^0s^0}. \end{pmatrix}.$

$$\begin{aligned} G^{-1}_{XX} &= -I - ig_1\mathcal{A}, \\ G^{-1}_{XY^0} = G^{-1}_{Y^0X} &= -i\sqrt{g_1g_2}\mathcal{A} \\ G^{-1}_{Xs^0} = G^{-1}_{s^0X} &= 2i\sqrt{g_1}v - 2i\sqrt{g_1g_2}G_{Ys}G_s^{(s)} \\ G^{-1}_{Y^0Y^0} &= -I - ig_2\mathcal{A}, \\ G^{-1}_{Y^0s^0} = G^{-1}_{s^0Y^0} &= 2i\sqrt{g_2}v - 2ig_2G_{Ys}G_s^{(s)} \\ G^{-1}_{s^0s^0} &= iD_0^{-1} - 2ig_2(G_s^{(s)}G_s^{(Y)} + G_{sY}G_{Ys}). \end{aligned}$$

The CJT effective potential:

$$V^{CJT}_{class}=n^2(m^2v^2+2h_0v)+\frac{1}{2}(X^2+(Y^0)^2)+i\sqrt{2}n(\sqrt{g_2}Y^0+\sqrt{g_1}X)v^2$$

$$\begin{aligned}V^{CJT}_{1-loop} &= -i\frac{n^2}{2}\int_k(\ln G_s^{(\pi)-1}(k)+D_0^{-1}(k)G_s^{(\pi)}(k))\\ &\quad -i\frac{n^2}{2}\int \mathrm{Tr}[\ln \mathcal{G}^{-1}_{(s,Y)}(k)+\mathcal{D}^{-1}_{(s,Y)}(k)\mathcal{G}_{(s,Y)}(k)]\end{aligned}$$

$$D_{ss}^{-1}(k)=iD_0^{-1},\quad D_{sY}^{-1}=D_{Ys}^{-1}=-2i\sqrt{g_2}v,\quad D_{YY}^{-1}=I.$$

$$\begin{aligned}V^{CJT}_{2-loop} &= i\frac{n^2}{4}g_2\int_k\int_p[(G_s^{(\pi)}(k)G_s^{(\pi)}(k+p)\\ &\quad +G_s^{(s)}(k)G_s^{(s)}(k+p))G_s^{(Y)}(p)+2G_{sY}(k)G_{Ys}(k+p)G_s^{(s)}(p)]\\ &\quad +i\frac{n^2}{2}g_2\int_k\int_p(G_s^{(Y)}(k)G_s^{(s)}(k+p)-G_{sY}(k)G_{Ys}(k+p))G_s^{(\pi)}(p).\end{aligned}$$

V_{ct}^{CJT} =???, **LEADING ORDER SOLUTION NOT KNOWN**

Approximation: (Assumed) heavy scalar dynamics neglected
All pions share the mass determined by the saddle point equations:

$$m^2(\pi^a,\ a = 0, 1, ..., n^2 - 1) = M^2.$$

Saddle point equations:

$$X = i\sqrt{\frac{g_1}{2}}n(2v^2 + T_d^{(2)} + (M^2 - M_0^2)T_d^{(0)} + T_\pi^F), \qquad \sqrt{g_1}Y^0 = \sqrt{g_2}X.$$

Construction of the corresponding counterterms is obvious
The gap equation for the determination of M^2:

$$X = i\sqrt{\frac{g_1}{2}}n(2v^2 + T_\pi^F), \qquad M^2 v = h_0.$$

Determinant equations for the dispersion relations of scalar excitations ($I_{\pi\pi}$ the pion bubble):

$$\mathrm{s}Y: \qquad k^2 = M^2 + 4g_2 v^2 \frac{1}{1 - \frac{g_2}{2} I_{\pi\pi}(k, M)}$$

$$\mathrm{X}Y^0 s^0: \qquad k^2 = M^2 + 4(g_1 + g_2) v^2 \frac{1}{1 - (g_1 + g_2) I_{\pi\pi}(k, M)}.$$

