# Calculating the QGP viscosity on the lattice

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# Outline

# Background

Elliptic flow measured at RHIC

- Results could largely be modelled by ideal hydrodynamics
- Viscous hydrodynamics gives a very small value for $\eta/s$
- Close to the conjectured lower bound from AdS/CFT
- The most perfect fluid known to humankind?

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- Shear viscosity $\eta$ has a minimum near the crossover
- Bulk viscosity $\zeta$ diverges at a second order phase transition

# Kubo formulae

The viscosity is given by the low-frequency limit of certain spectral functions:

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho_{ij}(\omega, \overrightarrow{0})}{\omega}$$

$$\zeta = \pi \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \overrightarrow{0})}{\omega}$$

The spectral functions $\rho_{ij}, \rho_p$ are the imaginary part of the retarded correlators of the traceless stress tensor and the pressure.

# Lattice simulations?

The spectral functions are related to euclidean correlators $C_{s,p}$,

$$C_{s,p}(\tau, \overrightarrow{p}) = \int_0^\infty \rho_{s,p}(\omega, \overrightarrow{p}) \frac{\cosh \omega(\tau - \frac{2}{T})}{\sinh \frac{\omega}{2T}}$$

with

$$C_s(\tau, \overrightarrow{p}) = \frac{1}{20} \int e^{-i\overrightarrow{p}\cdot(\overrightarrow{x}-\overrightarrow{y})} \langle T_{ij}(t, \overrightarrow{x}) T_{ij}(t+\tau, \overrightarrow{y}) \rangle d^3x$$

$$C_p(\tau, \overrightarrow{p}) = \frac{1}{18} \int e^{-i\overrightarrow{p}\cdot(\overrightarrow{x}-\overrightarrow{y})} \langle T_{ii}(t, \overrightarrow{x}) T_{jj}(t+\tau, \overrightarrow{y}) \rangle d^3x$$

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Relations between transport coefficients and correlators at nonzero momentum may help!

# Energy–momentum operators

In the continuum, the energy–momentum tensor is

$$
\begin{aligned}
T_{\mu\nu}(x) &= T^{G}_{\mu\nu} + T^{F}_{\mu\nu} \\
&= 2\,\mathrm{Tr}(F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma}) \\
&\quad + \frac{1}{4}\sum_{f}\left[\overline{\psi}_f D_\mu \gamma_\nu \psi_f + \overline{\psi}_f D_\nu \gamma_\nu \psi_f - \frac{1}{2}\delta_{\mu\nu}\overline{\psi}_f D_\rho \gamma_\rho \psi_f\right]
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Possible discretisations of $T^G_{\mu\nu}$

1. Plaquette definition, $F_{\mu\nu}F_{\mu\nu} = \frac{4}{g^2}(1 - \Re\,\mathrm{Tr}\,P_{\mu\nu})$
2. Clover definition of $F_{\mu\nu}$
3. 'Action' definition of $T_{00} = \varepsilon$, $T_{ii} = p$

# Shear correlators

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— but no prescription for the shear (off-diagonal) channel

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## Tensor decomposition of correlators

In general,

$$\begin{aligned}\langle T_{ij}(0) T_{kl}(\tau, \vec{x})\rangle = & A(\tau, x^2)\delta_{ij}\delta_{kl} + B(\tau, x^2)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jl}) \\ & + C(\tau, x^2)(x_i x_k \delta_{jl} + x_i x_l \delta_{jk} + x_j x_k \delta_{il} + x_j x_l \delta_{ik}) \\ & + D(\tau, x^2)(x_i x_j \delta_{kl} + x_k x_l \delta_{ij}) + E(\tau, x^2) x_i x_j x_k x_l\end{aligned}$$

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In the fluid rest frame we get additional relations between $A, B, C, D, E$ — only two independent form factors $A, B$ survive

$$\Longrightarrow \quad \langle T_{12}T_{12}\rangle = \langle T_{11}T_{11}\rangle - \langle T_{11}T_{22}\rangle$$

