The QCD phase diagram from an effective point of view

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on

QCD in Extreme Conditions

Trondheim, Norway

QCD Phase Transitions

QCD: two phase transitions:

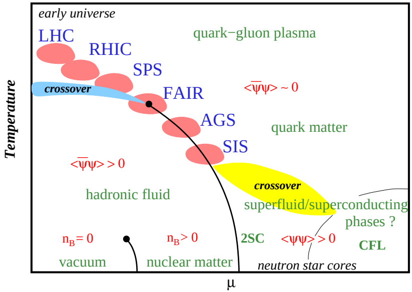
1. restoration of chiral symmetry

   $SU_{L+R}(N_f) \longrightarrow SU_L(N_f) \times SU_R(N_f)$

   order parameter:

   $$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken}, T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase}, T > T_c \end{cases}$$

   associate limit: $m_q \longrightarrow 0$

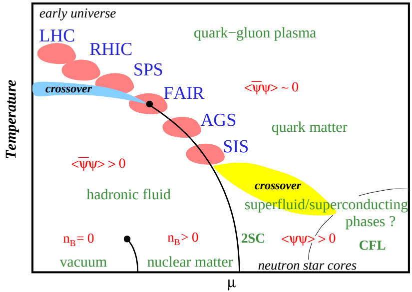


chiral transition: spontaneous restoration of global $SU_L(N_f) \times SU_R(N_f)$ at high T

QCD Phase Transitions

QCD: two phase transitions:

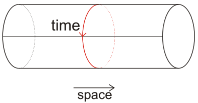
1. restoration of chiral symmetry
2. de/confinement (center symmetry)



order parameter:

$$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase}, & T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase}, & T > T_c \end{cases}$$

$$\Phi = \frac{1}{N_c} \langle \mathrm{tr}_c \mathcal{P} e^{i \int_0^\beta d\tau A_0(\tau, \vec{x})} \rangle$$



associate limit: $m_q \to \infty$

➜ related to free energy of a static quark state: $\Phi = e^{-\beta F_q}$

QCD Phase Transitions

QCD: two phase transitions:

1. restoration of chiral symmetry
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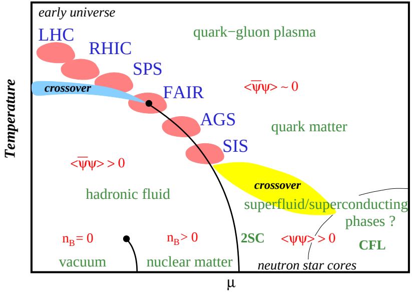
$$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase,} & T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase,} & T > T_c \end{cases}$$

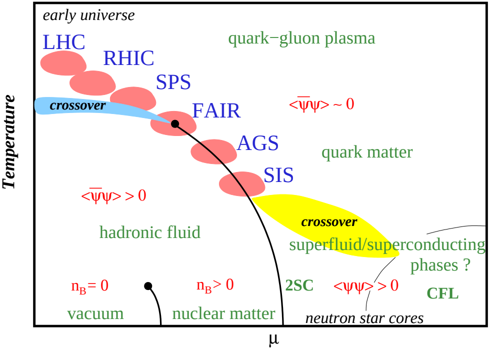


alternative:

➜ dressed Polyakov loop (or dual condensate)

it relates chiral and deconfinement transition to spectral properties of Dirac operator

The conjectured QCD Phase Diagram



At densities/temperatures of interest
only model calculations available

Open issues:

related to chiral & deconfinement transition

- ▷ existence of CEP?
- ▷ its location?
- ▷ additional CEPs? How many?
- ▷ coincidence of both transitions at $\mu = 0$?
- ▷ quarkyonic phase at $\mu > 0$?
- ▷ chiral CEP/ deconfinement CEP?
- ▷ so far only MFA results effect of fluctuations (e.g. size of crit. reg.)?
- ▷ ...