Renormalisations characteristic for $O(N)$ symmetric models:

$$\frac{1}{g_{2R}} = \frac{1}{g_2} - \frac{1}{2} I_{\pi\pi}^{div}, \qquad \frac{1}{(g_1 + g_2)_R} = \frac{1}{g_1 + g_2} - I_{\pi\pi}^{div}$$

Scalar masses selfconsistently larger than pseudoscalar's.
Retain $N = n^2$-vector pions and the SB-agent s^0:
$O(N)$-model for mesons + Yukawa coupling to the quarks

Renormalised QuarkNLO Approximation

Slightly modified notations with $N_f^2 = N,\ \lambda \leftrightarrow g_1 + g_2,\ \sigma \leftrightarrow s^0$:

$$
\begin{aligned}
L &= L_{O(N)} + L_Y, \\
L_{O(N)} &= \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi^a)^2 \\
&\quad -\frac{1}{2}m^2(\sigma^2 + (\pi^a)^2) - \frac{\lambda}{24N}(\sigma^2 + (\pi^a)^2)^2 - h\sqrt{N}\sigma, \\
L_Y &= i\bar{\psi}(x)\gamma_\mu\partial^\mu\psi(x) - \frac{g}{\sqrt{N}}\bar{\psi}(x)[\sigma(x) + i\sqrt{2N_f}\gamma_5 T^a\pi^a(x)]\psi(x).
\end{aligned}
$$

ChSB: $\sigma \to \sigma + \sqrt{N}v \quad \to$ constituent quark mass: gv

Single auxiliary field: $\alpha \sim \sigma^2 + (\pi^a)^2$

Free propagators

$$
iD_\pi^{-1}(k) = k^2 - m^2 + i\alpha, \qquad iD_\psi^{-1}(k) = \gamma_\mu k^\mu - gv
$$

$\mathcal{O}(1/\sqrt{N})$ accurate effective potential
for the 1- and 2-point Dyson-Schwinger equations

$$\begin{aligned} V[v,\alpha,G_\pi,G_\psi] &= \frac{1}{2}(m^2 - i\alpha)Nv^2 + \frac{3N}{2\lambda}\alpha^2 + hNv \\ &-\frac{i}{2}N\int_k (\ln G_\pi^{-1}(k) + D_\pi^{-1}(k)G_\pi(k)) \\ &+iN_f N_c \int_k \mathrm{tr}_D(\ln G_\psi^{-1}(k) + D_\psi^{-1}(k)G_\psi(k)) \\ &-\frac{i}{2}g^2 N_f N_c \int_p\int_q\int_r \mathrm{tr}_D(\gamma_5 G_\psi(p)\gamma_5 G_\psi(q))G_\pi(r) \\ &\qquad\qquad \times F_{\pi\psi\bar\psi}(r,p,q)\delta(r+p-q) \end{aligned}$$

Classical $\pi\bar\psi\psi$ vertex: $F_{\pi\psi\bar\psi}(r,p,q) = 1$ breaks chiral WI

Self-consistent fermion propagator

$$
\begin{aligned}
0 &= \frac{3N}{\lambda}\alpha - i\frac{N}{2}\left(v^2 + \int_k D_\pi(k)\right) \\
&\quad - iN_c\frac{g^2 N_f}{2}\int_k D_\pi^2(k)\int_p tr_D\left(\gamma_5 G_\psi(p)\gamma_5 G_\psi(k+p)\right), \\
0 &= (m^2 - i\alpha)Nv + Nh - gN_f N_c\int_k tr_D G_\psi(k), \\
0 &= \frac{N}{2}\left(G_\pi^{-1}(k) - D_\pi^{-1}(k)\right) \\
&\quad - N_c\frac{g^2 N_f}{2}\int_p tr_D\left(\gamma_5 G_\psi(p)\gamma_5 G_\psi(k+p)\right), \\
0 &= G_\psi^{-1}(k) - D_\psi^{-1}(k) + g^2\int_p \gamma_5 G_\psi(p)\gamma_5 D_\pi(k-p).
\end{aligned}
$$

