

P- and CP-odd effects in QCD in extreme conditions

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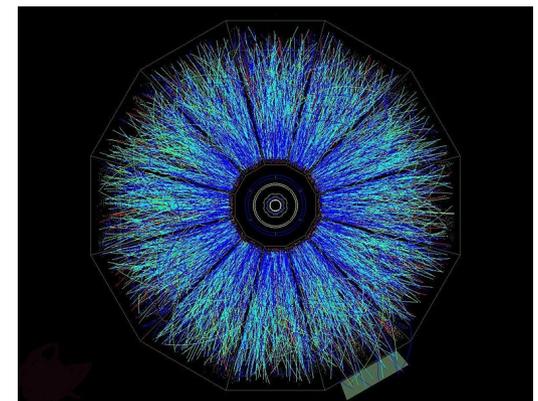
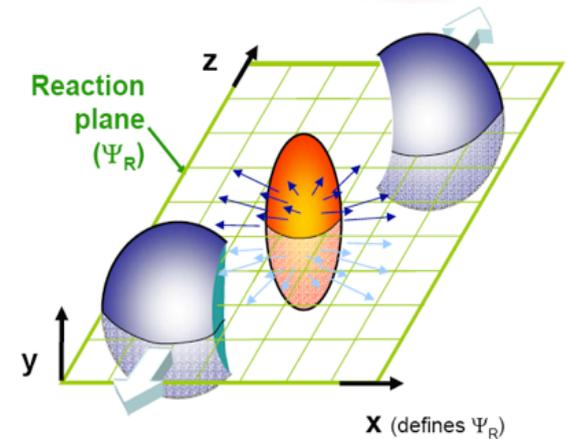
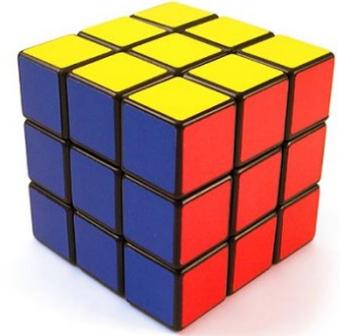
Collaborators: Kenji Fukushima, Dmitri Kharzeev and Larry McLerran.