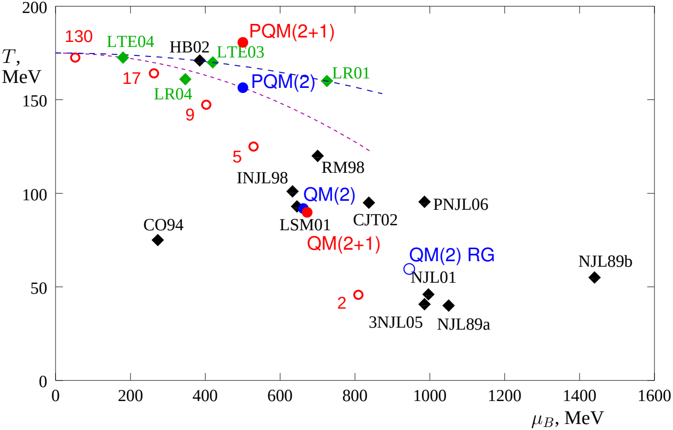
effective models:

1. Quark-meson model (renormalizable) — or other models e.g. NJL
2. Polyakov–quark-meson model — or PNJL models

Charts of QCD Critical End Points

model studies vs. lattice simulations

Black points: models   Lines & green points: lattice   Red points: Freezeout points for HIC



Stephanov '05 & '07

lattice methods:

- reweighting
- imaginary μ_B
- Taylor expansion around $\mu_B = 0$

Outline

- ○ **Three-Flavor Quark-Meson Model**
- ○ **...with Polyakov loop dynamics**
- ○ **Finite density extrapolations**

$N_f = 3$ Quark-Meson (QM) model

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

Quark part with Yukawa coupling g:

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial\!\!\!/ - g\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

Meson part: scalar σ_a and pseudoscalar π_a nonet

fields: $\phi = \sum_{a=0}^{8} \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$

$$\mathcal{L}_{\text{meson}} = \text{tr}[\partial_\mu\phi^\dagger\partial^\mu\phi] - m^2\text{tr}[\phi^\dagger\phi] - \lambda_1(\text{tr}[\phi^\dagger\phi])^2 - \lambda_2\text{tr}[(\phi^\dagger\phi)^2] + c[\det(\phi) + \det(\phi^\dagger)] + \text{tr}[H(\phi + \phi^\dagger)]$$

- explicit symmetry breaking matrix: $H = \sum_a \frac{\lambda_a}{2} h_a$
- $U(1)_A$ symmetry breaking implemented by 't Hooft interaction

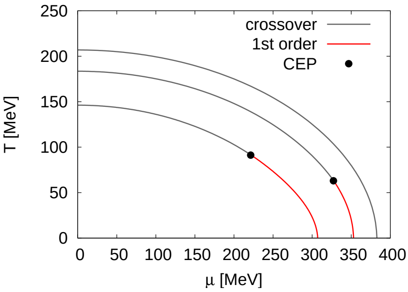
Phase diagram for $N_f = 2 + 1 \quad (\mu \equiv \mu_q = \mu_s)$

- Model parameter fitted to (pseudo)scalar meson spectrum:
- PDG: $f_0(600)$ mass=$(400 \ldots 1200)$ MeV → broad resonance

➜ existence of CEP depends on m_σ!

Example: $m_\sigma = 600$ MeV (lower lines), 800 and 900 MeV (here mean-field approximation)

with $U(1)_A$ **[BJS, M. Wagner '09]**



Mass sensitivity

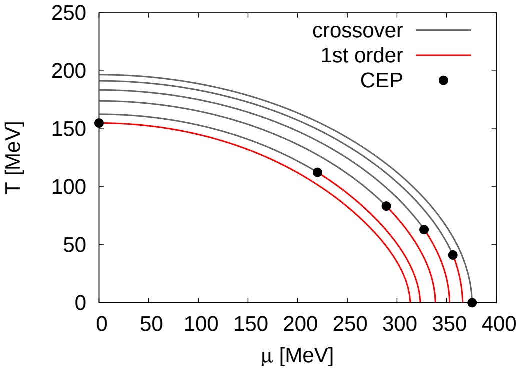
Chiral limit: RG arguments $\rightarrow$ for $N_f \geq 3$ first-order [Pisarski, Wilczek '84]