Parametrisation: $$iG_\psi^{-1}(k)=\gamma_\mu k^\mu S_1(k^2)-gvS_2(k^2).$$

Normalisation: $$iG_\psi^{-1}(k=0)=-gv,\qquad \frac{\partial iG_\psi^{-1}}{\partial k^\mu}\bigg|_{k=0}=\gamma_\mu,$$

Renormalised equations:

$$S_1(k^2)+\frac{k^2}{2}\frac{dS_1(k^2)}{dk^2}=1-i\frac{g^2}{2}\int_p\frac{S_1(p^2)}{p^2S_1^2(p^2)-(gv)^2S_2^2(p^2)}\times\left[\frac{p(k+p)}{((k+p)^2-M^2)^2}-\frac{p^2}{(p^2-M^2)^2}\right],$$

$$S_2(k^2)=1-ig^2\int_p\frac{S_2(p^2)}{p^2S_1^2(p^2)-(gv)^2S_2^2(p^2)}\times\left[\frac{1}{(k+p)^2-M^2}-\frac{1}{p^2-M^2}\right].$$

Wick rotation & Euclidean angular integration:

$$S_1(k_E^2)+\frac{1}{2}k_E^2S_1'(k_E^2)=1-\frac{g^2}{16\pi^2}\int_0^\infty dp_E\frac{p_E^3S_1(p_E^2)}{p_E^2S_1^2(p_E^2)+(gv)^2S_2^2(p_E^2)}$$
$$\times\left[\frac{p_E^2}{(p_E^2+M^2)^2}-\frac{(\sqrt{A}-1)^2}{4k_E^2\sqrt{A}}+\frac{(\sqrt{A}-1)^4[(k_E+p_E)^2+M^2]}{16k_E^2p_E^2\sqrt{A}}\right],$$
$$S_2(k_E^2)=1-\frac{g^2}{8\pi^2}\int_0^\infty dp_E\frac{p_E^3S_2(p_E^2)}{p_E^2S_1^2(p_E^2)+(gv)^2S_2^2(p_E^2)}$$
$$\times\left[\frac{1}{p_E^2+M^2}-\frac{(\sqrt{A}-1)^2[(k_E+p_E)^2+M^2]}{4k_E^2p_E^2}\right],$$

$$A=\frac{(k_E-p_E)^2+M^2}{(k_E+p_E)^2+M^2}$$

Chiral case $h = 0 \rightarrow M^2 \sim \mathcal{O}(1/\sqrt{N})$

For the SPE and EoS one needs:
– Numerical solution for arbitrary v
– **Asymptotic behavior** for renormalising the fermion tadpole and bubble! (strict 2PI: Reinosa (2006))

Exact equations

$$S_1 + \frac{1}{4} k_E \frac{dS_1}{dk_E} = 1 - \frac{g^2}{16\pi^2} \int_0^{k_E} dp_E \frac{p_E S_1}{p_E^2 S_1^2 + m_\psi^2 S_2^2},$$

$$S_2 = 1 - \frac{g^2}{8\pi^2} \int_0^{k_E} dp_E \frac{p_E^3 S_2}{p_E^2 S_1^2 + m_\psi^2 S_2^2} \left(\frac{1}{p_E^2} - \frac{1}{k_E^2} \right).$$

Search for the asymptotic solution when $k >> m_\psi = gv$

$$S_{i,a} \sim S_i^{(0)}[\log(k_E/\mu)] + \frac{m_\psi^2}{k_E^2} S_i^{(-1)}[\log(k_E/\mu)]$$

The leading asymptotic equations:

$$\Delta S_1^{(0)} + \frac{1}{4}\frac{d\Delta S_1^{(0)}}{dx} = -\frac{g^2}{16\pi^2}\int_{x_0}^{x} dy \frac{1}{1+\Delta S_1^{(0)}}, \quad \Delta S_1^{(0)} = S_1^{(0)} - 1$$

$$S_2^{(0)} = 1 - \frac{g^2}{8\pi}\int_{k_0}^{k_E}\frac{dp}{p}\frac{S_2^{(0)}}{(S_1^{(0)})^2}\left(1-\frac{p^2}{k^2}\right), \quad x = \log(k_E/\mu).$$