P- and CP-odd effects in QCD in extreme conditions

I. QCD, Topology, P and CP

II. Heavy ion collisions

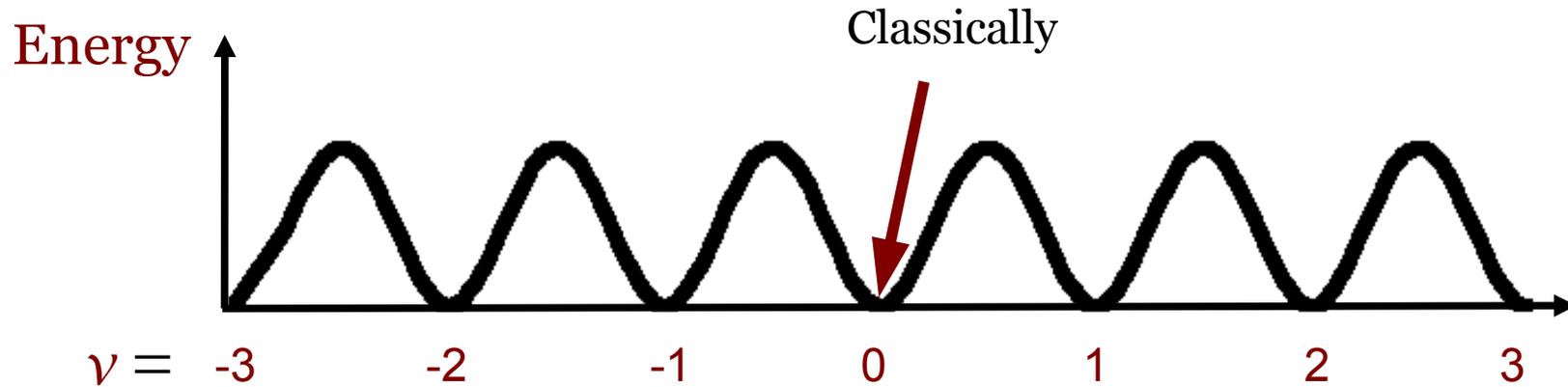
III. How to observe I in II



QCD = SU(3) Yang-Mills + Quarks

SU(N) Yang-Mills theories have interesting vacuum

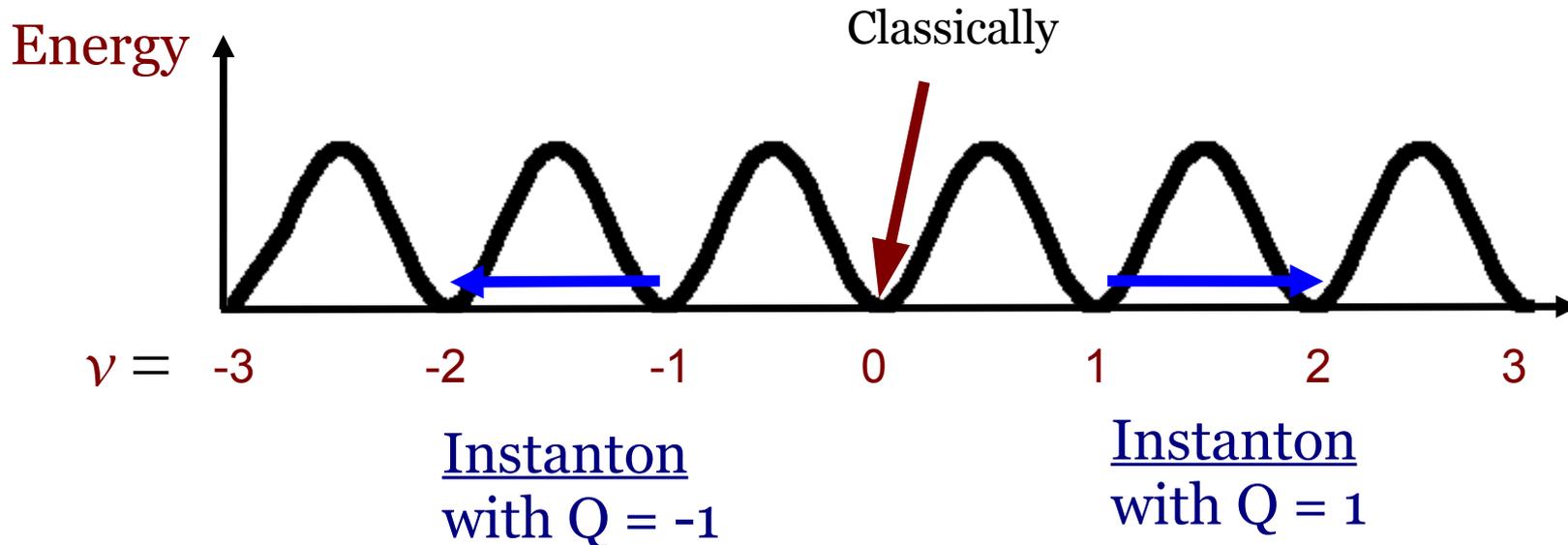
Belavin et. al. ('75), Jackiw & Rebbi ('76), Callan et al. ('76), 't Hooft ('76)



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Quantum tunneling: Instanton, (Belavin et al. 't Hooft, ...)

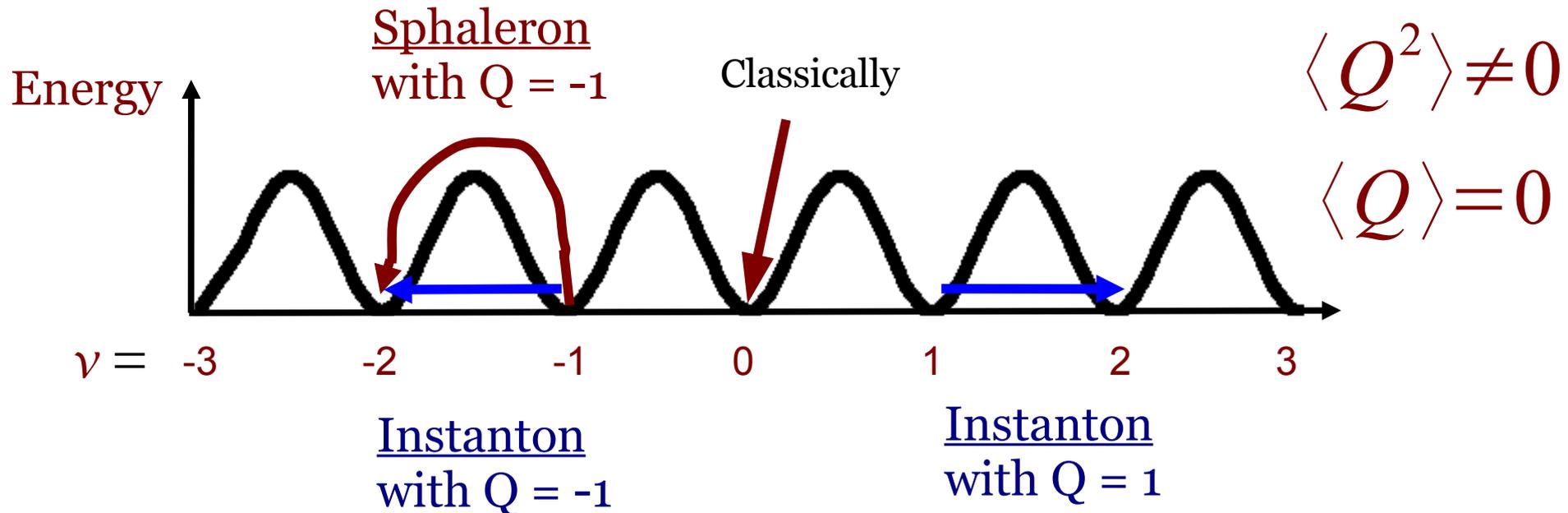
Q = topological charge = change in winding number

$$Q = \frac{g^2}{32\pi^2} \int d^4x F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} = \Delta N_{CS}$$

QCD = SU(3) Yang-Mills + Quarks

SU(N) Yang-Mills theories have interesting vacuum

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Quantum tunneling: Instanton, (Belavin et al. 't Hooft, ...)

Caloron, (finite T. instanton) (Gross, Pisarski, & Yaffe, Kraan & Van Baal, ..)

Thermal activation: Sphaleron, (Klinkhamer & Manton ...)

