Perturbative QCD with Finite Masses and Densities

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Work in collaboration with Aleksi Kurkela (ETH Zürich) and Paul Romatschke (INT, Seattle); arXiv:0912.1856, ...

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Tools for finite density QCD

- At $T \neq 0$, $\mu \lesssim T$, lattice QCD is the definitive non-perturbative method
- However: At $\mu \gtrsim T$ simulations hampered by the *sign problem*...
  - ... unless one switches to a model that can be solved
- Alternatives for describing QCD
  - At intermediate densities: Quantum many-body theory
  - At asymptotic densities: Perturbation theory

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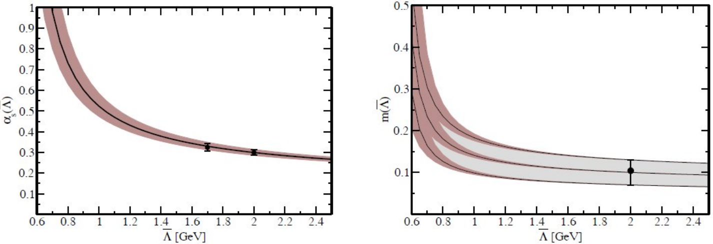

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- Important difference to high T: Purely gluonic sector vanishes at $T = 0$ $\Rightarrow$ Expect much better convergence at high μ!

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Status of equation of state calculations

- With $m_{\mathrm{quark}} = 0$, weak coupling expansion of p_{QCD} known to
  - High T, $\mu \lesssim T/\sqrt{\alpha_s}$: $\mathcal{O}(\alpha_s{}^3 \ln \alpha_s)$ (Kajantie, Laine, Rummukainen, Schröder; AV)
    - Severe IR problems at four loops $\Rightarrow$ Requires extensive use of dimensionally reduced effective theories
    - Progress at $\mathcal{O}(\alpha_s^3)$: Contributions of soft scales and entire $N_f^3 \alpha_s^3$ term known (de Renzo, Laine, Schröder, ...; Gynther, Kurkela, AV)
  - High μ, $T = 0$: $\mathcal{O}(\alpha_s^2)$ (Freedman, McLerran)
    - Three-loop vacuum graphs and numerical plasmon sum

Status of equation of state calculations

- With finite quark masses $m_{\mathrm{quark}} \neq 0$
  - High T, $\mu = 0$: $\mathcal{O}(\alpha_s)$ (Laine, Schröder)
    - Dependence on m_q well modeled by Stefan-Boltzmann result
    - $m_s \ll T$ in the region where perturbation theory convergent
  - High μ, $T = 0$: $\mathcal{O}(\alpha_s)$ (Fraga, Romatschke)
    - Better convergence properties $\Rightarrow$ Dependence of result on m_s both sizable and non-trivial
    - Renormalization scale dependence sizable $\Rightarrow$ Need to go to $\mathcal{O}(\alpha_s^2)$

Perturbative evaluation of the grand potential

Thermodynamics defined by the grand potential

$$\Omega(\mu_u,\mu_d,\mu_s,m_s) = -T\log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^4x\mathcal{L}_{\text{QCD}}},$$

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}+\bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i$$

All equilibrium thermodynamical quantities can be derived from $\Omega(\mu_u,\mu_d,\mu_s,m_s)$

$$pV = -\Omega(\mu_u,\mu_d,\mu_s,m_s)$$

$$n_i = -\partial_{\mu_i}\Omega(\mu_u,\mu_d,\mu_s,m_s)$$

$$\varepsilon = -p+\mu_i n_i$$

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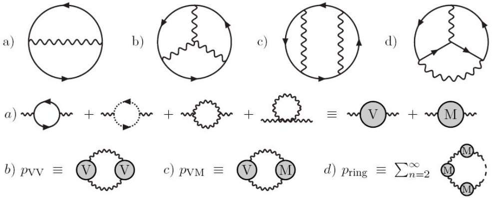
$\Omega(\mu_u, \mu_d, \mu_s, m_s)$ available through 1PI graphs



After scalarization end up with $4d$ Euclidean integrals constructed from μ-dependent fermion propagators

$$\Delta(p) = \frac{1}{(p_0 + i\mu)^2 + \mathbf{p}^2 + m_s^2}$$

- Technical challenge: Multiscale problem!
  - $\mu = 0$: Extensive literature on integral reductions, IBP, ...
    - Ready-to-use applications FIRE, Tarcer, ...
  - $m = 0$: $3d$ Fourier transformation technology
  - Combination of these problematic
- Our strategy: Reduce graphs to on-shell vacuum amplitudes integrated over μ-dependent kinematic measures

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  1. "Cutting" of fermion lines, *i.e.* organizing result of p_0 integration
  2. Reduction of amplitudes with FIRE (master integrals from literature)
  3. Implementation of renormalization
  4. Numerical evaluation of finite kinematic integrals

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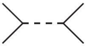
Example:

 $\rightarrow$  $-\,2\int \frac{d^3\vec{p}}{(2\pi)^3}\frac{\theta(\mu - E(\vec{p}))}{2E(\vec{p})}$ 

$+\int \frac{d^3\vec{p}}{(2\pi)^3}\frac{\theta(\mu - E(\vec{p}))}{2E(\vec{p})}\int \frac{d^3\vec{q}}{(2\pi)^3}\frac{\theta(\mu - E(\vec{q}))}{2E(\vec{q})}$ 

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Warmup: Massless quarks with $\mu_i = \mu$