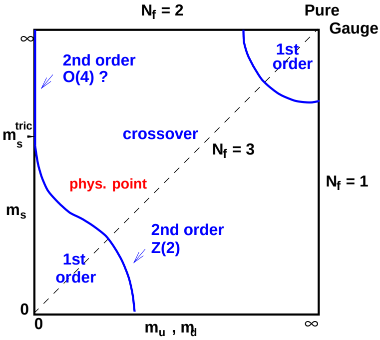
- variation of m_π and m_K:

  $m_\pi / m_\pi^* = 0.49$ (lower line), $0.6, 0.8 \ldots, 1.36$ (upper line)

  $m_\pi^* = 138$ MeV , $m_K^* = 496$ MeV , fixed ratio $m_\pi / m_K = m_\pi^* / m_K^*$

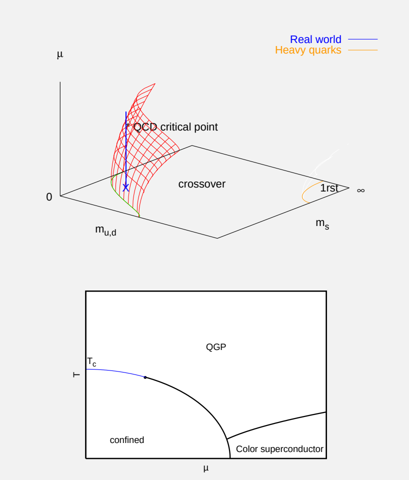
with $\mathbf{U(1)_A}$, $\mathbf{m_\sigma = 800}$ MeV



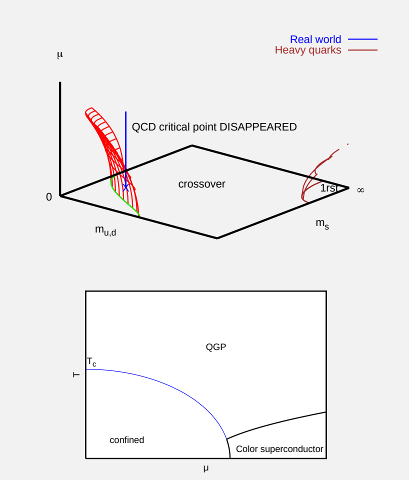


Mass sensitivity (lattice, $N_f = 3, \mu_B \neq 0$)

Standard scenario: $m_c(\mu)$ **increasing**



Nonstandard scenario: $m_c(\mu)$ **decreasing**

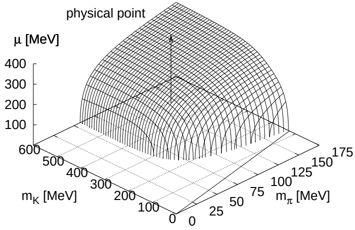


[de Forcrand, Philipsen: hep-lat/0611027]

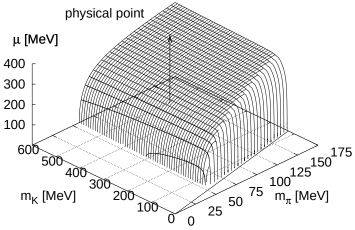
Chiral critical surface ($m_\sigma = 800$ MeV)

➜ standard scenario for $m_\sigma = 800$ MeV (as expected)

with $U(1)_A$



without $U(1)_A$



[BJS, M. Wagner, '09]

Note: 't Hooft coupling μ-independent

PNJL with (unrealistic) large vector int. $\longrightarrow$ bending of surface

Outline

- Three-Flavor Quark-Meson Model
- ...with Polyakov loop dynamics
- Finite density extrapolations

Polyakov-quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

- **polynomial Polyakov loop potential:** **Polyakov 1978, Meisinger 1996, Pisarski 2000**

$$\frac{\mathcal{U}(\phi, \bar{\phi})}{T^4} = -\frac{b_2(T, T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

with

$$b_2(T, T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

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- **logarithmic potential:** **Rössner et al. 2007**

$$\frac{\mathcal{U}_{\text{log}}}{T^4} = -\frac{1}{2}a(T)\bar{\phi}\phi + b(T)\ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]$$

with

$$a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 \quad \text{and} \quad b(T) = b_3(T_0/T)^3$$