Q = topological charge = change in winding number

$$Q = \frac{g^2}{32\pi^2} \int d^4x F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} = \Delta N_{CS}$$

Part I: Topological charge induces chirality

This is the P- and CP-odd effect

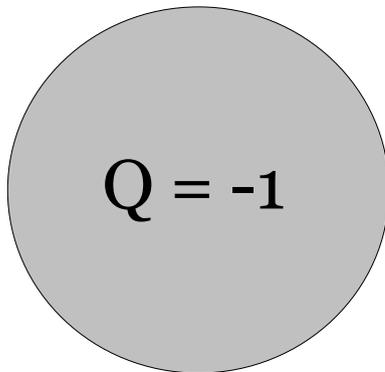
Chirality: $N_5 =$ difference # quarks + antiquarks with R & L-handed helicity

momentum

$$N_5 = \# \begin{array}{c} \text{spin} \\ \nearrow \\ \text{q}_R \\ \searrow \\ \text{momentum} \end{array} + \# \begin{array}{c} \nearrow \\ \text{q}_R \\ \searrow \\ \text{spin} \end{array} - \# \begin{array}{c} \searrow \\ \text{q}_L \\ \nearrow \\ \text{momentum} \end{array} - \# \begin{array}{c} \searrow \\ \text{q}_L \\ \nearrow \\ \text{spin} \end{array}$$

Relativistic fermions

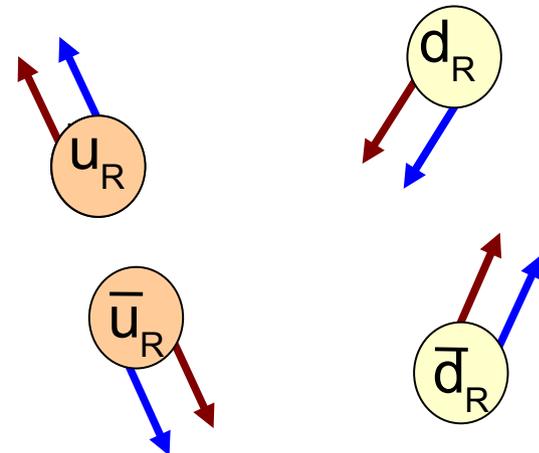
U(1) axial anomaly: $\partial_\mu J_5^\mu = -\frac{g^2}{16\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu}$ $\Delta N_5 = -2Q$