Counterterm function renormalising the EoS (quark-tadpole)

$$\delta V_{ct,1} = \frac{N}{2}\delta m^2 v^2 + \frac{N}{24}\delta\lambda v^4,$$

$$\delta m^2 = -\frac{4g^2 N_c}{\sqrt{N}}\int_k \left(\frac{S_2^{(0)}}{k^2(S_1^{(0)})^2 + M_0^2} + M_0^2\frac{S_2^{(0)}}{(k^2(S_1^{(0)})^2+M_0^2)^2}\right),$$

$$\frac{\delta\lambda}{24} = -\frac{g^4 N_c}{\sqrt{N}}\int_k \left(\frac{S_2^{(0)}(2S_1^{(0)}S_1^{(-1)} + (S_2^{(0)})^2) + S_2^{(-1)}}{(k^2(S_1^{(0)})^2+M_0^2)^2}\right).$$

Counterterm functional renormalising the pion propagator (quark bubble):

$$\delta V_{ct,2} = \frac{N}{2}\int_k \left(-k^2\delta Z + \delta m_2^2 + \frac{\delta\lambda_2}{6}v^2\right) G_\pi(k),$$

$$-\delta Z = \frac{g^2 N_c}{\sqrt{N}}\int_k \frac{(S_1^{(0)})^2}{(k^2(S_1^{(0)})^2 + M_0^2)^2},$$

$$\delta m_2^2 = -\frac{4g^2 N_c}{\sqrt{N}}\int_k \left[\frac{1}{k^2(S_1^{(0)})^2 + M_0^2} + \frac{M_0^2}{(k^2(S_1^{(0)})^2 + M_0^2)^2}\right],$$

$$\frac{\delta\lambda_2}{24} = -\frac{g^4 N_c}{\sqrt{N}}\int_k \frac{2S_1^{(0)}S_1^{(-1)} + (S_2^{(0)})^2}{(k^2(S_1^{(0)})^2 + M_0^2)^2}.$$

– „Back-reaction" on EoS (with $M_{LO}^2 = 0$) modifies only δm^2!
– $\delta m^2 \neq \delta m_2^2, \delta\lambda \neq \delta\lambda_2$ reflects violation of WI.

Counterterm function renormalising the saddle point equation:

$$0 = \frac{3N}{\lambda}(-i\alpha) - \frac{N}{2}\left(v^2 + \int_k \frac{i}{k^2}\right) - \frac{iN}{2}\int_k \frac{1}{k^4}\left[M^2 - \frac{g^2 N_c}{\sqrt{N}} B^F_{\psi\psi}(k)\right]$$

$$B_{\psi\psi} = -i\int_p \mathrm{tr}_D[\gamma_5 G_\psi(p)\gamma_5 G_\psi(k-p)]$$

Overall divergence of the second integral arises from $\sim M^2$ and the part of the renormalised fermion bubble integrals $\sim Ak^2 + B(gv)^2$

Corresponding counterterm expression: $\sim \delta\kappa_s\alpha + \delta\kappa_{\alpha v}\alpha v^2$

„Back reaction" on EoS: $\rightarrow 2\delta\kappa_{\alpha v}\alpha v$

At LO $\alpha = -im^2 \rightarrow$ mere modification of δm^2 in EoS.

The quark-meson model is renormalised consistently in the chiral limit at NLO

The problem: Ward Identity and the Goldstone-theorem

$$\sqrt{\frac{2}{\sqrt{N}}} v\Gamma^3_{\pi^a\psi\bar{\psi}}(0,p,p) = iT^a[\gamma_5, iG_\psi^{-1}(p)]_+,$$

$$F_{\pi\psi\bar{\psi}}(0,k,k) = S_2(k)$$

$$-\frac{h}{v} = M^2 - \frac{4ig^2N_c}{\sqrt{N}}\int_k \frac{S_2(k)}{k^2S_1^2(k)-(gv)^2S_2^2(k)}$$

$$-iG_\pi^{-1}(0) = M^2 - \frac{4ig^2N_c}{\sqrt{N}}\int_k \frac{F_{\pi\psi\bar{\psi}}(0,k,k)}{k^2S_1^2(k)-(gv)^2S_2^2(k)}$$

Non-perturbative renormalisation of the *ad hoc* modified theory (e.g. $F_{\pi\psi\bar{\psi}}(p,k,k+p) \equiv S_2(k)$) can proceed along the same lines as above.