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- **Fukushima** **Fukushima 2008**

$$\mathcal{U}_{\text{Fuku}} = -bT\left\{54e^{-a/T}\phi\bar{\phi} + \ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]\right\}$$

with

a controls deconfinement $\quad$ b strength of mixing chiral & deconfinement

Polyakov-quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

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with

$$b_2(T, T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

in presence of dynamical quarks: $T_0 = T_0(N_f, \mu)$ **BJS, Pawlowski, Wambach, 2007**

N_f	0	1	2	$2+1$	3
T_0 [MeV]	270	240	208	187	178

$\mu \neq 0 :\quad \bar{\phi} > \phi$

since $\bar{\phi}$ is related to free energy gain of antiquarks

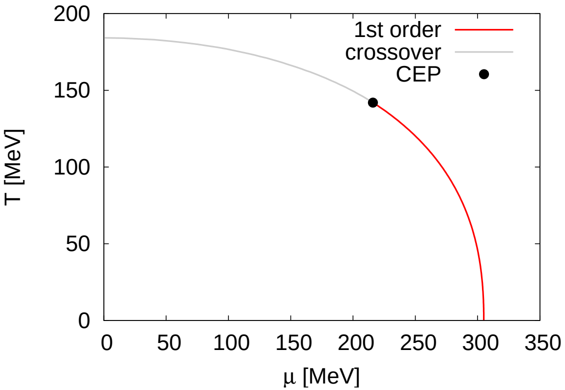
in medium with more quarks $\rightarrow$ antiquarks are more easily screened.

Phase diagrams $N_f = 2$

[BJS, Pawlowski, Wambach '07]

in mean field approximation

chiral transition and 'deconfinement' coincide



- for PQM model
  $N_f = 2$

Phase diagrams $N_f = 2$

[BJS, Pawlowski, Wambach '07]

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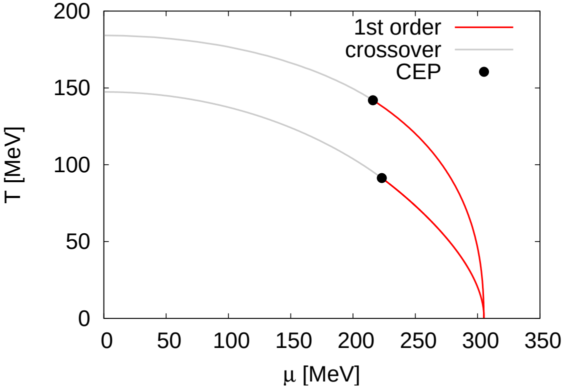

- for PQM model $N_f = 2$
- for QM model $N_f = 2$ (lower lines)

Phase diagrams $N_f = 2$

[BJS, Pawlowski, Wambach '07]

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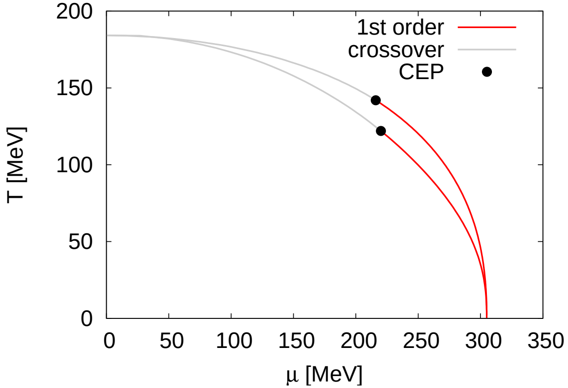


- for PQM model
  $N_f = 2$
- for PQM model
  $N_f = 2$

  with

  $T_0(\mu)$-modification in Polyakov loop potential

  (lower lines)