Some gluon field configuration with $Q = -1$

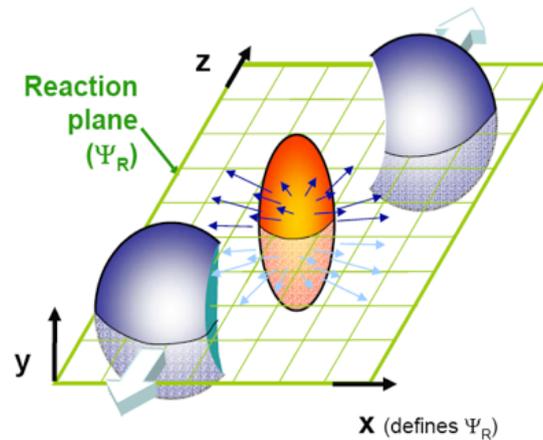
$$\Delta N_5 = 2$$

→



No preferred direction

II. Heavy ion collisions

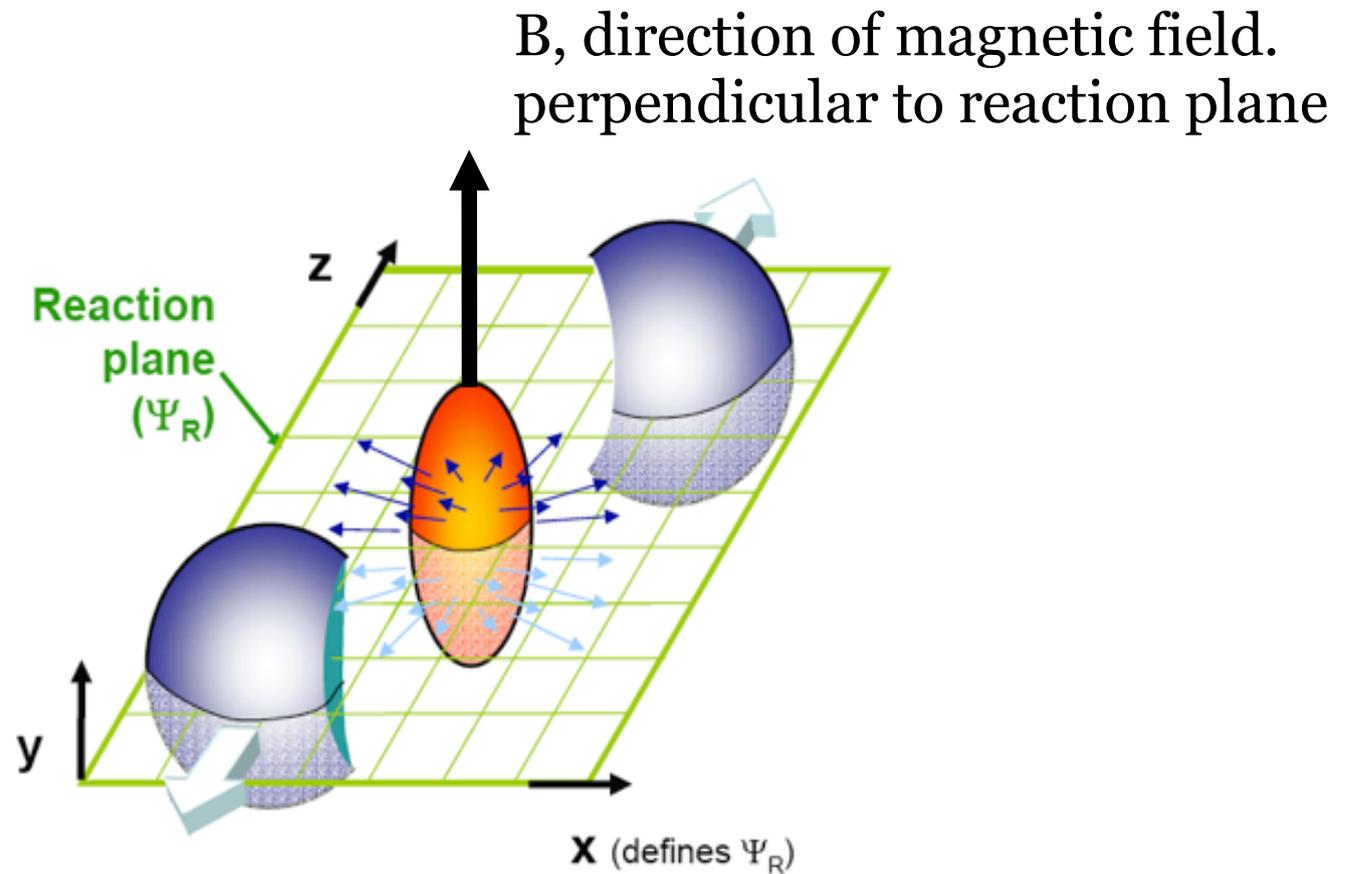


For example:

Gold-Gold @ $\sqrt{s} = 200$ GeV per nucleon pair at RHIC

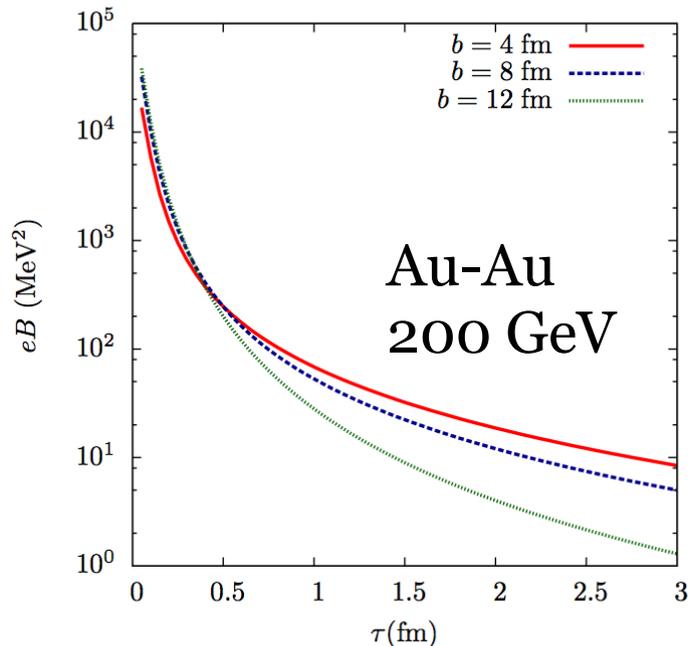
Lead-Lead @ $\sqrt{s} = 5.5$ TeV per nucleon pair at LHC

Ultra high-energy heavy ion collisions = Ultra strong (EM) magnetic fields

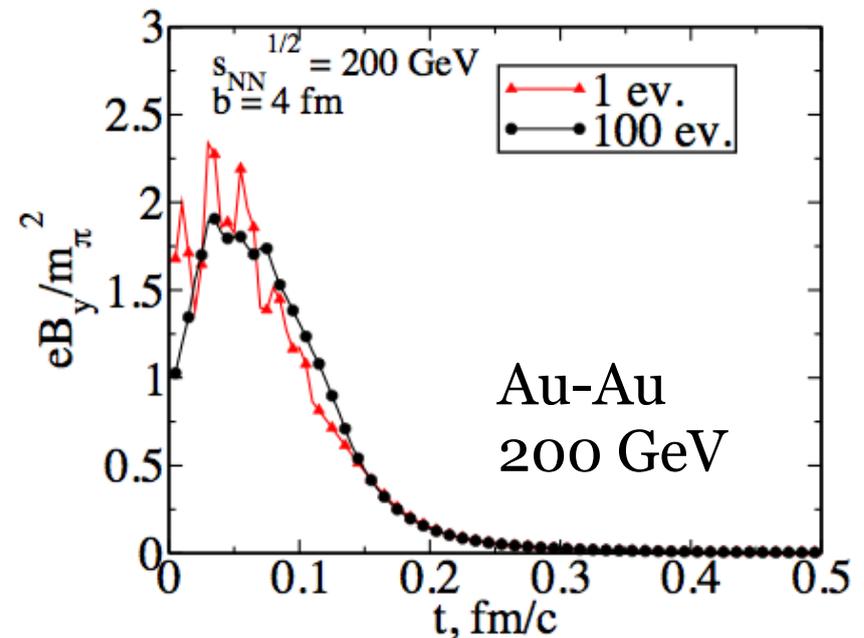


Gold – Gold collision: two currents which carry 79 charges each

Ultra high-energy heavy ion collisions = Ultra strong (EM) magnetic fields



Pancake approximation
 Kharzeev, McLerran & HJW ('08)