# Energy and pressure operators

$\varepsilon, p$ are naturally defined on an anisotropic lattice, with $a_\sigma = \xi a_\tau$:

$$\varepsilon = -\frac{T^2}{V}\frac{\partial \ln Z}{\partial T} = \frac{T}{V}\left\langle \frac{\partial S}{\partial a_\tau} \right\rangle = -\frac{T}{V}\left\langle \left.\frac{\partial S}{\partial \xi}\right|_{a_\sigma} \right\rangle$$

$$p = \frac{\partial \ln Z}{\partial V} = -\frac{T}{3V}\left\langle \frac{\partial S}{\partial a_\sigma} \right\rangle$$

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The derivatives $\langle \frac{\partial S}{\partial a_{\sigma,\tau}} \rangle$ involve the derivatives of the bare couplings $\beta_s, \beta_t, \kappa_s, \kappa_t$ etc wrt the lattice spacings $a_{\sigma,\tau}$, or equivalently, wrt $a = a_\sigma, \xi$.

These are the Karsch coefficients

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The $\xi$-derivatives cancel in the trace anomaly $\Theta = \varepsilon - 3p$,

$$\Theta = \frac{T}{V}\left\langle \frac{\partial S}{\partial a}\Big|_{\xi} \right\rangle$$

# Pure gauge, Wilson action

The anisotropic Wilson action is

$$S = \sum_x \left[ \frac{\beta}{\xi_0} \sum_{i<j} \left(1 - \Box_{ij}(x)\right) + \xi_0 \beta \sum_i \left(1 - \Box_{i0}(x)\right) \right]$$

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$$S = \sum_x \left[ \frac{\beta}{\xi_0} \sum_{i<j} \left(1 - \square_{ij}(x)\right) + \xi_0 \beta \sum_i \left(1 - \square_{i0}(x)\right) \right]$$

The energy and pressure operators are, after vacuum subtraction

$$\varepsilon = \frac{3T}{V} \left[ \left( \frac{1}{\xi_0} \frac{\partial \beta}{\partial \xi} - \frac{\beta}{\xi_0^2} \frac{\partial \xi_0}{\partial \xi} \right) \square_s + \left( \xi_0 \frac{\partial \beta}{\partial \xi} + \beta \frac{\partial \xi_0}{\partial \xi} \right) \square_t \right]$$

$$\varepsilon - 3p = \frac{3T}{V} \left[ a \frac{\partial \beta}{\partial a} \left( \frac{\square_s}{\xi_0} + \xi_0 \square_t \right) + \beta a \frac{\partial \xi_0}{\partial a} \left( \square_t - \frac{\square_s}{\xi_0^2} \right) \right]$$

where $\square_{s,t} \equiv \langle \square_{s,t} \rangle - \langle \square_{s,t} \rangle_0$

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- A large number of points in time direction required
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- Far too expensive with isotropic lattices $a_s = a_t$!
- Introduces 2 additional parameters
- Non-trivial tuning problem: bare quark and gluon anisotropies must be tuned simultaneously
  [PRD **74** 014505 (2006)]

# Action

Two-plaquette Symanzik Improved

$$S_g = \frac{\beta}{\xi_g}\left\{\frac{5(1+\omega)}{3u_s^4}\Omega_s - \frac{5\omega}{3u_s^8}\Omega_s^{(2t)} - \frac{1}{12u_s^6}\Omega_s^{(R)}\right\} + \beta\xi_0\left\{\frac{4}{3u_s^2u_t^2}\Omega_t - \frac{1}{12u_s^4u_t^2}\Omega_t^{(R)}\right\}$$

$$\Omega_s = \sum_{x,i>j}\Big[1 - P_{ij}(x)\Big], \quad P_{ij} = \text{spatial plaquette}$$

$$\Omega_t = 1 - \text{timelike plaquette}$$

$$\Omega_s^{(2t)} = \frac{1}{2}\sum_{x,i>j}\Big[1 - P_{ij}(x)P_{ij}(x+\hat{t})\Big]$$

$$\Omega_{s/t}^{(R)} = 2\times 1 \text{ rectangle in } (i, j/t) \text{ planes}$$

[Morningstar & Peardon, NPBP **83–84**, 887 (2000)]