Approximate QNLO

$$iG_\psi^{-1} \longrightarrow \quad \text{tree level} \quad D_\psi^{-1} = k_\mu \gamma^\mu - gv$$

Chiral limit: Jakovác *et al.* (2004)

Physical pion + Polyakov-background:
Markó & Szép (in preparation)

– Extra contributions to EoS: $\left.\frac{\delta V}{\delta G_\psi}\right|_{G_\psi = D_\psi} \times \frac{\partial D_\psi}{\partial v} \neq 0$

– Evaluation of quark tadpole and quark bubble provides explicit counterterms

Saddle Point Equation

$$V_{ct}^{\alpha} = \frac{3N}{2}\delta\left(\frac{1}{\lambda}\right)\alpha^2 + \delta\kappa\alpha + \frac{i}{2}N\delta\kappa_\alpha \alpha v^2$$

$$\begin{aligned} 0 \;=\; & \left(\frac{6}{\lambda} + \delta\left(\frac{1}{\lambda}\right)\right) i\alpha + i\frac{2}{N}\delta\kappa_1 + v^2(1+\delta\kappa_\alpha) + \int_k \frac{i}{k^2 - M^2} \\ & - i\frac{2g^2 N_c}{\sqrt{N}} \int_k \frac{2T_\psi^F - k^2 I_\psi^F(k)}{(k^2 - M^2)^2} \end{aligned}$$

T_ψ^F, $I_\psi^F(k)$: finite parts of the bosonic tadpole and bubble with m_ψ

Renormalisation conditions:

$$\delta\left(\frac{1}{\lambda}\right) = T_d^{(0)}, \qquad \delta\kappa_\alpha = \frac{2g^4 N_c}{N_f} T_d^{(I)}$$

$$\delta\kappa = -\frac{N}{2} T_d^{(2)} - g^2\sqrt{N} N_c T_d^{(2,I)} - \frac{N}{2}(m^2 - M_0^2) T_d^{(0)}$$

Cut-off integrals:

$$T_d^{(2)} = i\int_k \frac{1}{k^2 - M_0^2}, \qquad T_d^{(0)} = i\int_k \frac{1}{(k^2 - M_0^2)^2},$$

$$T_d^{(2,I)} = \frac{i}{16\pi^2}\int_k \frac{k^2}{(k^2 - M_0^2)^2} \ln\frac{|k|^2}{2M_0^2},$$

$$T_d^{(0,I)} = \frac{i}{8\pi^2}\int_k \frac{1}{(k^2 - M_0^2)^2} \ln\frac{e|k|^2}{M_0^2}$$

Pion Propagator

$$iG_\pi^{-1}(k) = k^2\left(1 - \frac{2g^2 N_c}{\sqrt{N}} I_\psi^F(k)\right) - M^2 + \frac{4g^2 N_c}{\sqrt{N}} T_\psi^F,$$

$$V_{ct}^\pi = -\frac{N}{2}\int_k (\delta Z k^2 - \delta m^2 + i\alpha\delta\kappa_0 + \delta\kappa_v v^2) G_\pi(k)$$

$$\delta Z_\pi = \frac{2g^2 N_c}{\sqrt{N}} T_d^0, \quad \delta m^2 = \frac{4g^2 N_c}{\sqrt{N}} (T_d^{(2)} - M_0^2 T_d^0)$$

$$\delta\kappa_0 = 0, \quad \delta\kappa_v = -\frac{4g^2 N_c}{\sqrt{N}} T_d^0.$$

Equation of State

$$
\begin{aligned}
-hN &= Nv(m^2+\delta m_2^2 - i\alpha(1+\delta\kappa_\alpha)) + \delta\kappa_4 v^3 - Nv\delta\kappa_v \int_k D_\pi(k) \\
&\quad -\sqrt{N}N_c(g+\delta g)\int_k \mathrm{tr}_D D_\psi(k).
\end{aligned}
$$

$$
\delta g = 0, \quad \delta\kappa_4 = \frac{4N_c g^4}{\sqrt{N}} T_d^{(0)}, \quad \delta m_2^2 = \delta\kappa_v (T_d^{(2)} + (m^2 - M_0^2)T_d^{(0)})
$$