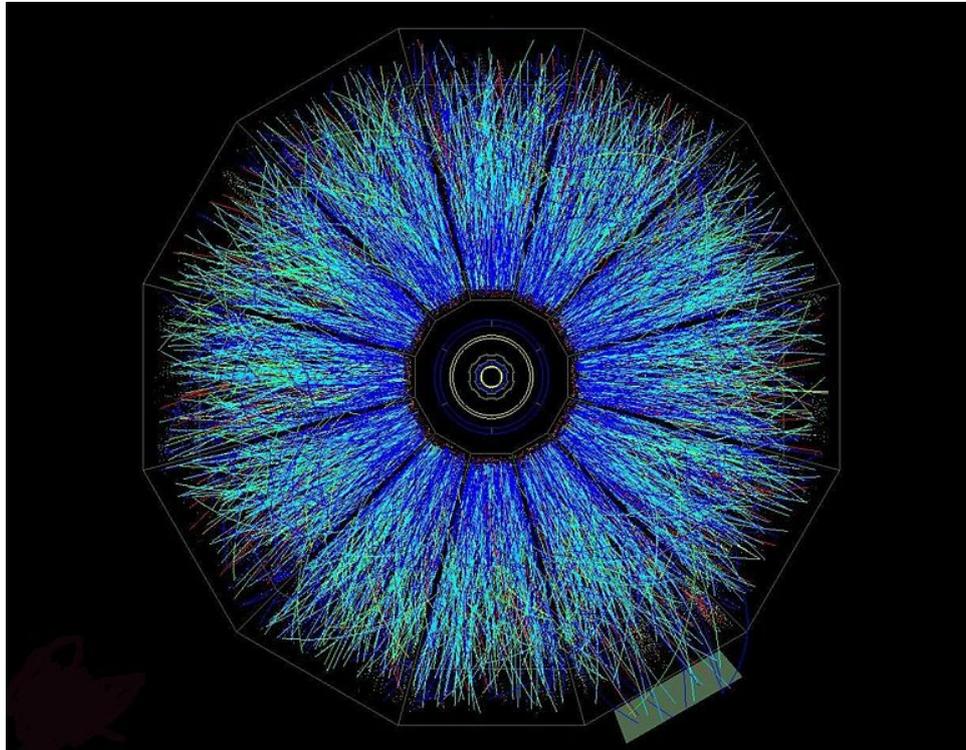


URQMD calculation
 Skokov, Illarionov, Toneev ('09)

$$eB(\tau = 0.2 \text{ fm/c}) \approx 10^3 \sim 10^4 \text{ MeV}^2 \approx 10^{18} \text{ G}$$

See also Minakata and Müller ('96)