# Action (II)

fine Wilson coarse Hamber–Wu

$$
\begin{aligned}
M\psi(x) = \frac{1}{a_t}\Big\{ & \big(\mu_r m a_t + \frac{18s}{\xi_q} + r\big)\psi(x) \\
& - \frac{1}{2u_t}\Big[(r-\gamma_t)U_t(x)\psi(x+\hat{t}) + (r+\gamma_t)U_t^\dagger(x-\hat{t})\psi(x-\hat{t})\Big] \\
-\frac{1}{\xi_q}\sum_i \Big[ & \frac{1}{u_s}\big(4s-\frac{2}{3}\mu_r\gamma_i\big)U_i(x)\psi(x+\hat{\imath}) + \frac{1}{u_s}\big(4s+\frac{2}{3}\mu_r\gamma_i\big)U_i^\dagger(x-\hat{\imath})\psi(x-\hat{\imath}) \\
& - \frac{1}{u_s^2}\big(s-\frac{1}{12}\mu_r\gamma_i\big)U_i(x)U_i(x+\hat{\imath})\psi(x+2\hat{\imath}) \\
& - \frac{1}{u_s^2}\big(s+\frac{1}{12}\mu_r\gamma_i\big)U_i^\dagger(x-\hat{\imath})U_i^\dagger(x-2\hat{\imath})\psi(x-2\hat{\imath})\Big]\Big\}.
\end{aligned}
$$

where $\mu_r = 1 + \frac{1}{2}ma_t$
[Foley *et al*, hep-lat/0405030]

# Simulation parameters

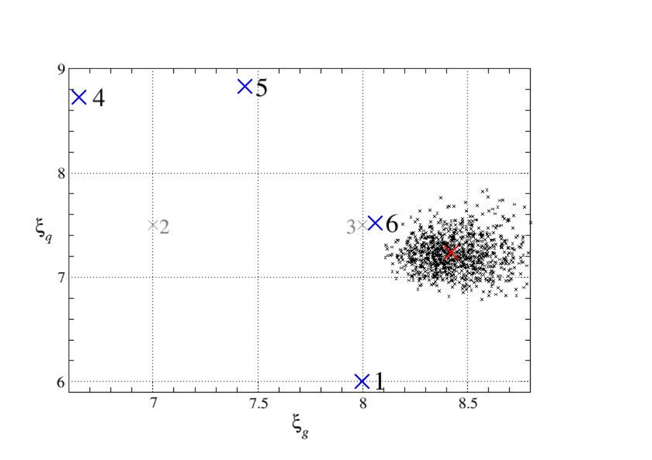
| Run | $\xi_g^0$ | $\xi_q^0$ | $u_s$ | $\beta$ | $m_0$ |
|---|---|---|---|---|---|
| 1 | 8.00 | 6.00 | 0.32 | 1.513 | -0.057 |
| 4 | 6.65 | 8.72 | 0.32 | 1.544 | -0.057 |
| 5 | 7.44 | 8.83 | 0.32 | 1.522 | -0.057 |
| 6 | 8.08 | 7.52 | 0.32 | 1.514 | -0.057 |
| 7 | 8.42 | 7.43 | 0.32 | 1.508 | -0.057 |
| 8 | 8.42 | 7.43 | 0.32 | 1.514 | -0.050 |
| 9 | 8.42 | 7.43 | 0.31 | 1.458 | -0.057 |

## Physical parameters

| | | |
|---|---|---|
| Light quarks | $m_\pi / m_\rho$ | 0.54 |
| Anisotropy | $\xi$ | 6 |
| Lattice spacing | $a_t$ | 0.027fm |
| | $a_s$ | 0.17 fm |
| Lattice volume | $N_s^3$ | $12^3$ |

# Tuning the anisotropies

Assume linear relation between bare and physical anisotropies, interpolate:



# Computing Karsch coefficients

[With Richie Morrin]

We have 4 input parameters $\beta$ (or $u_s$), $\xi_g^0, \xi_q^0, m_0$. The (generalised) Karsch coefficients are given by an expansion about the physical point where $\xi_g = \xi_q = \xi$,