Phase diagram $N_f = 2 + 1$

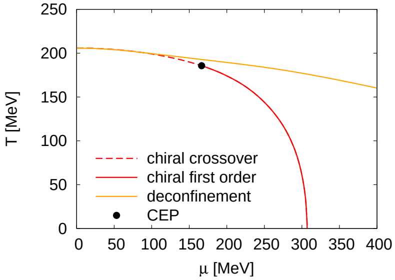
[BJS, M. Wagner; in preparation '10]

influence of Polyakov loop

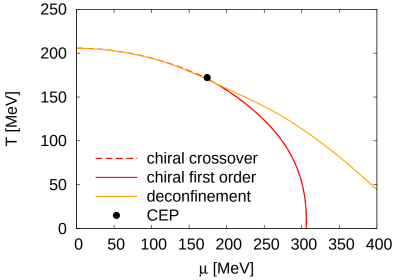
Logarithmic Polyakov loop potential

Mean-field approximation

$T_0 = 270$ MeV (constant)



$T_0(\mu)$ (i.e. with μ corrections)



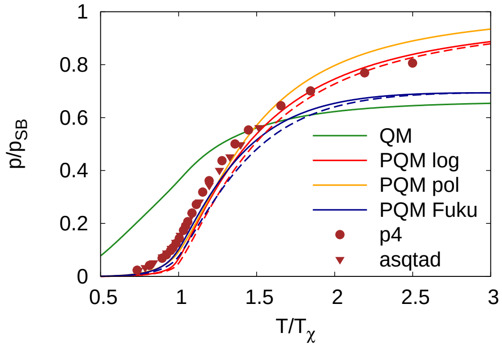
shrinking of possible quarkyonic phase

QCD Thermodynamics $N_f = 2 + 1$

[BJS, M. Wagner, J. Wambach; arXiv:0910.5628]

SB limit: $\frac{p_{SB}}{T^4} = 2(N_c^2 - 1)\frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$

(P)QM models (three different Polyakov loop potentials) versus QCD lattice simulations



- ▷ solid lines: PQM with lattice masses (HotQCD)

  $m_\pi \sim 220, m_K \sim 503$ MeV
- ▷ dashed lines: (P)QM with realistic masses

lattice data: [Bazavov et al. '09]

QCD Thermodynamics $N_f = 2 + 1$

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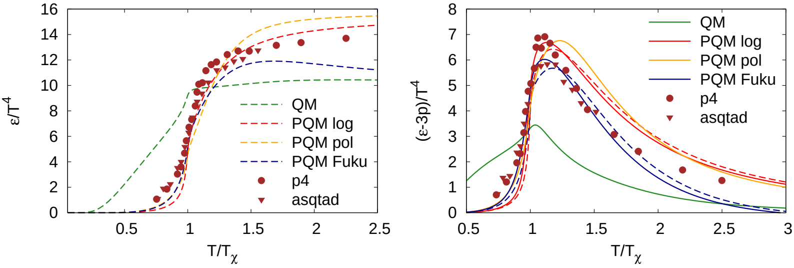

solid lines: $m_\pi \sim 220, m_K \sim 503$ MeV (HotQCD)

[Bazavov et al. '09]

Outline

- Three-Flavor Quark-Meson Model
- ...with Polyakov loop dynamics
- **Finite density extrapolations**

Finite density extrapolations $\mathbf{N_f = 2+1}$

Taylor expansion:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n \left(p(T,\mu)/T^4\right)}{\partial \left(\mu/T\right)^n} \Bigg|_{\mu=0}$$

high temperature limits:

$$c_0(T \to \infty) = \frac{7 N_c N_f \pi^2}{180} \; ,$$
$$c_2(T \to \infty) = \frac{N_c N_f}{6} \; ,$$
$$c_4(T \to \infty) = \frac{N_c N_f}{12\pi^2}$$
$$c_n(T \to \infty) = 0 \text{ for } n > 4.$$

Finite density extrapolations $N_f = 2+1$

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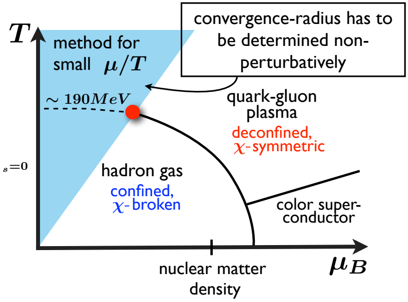