III. How to observe P- and CP-odd effects in QCD in extreme conditions

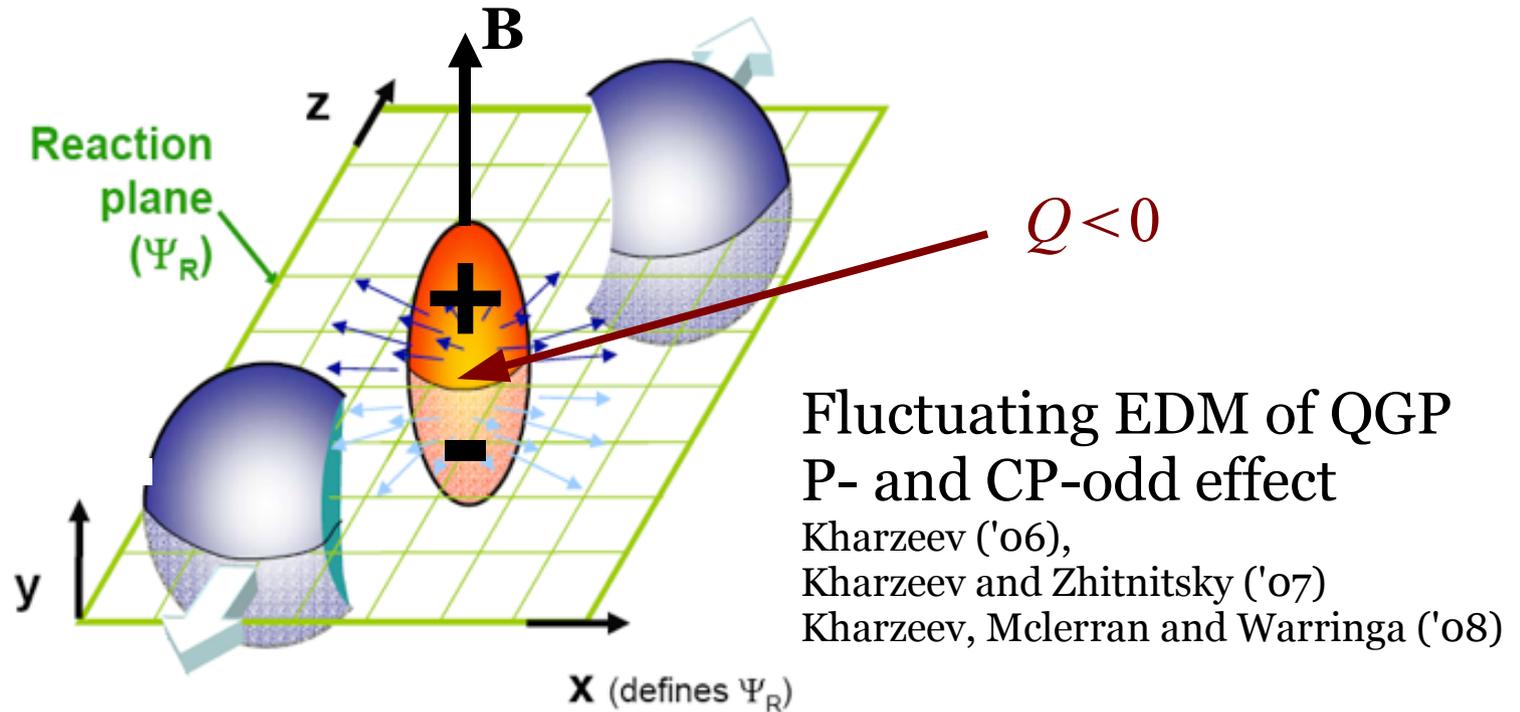


I will explain you that

Topological charge + Magnetic Field =

$$\langle Q \rangle = 0$$

$$\langle Q^2 \rangle \neq 0$$

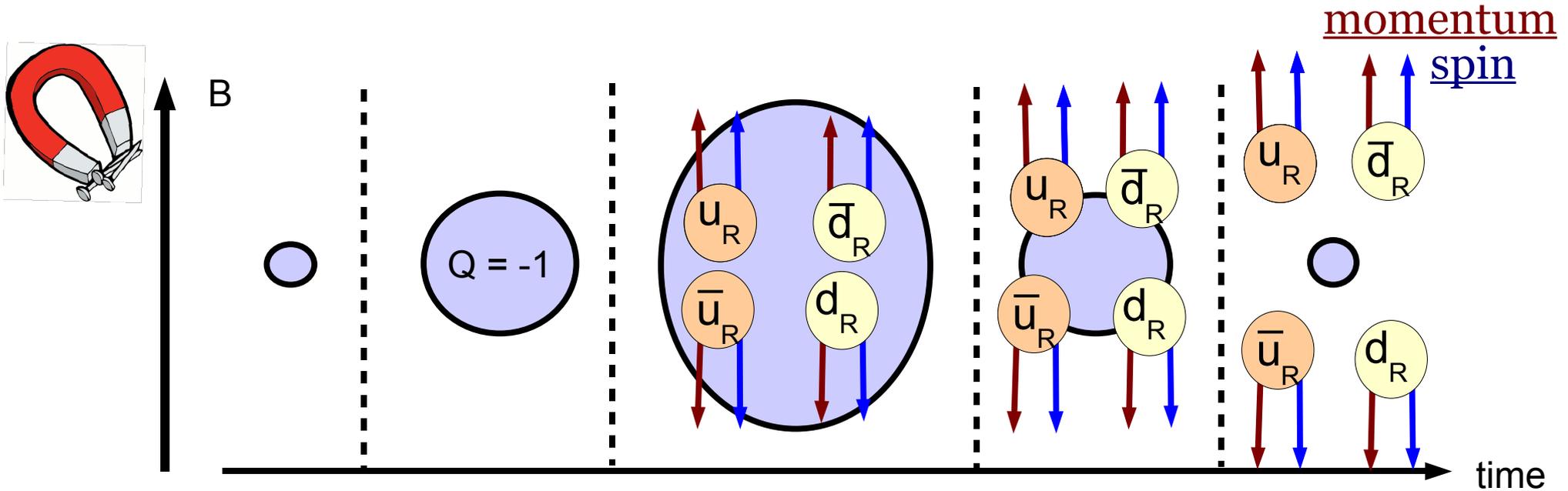


Charge separation

This can potentially be observed in experiment by charge correlation study [Voloshin ('04)]

IIIa. A qualitative explanation of Charge Separation

Topological Charge + Magnetic field = Chirality + Polarization =

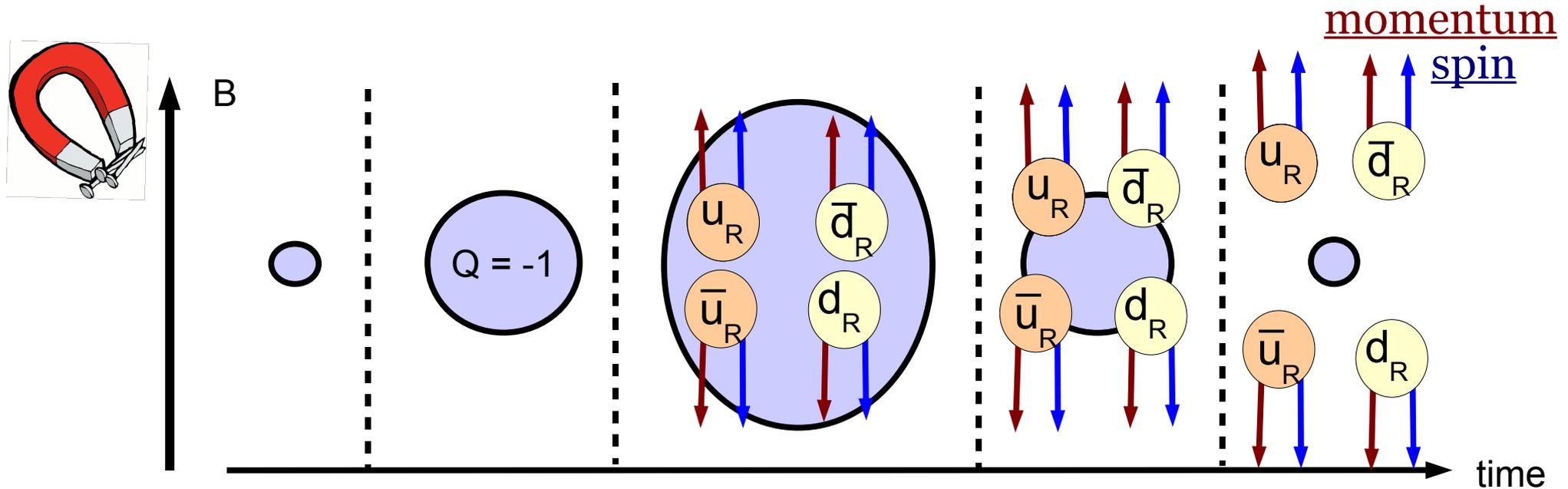


$Q < -1$: Positively charged particles move parallel to magnetic field,
negatively charged antiparallel

... = **Electromagnetic Current**

P- and CP-odd effect --> Chiral Magnetic Effect: Kharzeev, McLerran & HJW ('08)