$$\begin{pmatrix} \Delta\xi_g^0 \\ \Delta\xi_q^0 \\ \Delta\beta \\ \Delta m_0 \end{pmatrix} = \begin{pmatrix} \frac{\partial\xi_g^0}{\partial\xi} & \frac{\partial\xi_g^0}{\partial a} & \frac{\partial\xi_g^0}{\partial\xi^-} & \frac{\partial\xi_g^0}{\partial M} \\ \frac{\partial\xi_q^0}{\partial\xi} & \frac{\partial\xi_q^0}{\partial a} & \frac{\partial\xi_q^0}{\partial\xi^-} & \frac{\partial\xi_q^0}{\partial M} \\ \frac{\partial\beta}{\partial\xi} & \frac{\partial\beta}{\partial a} & \frac{\partial\beta}{\partial\xi^-} & \frac{\partial\beta}{\partial M} \\ \frac{\partial m_0}{\partial\xi} & \frac{\partial m_0}{\partial a} & \frac{\partial m_0}{\partial\xi^-} & \frac{\partial m_0}{\partial M} \end{pmatrix} \begin{pmatrix} \Delta\xi \\ \Delta a \\ \Delta\xi^- \\ \Delta M \end{pmatrix}$$

where we have defined the 'physical' quantities

$$\xi = \frac{1}{2}(\xi_g + \xi_q)\,, \quad \xi^- = \frac{1}{2}(\xi_g - \xi_q)\,, \quad M = \left(\frac{m_\pi}{m_\rho}\right)^2 .$$

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We need the first two columns of this matrix

# Computing Karsch coefficients (II)

We can readily fit the measured quantities $f_i = \xi, a, \xi^-, M$ as functions of the input parameters $c_i = \xi_g^0, \xi_q^0, \beta/u_s, m_0$ and obtain the derivatives $\partial f_i/\partial c_j$

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## Method 1

Iterative procedure: fit $\frac{\partial f_i}{\partial c_j}$ by guessing initial variances for $f_i$ [Levkova, Manke, Mawhinney (2006)]

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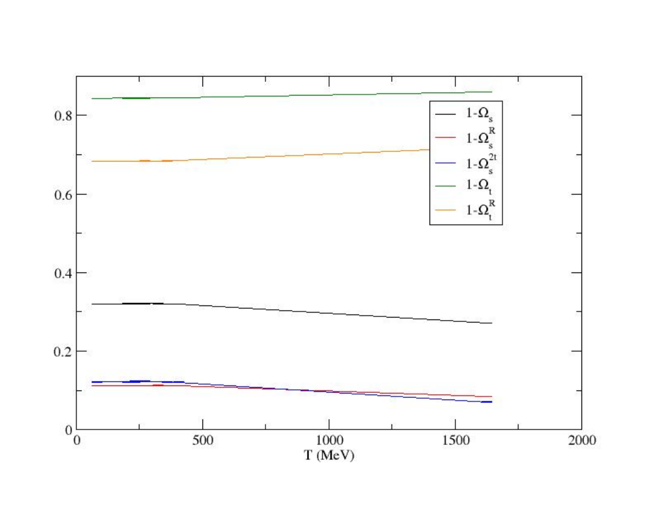
## Method 2

Direct method: fit $\frac{\partial c_i}{\partial f_j}$, invert the resulting matrix to obtain $\frac{\partial f_i}{\partial c_j}$
[Morrin, JIS (2008)]