[C. Schmidt '09]

convergence radii:

limited by first-order line?

$$\rho_{2n} = \left|\frac{c_2}{c_{2n}}\right|^{1/(2n-2)}$$

Finite density extrapolations $\mathbf{N_f = 2 + 1}$

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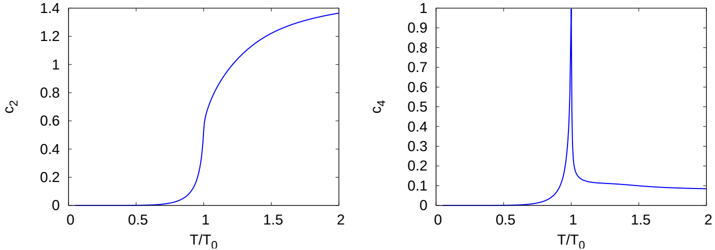
<u>first three coefficients:</u>

c_0: pressure at $\mu = 0$



Finite density extrapolations $N_f = 2 + 1$

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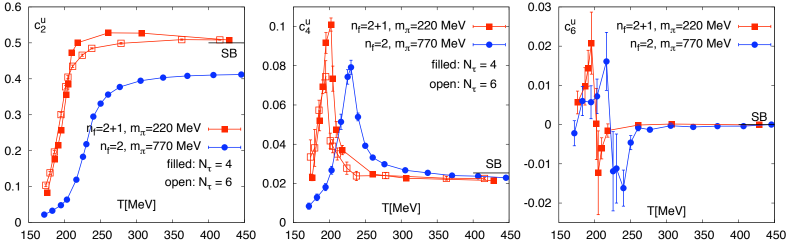
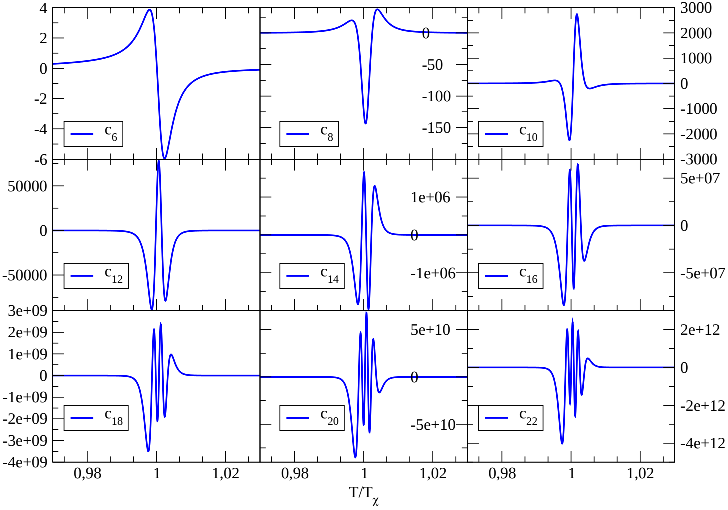


[Miao et al. '08]

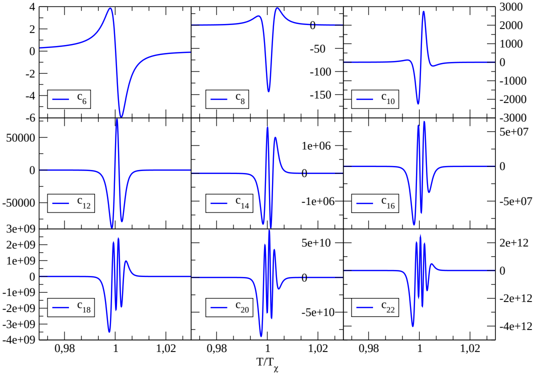
New method: based on algorithmic differentiation

[M. Wagner, A. Walther, BJS, CPC '10]

Taylor coefficients c_n numerically known to high order, e.g. $n = 22$



Finite density extrapolations $N_f = 2 + 1$



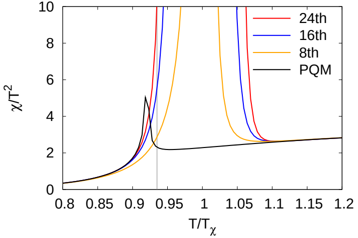
- ▷ this technique applied to PQM model
- ▷ investigation of convergence properties of Taylor series
- ▷ properties of c_n
  - oscillating
  - increasing amplitude
  - no numerical noise
  - small outside transition region
  - number of roots increasing
  - 26th order