Construct thermodynamics from quark number density

$$n^{(2)}(\mu,\bar{\Lambda}) = n^{(0)}(\mu)\Bigg[1 - 2\,\frac{\alpha_s(\bar{\Lambda})}{\pi} - \left(\frac{\alpha_s(\bar{\Lambda})}{\pi}\right)^2 \left(\frac{61}{4} - 11\ln 2 - 0.369165\,N_f + N_f \ln\frac{N_f\alpha_s}{\pi} + \beta_0 \ln\frac{\bar{\Lambda}}{\mu}\right)\Bigg]$$

- Explicit dependence on renormalization scale $\bar{\Lambda}$ cancels against running of α_s
- Residual dependence parametrically higher order
  - Variation of $\bar{\Lambda} = 1 \ldots 4\mu$ quantifies systematic uncertainty in the results

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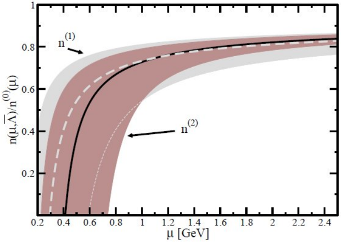

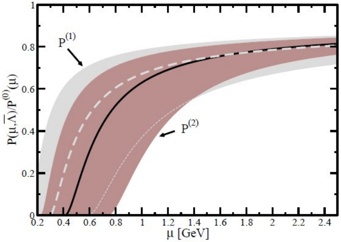

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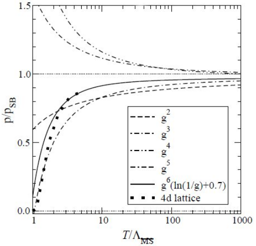
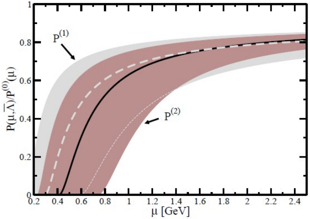
Convergence excellent in comparison with high T:





Physical case: 2+1 flavors, $m_s \neq 0$, $\mu_i \neq \mu$

In any physically realistic setup, quark matter is

- In β-equilibrium:

$$\begin{aligned} d \rightarrow u + e + \bar{\nu}_e, &\qquad u + e \rightarrow d + \nu_e, \\ s \rightarrow u + e + \bar{\nu}_e, &\qquad u + e \rightarrow s + \nu_e, \\ s + u \quad \leftrightarrow &\quad d + u, \end{aligned}$$

leading to a constraint for quark chemical potentials

$$\mu \equiv \mu_s = \mu_d = \mu_u + \mu_e$$

- Charge neutral:

$$2/3 n_u - 1/3 n_s - 1/3 n_d - n_e \;=\; 0 \;\Rightarrow\; \mu_e \;=\; \mu_e(\mu)$$

Physical case: 2+1 flavors, $m_s \neq 0$, $\mu_i \neq \mu$

Also have to recall that

- The perturbative computation gives the pressure only up to an additional constant

$$P(\mu,\bar{\Lambda}) = -B + \int_{\mu_0(\bar{\Lambda})}^{\mu} \mathrm{d}\mu \; n(\mu,\bar{\Lambda})$$

- The theory has a paring instability $\Rightarrow$ Non-perturbative term in the EoS

$$P_{\rm CSC} \quad \equiv \quad \frac{\Delta^2 \left(\mu_u + \mu_d + \mu_s\right)^2}{3\pi^2}$$

  - Asymptotically $\Delta \sim \frac{\mu}{g^5}\,\mathrm{e}^{-3\pi^2/(2\sqrt{g})}$

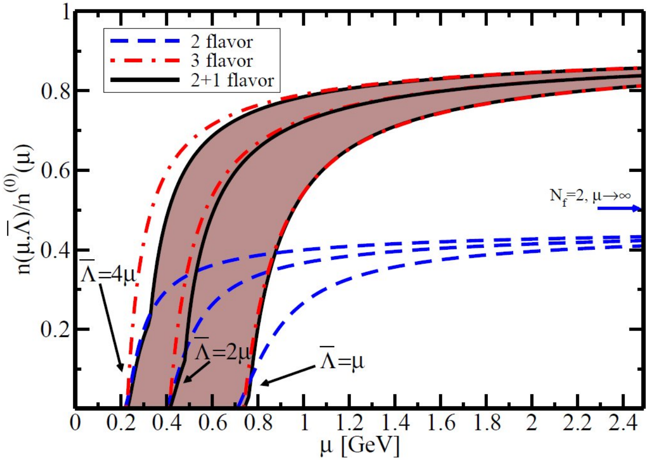
Physical case: 2+1 flavors, $m_s \neq 0$, $\mu_i \neq \mu$

... which add to the uncertainties of our result:

- Experimental:
  - Value (and running) of $\alpha_s \Leftrightarrow \Lambda_{\overline{\text{MS}}} = 378 \pm 34$ MeV
  - Mass of the strange quark m_s(2 GeV)$= 100 \pm 30$ MeV
- Theoretical:
  - Truncation of the perturbative series: $\bar{\Lambda}$ dependence
  - Bag constant $B \sim 200$ MeV?
  - CSC Effects — $\Delta \lesssim 100$ MeV

Solution: Vary all parameters ("quantify our ignorance") and try to find constraints...

Result: Mass-dependent EoS



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Conclusions

- Perturbative methods needed in regions of the phase diagram where lattice QCD inapplicable
  - Very high temperatures $T \gg T_c$
  - High densities $\mu \gtrsim T$
- Strange quark mass effects significant — and non-trivial — at high μ and small T
  - Three-loop EoS of cold quark matter computed as function of μ and m_s
  - Renormalization scale dependence $\Rightarrow$ Expansion converges at $\mu \gtrsim 1$ GeV
- For applications, please hold your breath...