# Results

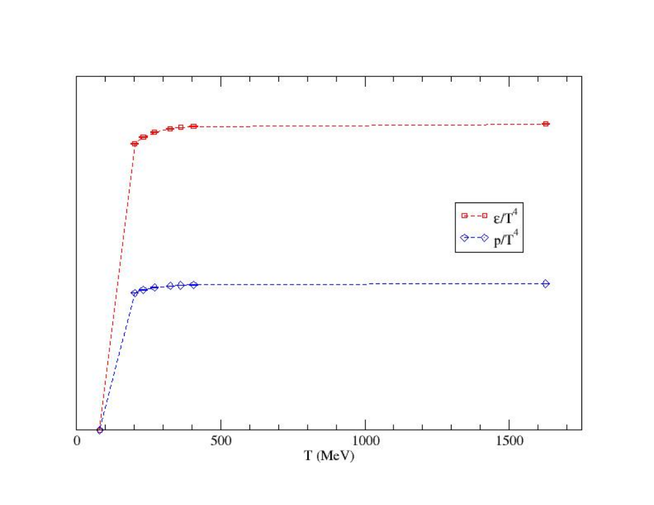
| | Method 1 | Method 2 | |
|---|---|---|---|
| | | $u_s$ | $\beta$ |
| $\frac{\partial \xi_g^0}{\partial \xi}$ | $3.00^{+53}_{-51}$ | $2.64^{+52}_{-41}$ | $2.64^{+52}_{-42}$ |
| $\frac{\partial \xi_q^0}{\partial \xi}$ | $1.58^{+51}_{-53}$ | $1.55^{+47}_{-39}$ | $1.51^{+48}_{-39}$ |
| $\frac{\partial u_s}{\partial \xi}$ | $-0.0315^{+61}_{-60}$ | $-0.0352^{+55}_{-70}$ | |
| $\frac{\partial \beta}{\partial \xi}$ | $-0.177^{+51}_{-54}$ | | $-0.224^{+35}_{-44}$ |
| $\frac{\partial m_0}{\partial \xi}$ | $0.0075^{+71}_{-68}$ | $0.0014^{+26}_{-26}$ | $0.0011^{+26}_{-25}$ |
| $\frac{\partial \xi_g^0}{\partial a}$ | $-4.29^{+4.19}_{-4.38}$ | $-5.24^{+4.35}_{-4.65}$ | $-5.39^{+4.26}_{-4.65}$ |
| $\frac{\partial \xi_q^0}{\partial a}$ | $-14.64^{+8.55}_{-8.16}$ | $2.51^{+5.72}_{-4.74}$ | $1.79^{+5.88}_{-4.63}$ |
| $\frac{\partial u_s}{\partial a}$ | $-0.310^{+59}_{-54}$ | $-0.380^{+63}_{-74}$ | |
| $\frac{\partial \beta}{\partial a}$ | $-1.95^{+38}_{-38}$ | | $-1.97^{+38}_{-44}$ |
| $\frac{\partial m_0}{\partial a}$ | $-0.141^{+67}_{-73}$ | $-0.233^{+34}_{-39}$ | $-0.233^{+34}_{-40}$ |

# Raw plaquette data



# Equation of state

PRELIMINARY!



# Correlators

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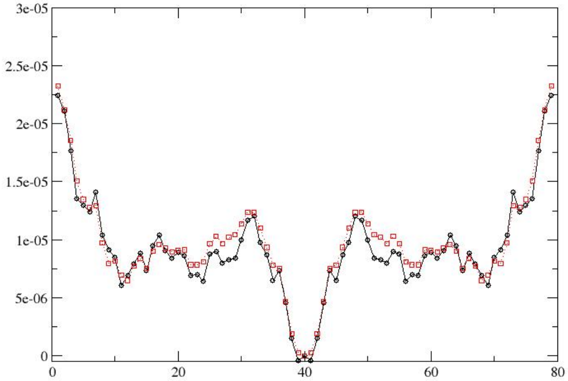

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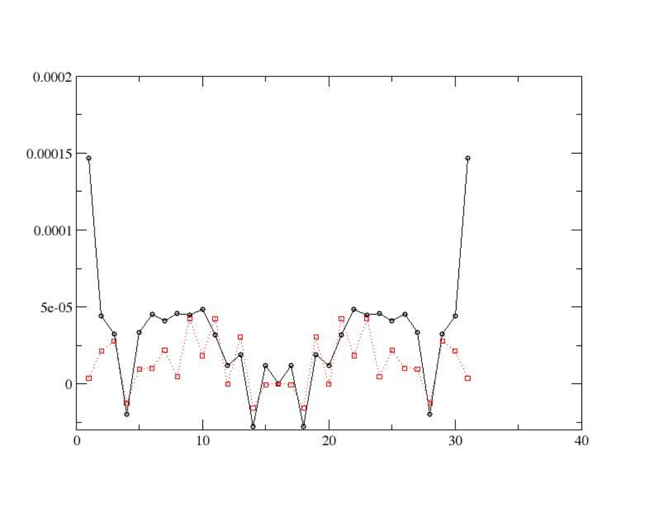

# Summary and outlook

- Nonperturbative energy–momentum renormalisation computed
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- Application to equation of state up to $\sim 10\,T_c$

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## Outlook

- Fermion contribution underway
- Improved energy–momentum operators
  - $\longrightarrow$ Reduce discretisation and statistical errors
  - $\longrightarrow$ Need separate (re)normalisation
- Correlators at nonzero momentum
- Improved determination of Karsch coefficients required?
- Simulations on finer lattices underway