[F. Karsch, BJS, M. Wagner, J. Wambach; in preparation ’10]

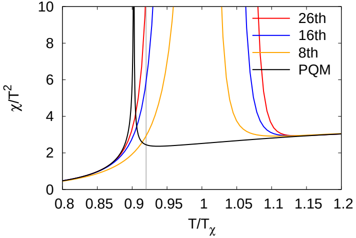
Can we locate the QCD critical endpoint with the Taylor expansion ?

Susceptibility $N_f = 2 + 1$ PQM model

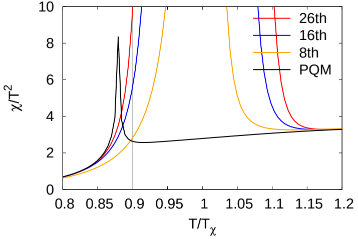
$\mu/T = 0.8$



$\mu/T = \mu_c/T_c$



$\mu/T = 3.$



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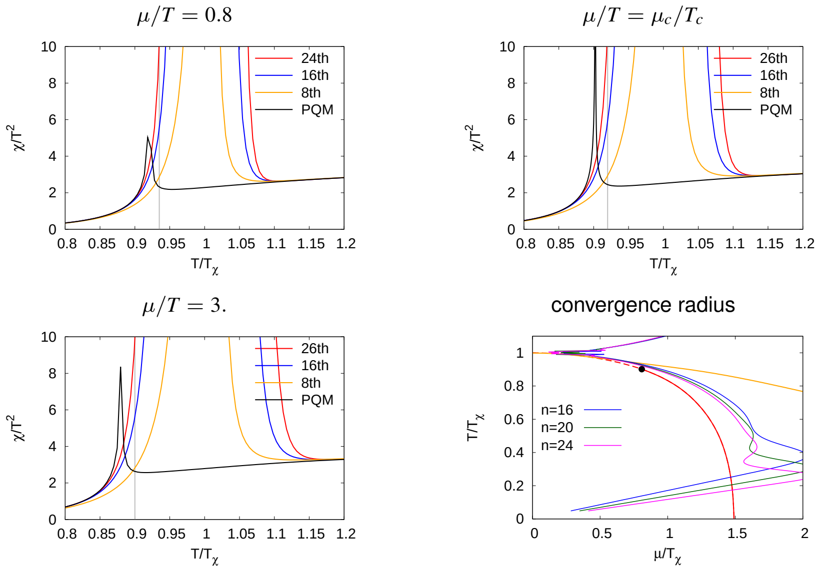

Susceptibility $N_f = 2 + 1$ PQM model

Findings:

- simply Taylor expansion: slow convergence
  high orders needed
  disadvantage for lattice simulations
- Taylor applicable within convergence radius
  also for $\mu/T > 1$
- but 1st order transition not resolvable
  expansion around $\mu = 0$

Summary

- $N_f = 2$ and $N_f = 2+1$ chiral (Polyakov)-quark-meson model study

→ Mean-field approximation and FRG

with and without axial anomaly

- novel AD technique: high order Taylor coefficients, here: $\mathbf{n = 26}$

Findings:

▷ Parameter in Polyakov loop potential:
$T_0 \Rightarrow T_0(N_f, \mu)$

▷ Chiral & deconfinement transition possibly **coincide** for $N_f = 2$ with $T_0(\mu)$-corrections but possibly not for $N_f = 2+1$

▷ Mean-field approximation encouraging

but effects of Dirac term point to interesting physics if fluctuations are considered

$\longrightarrow$ FRG with PQM truncation

▷ Taylorcoefficient $c_n(T) \longrightarrow$ **high order**

⇒ **convergence properties** of Taylor expansion

Outlook:

- include glue dynamics with FRG $\longrightarrow$ full QCD