Topological Charge + Magnetic field = Chirality + Polarization =



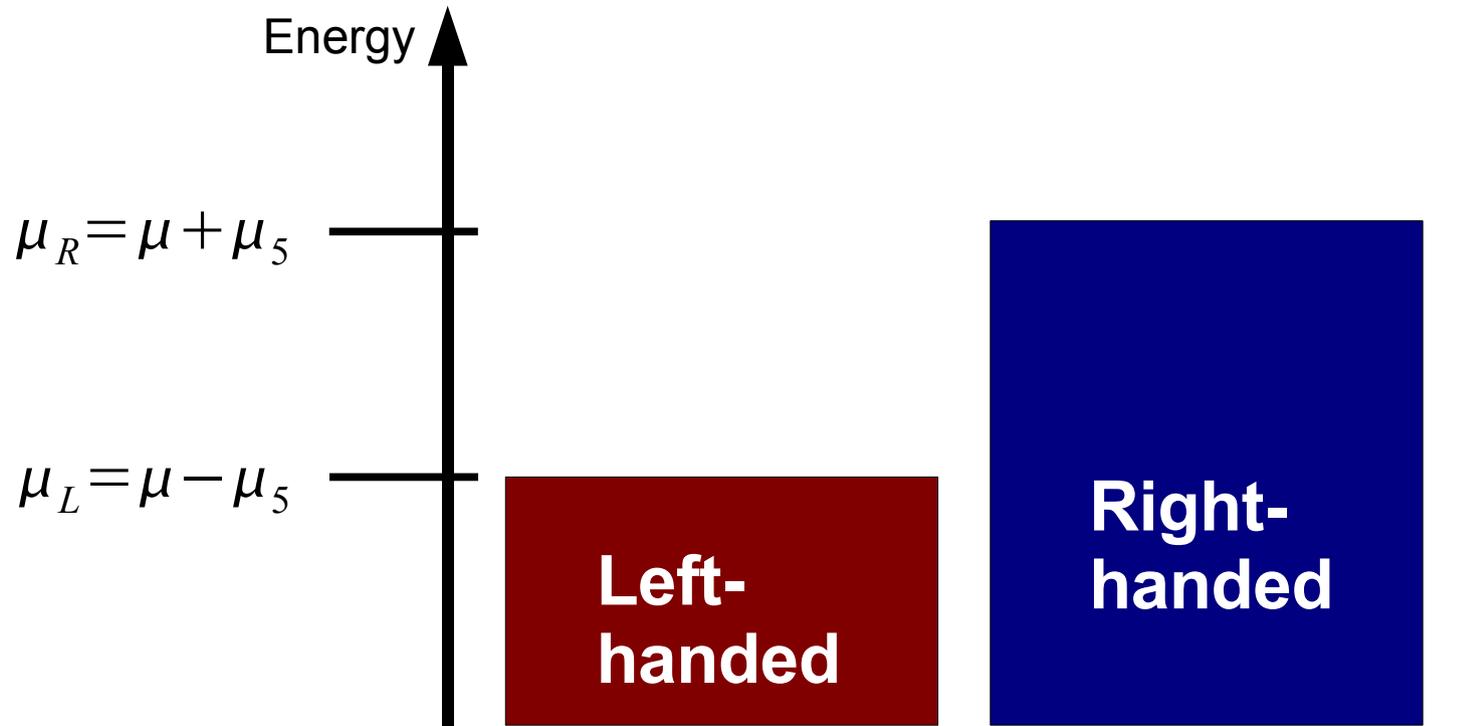
Size of Current: $J = \int d^3 x \langle \bar{\psi} \gamma^3 \psi \rangle = -2Q \sum_f |q_f|$

Valid for full polarization, what about smaller fields?

P- and CP-odd effect --> Chiral Magnetic Effect: Kharzeev, McLerran & HJW ('08)

IIIb. A static quantitative calculation of the Chiral Magnetic Effect

Static calculation: Introduce chirality by hand



Nonzero Chirality: Nonzero chiral chemical potential μ_5

$$H \rightarrow H - \mu_5 \int d^3 x \bar{\psi} \gamma^0 \gamma^5 \psi$$

Obtain induced EM current in magnetic field

Magnitude of the induced current

Alekseev, Cheianov, Fröhlich ('98), Fukushima, Kharzeev and HJW ('08)

1. Energy conservation $j = \frac{N_c \sum_f q_f^2}{2\pi^2} \mu_5 B$
Nielsen and Ninomiya ('83)

2. Density in Lowest Landau Level $j = \frac{N_c \sum_f q_f^2}{2\pi^2} \mu_5 B$
See also Metlitsky and Zhitnitsky ('06)

3. Chern-Simons term $j = \frac{N_c \sum_f q_f^2}{2\pi^2} \mu_5 B$

4. Thermodynamic potential $j = \frac{N_c \sum_f q_f^2}{2\pi^2} \mu_5 B$

5. Linear response $j = \frac{N_c \sum_f q_f^2}{2\pi^2} \mu_5 B$

Chiral Magnetic Conductivity: LO pert QCD Kharzeev and HJW ('09), AdS/CFT: H-u. Yee ('09)