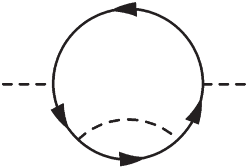
Contradictory renormalisation conditions:

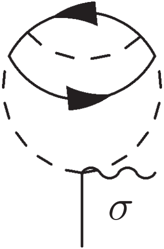
$$
\delta\kappa_\alpha = \delta\kappa_v T_d^{(0)}, \qquad \delta\kappa_v = 0
$$

The contribution from $\delta V/\delta G$ does not modify the conclusion: $\mathcal{O}(g^4)$ violation of renormalisability if D_ψ is used.

G. Markó & Zs. Szép:

Conjectured renormalisation by including $\mathcal{O}(g^4)$ contributions to the pion propagator and EoS:



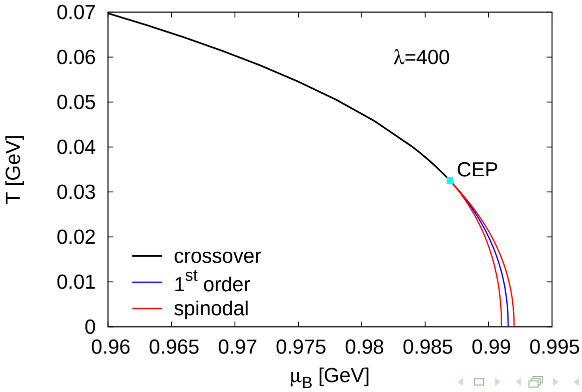


Regularized equations at finite T and ρ_B are used to explore phase structure of the model at $\sqrt{N} = 2$.

$\mu_B - T$ phase diagram using approximate pion propagator

- $N_f = 2$ pion propagator $iG_\pi^{-1}(k) = k^2 - M_\pi^2$

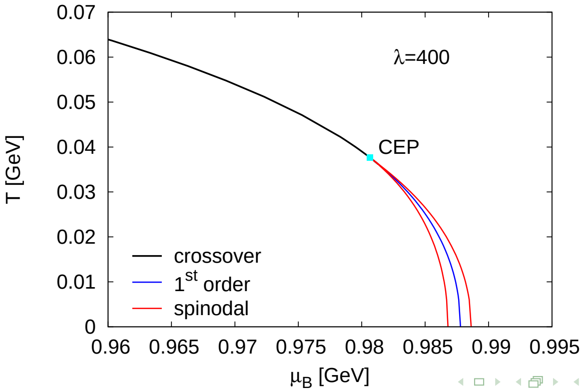
a., $M_\pi^2 = M^2[M_\pi^2] - \frac{4g^2N_c}{\sqrt{N}} T_\psi^F$



$\mu_B - T$ phase diagram using approximate pion propagator

- $N_f = 2$ pion propagator $iG_\pi^{-1}(k) = k^2 - M_\pi^2$

$$b.,\quad M_\pi^2 = M^2[M_\pi^2] - \frac{4g^2N_c}{\sqrt{N}}T_\psi^F + \frac{2M_\pi^2 g^2}{\sqrt{N}} I_\Psi^F(M_\pi)$$



Effective action for NLO DSE's of the **O(N) model**

$\mathcal{G}$: propagator matrix of the coupled $\sigma - \alpha$ sector

$$V[\alpha, v, G_\pi, \mathcal{G}] = \frac{1}{2}\left(m^2 - i\alpha\right) N v^2 + \frac{3N}{2\lambda}\alpha^2$$

$$- \quad \frac{i}{2}\int_k \Big[(N-1)\left(\ln G_\pi^{-1}(k) + D_\pi^{-1}(k) G_\pi(k)\right) + \mathrm{Tr}\ln \mathcal{G}^{-1}(k)$$

$$+\mathrm{Tr}\left(\mathcal{D}^{-1}(k)\mathcal{G}(k)\right)\Big]$$

$$+ \quad \frac{i\lambda}{12N}\int_k \int_p \Big[G_{\alpha\alpha}(k)\Big((N-1) G_\pi(p) G_\pi(p+k) + G_{\sigma\sigma}(p) G_{\sigma\sigma}(p+k)\Big)$$

$$+2 G_{\alpha\sigma}(p) G_{\sigma\sigma}(k) G_{\sigma\alpha}(p+k)\Big] + \Gamma_{ct}[\alpha, v, G_\pi, \mathcal{G}].$$

NLO-counterterm construction

LO propagator expressions/relations are exploited!

Pion propagator:

$$iG_\pi^{-1}(k)=iD_\pi^{-1}(k)-i\frac{\lambda}{3N}\int_p G^{(0)}_{\alpha\alpha}(p)D_\pi(p+k)+c.t.\ .$$

$$V^\pi_{ct}=-\frac{\lambda}{6}\left[T_d^{(2,I)}-\frac{\lambda}{2}(\mathbf{M}^2-M_0^2)T_d^{(0,I)}\right]\int_k G_\pi(k).$$

EoS:

$$-h=NvM^2-i\sqrt{\frac{\lambda}{3}}\int_k G^{(0)}_{\alpha\sigma}(k)+c.t.$$

$$V^v_{ct}=-\frac{\lambda}{6}v^2\left[T_d^{(2,I)}-\frac{\lambda}{2}(\mathbf{M}^2-M_0^2)T_d^{(0,I)}\right].$$

Counterterm-consistency

Renormalisation of the saddle point equation

$$\begin{aligned} 0 \quad = \quad & \frac{3N}{\lambda}\alpha - i\frac{N}{2}\left(v^2 + \int_k G_\pi(k)\right) - \frac{i}{2}\int_k \Big(G_{\sigma\sigma}(k) - G_\pi(k)\Big) \\ & + i\frac{N}{2}\left[T_d^{(2)} + (M^2 - M_0^2)T_d^{(0)}\right] \\ & - i\frac{\lambda^2}{12}T_d^{(0,I)}\left(v^2 + \int_k G_\pi(k)\right) + ct. \end{aligned}$$

In RED: „back-reaction" of the counterterms relaed to G_π and EoS
Consistently cancelled by part of the NLO-divergences

Only α-dependent counterterm is generated by the saddle point equation: $V_{ct}^{\alpha}[\alpha]$.

Full NLO counter-functional

$$
\begin{aligned}
V_{ct}[\alpha, v, G_\pi, \mathcal{G}] &= \frac{1}{2}\left(\delta m^2 - i\delta g\alpha\right) v^2 + i\delta\kappa_1\alpha + \delta\kappa_2\alpha^2 \\
&+ \frac{1}{2}(\delta m^2 - i\delta g\alpha)\int_k G_\pi(k) + \frac{1}{2}\delta\kappa_0\int_k G_{\alpha\alpha}(k)
\end{aligned}
$$

Remarkable:

$$
\frac{6}{\lambda_B} = \frac{6}{\lambda} + \frac{4}{N}\delta\kappa_2 = \frac{6}{\lambda} + \frac{N+8}{N} T_d^{(0)} + 2\frac{\lambda}{N} T_d^{(0,I)}\left(1 + \frac{\lambda}{6}T_d^{(0)} + \frac{\lambda}{64\pi^2}\right).
$$

Details: G. Fejős, A.P,. Zs. Szép: PRD**80** 025015

Conclusions

- It is relatively easy to regularize (partially) resummed perturbative series
- Renormalisation is harder: both renormalisable and (consistently) non-renormalisable approximations to the quark-meson effective model were presented
- Only renormalised approximations merit comparison with results of other (mostly lattice field theoretical) approaches