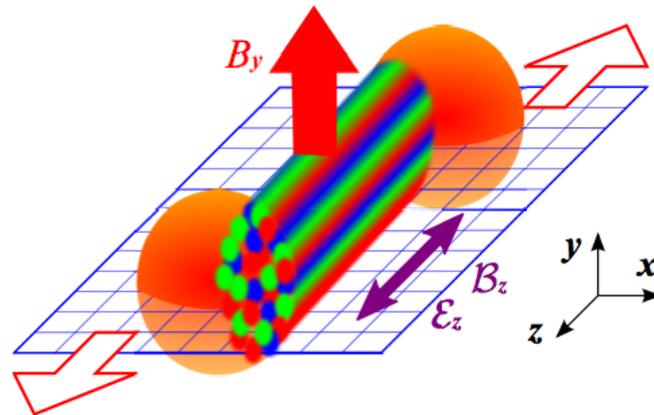
6. Propagator in magnetic field $j = \frac{N_c \sum_f q_f^2}{2\pi^2} \mu_5 B$

Result follows from EM axial anomaly. Therefore exact and independent of coupling strength.
Anomaly induced currents: c.f. Goldstone and Wilczek ('81) AdS/CFT: Rebhan, Schmidt and Stricker ('09)

IIIc. *A dynamic calculation of the Chiral Magnetic Effect*

Chiral Magnetic Effect in Color Flux Tube

Heavy ion collision: Perpendicular magnetic field to color flux tube



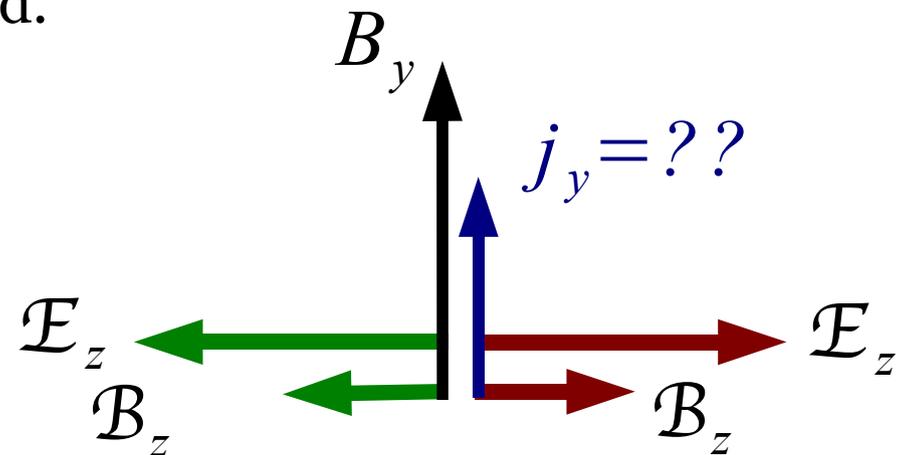
Flux tubes naturally arise in initial state collision, plasma instabilities: Glasma

Krasnitz et al. ('02),
Rebhan, Romatschke, Strickland ('05)
Lappi & McLerran, ('06), ...

Setup: Homogeneous flux tube + mag field.
Flux tube: infinite topological charge,
generates chirality dynamically

$$\partial_\mu J_5^\mu = - \frac{g^2}{16\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} = \frac{g^2}{4\pi^2} \vec{\mathcal{E}} \cdot \vec{\mathcal{B}}$$

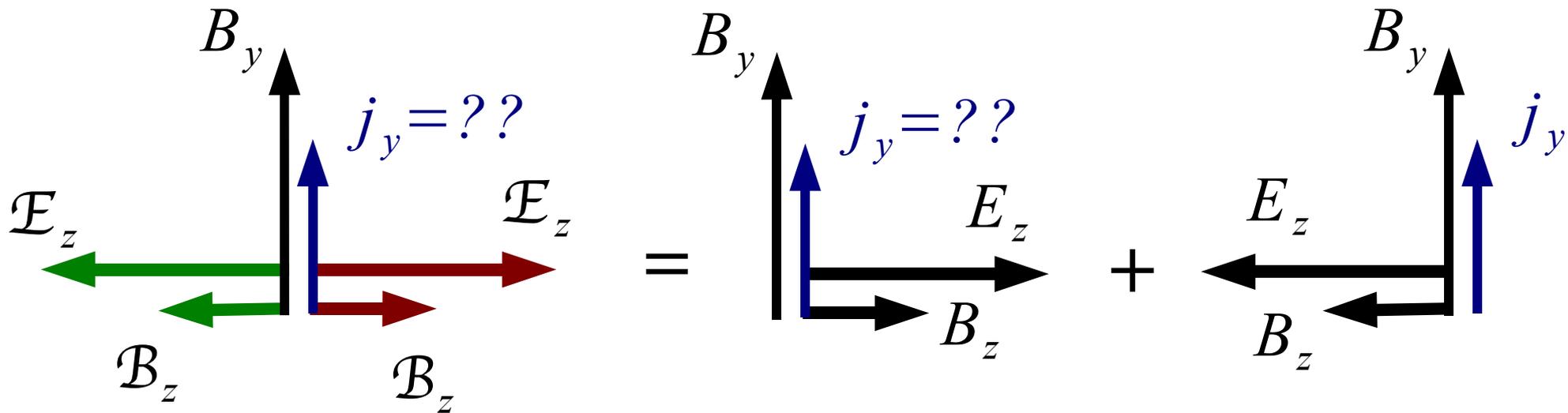
Goal: Current in y-direction
Verify Chiral Magnetic effect



Chiral Magnetic Effect in Color Flux Tube

We choose Abelianized flux-tube: $F_a^{\mu\nu} = \mathcal{F}^{\mu\nu} n_a$ $n_{a=3} = 1, n_{a \neq 3} = 0$

$$g A_\mu^a t^a - q A_\mu = \text{diag} \left(\frac{1}{2} g A_\mu^{a=3} - q A_\mu, -\frac{1}{2} g A_\mu^{a=3} - q A_\mu, 0 \right)$$

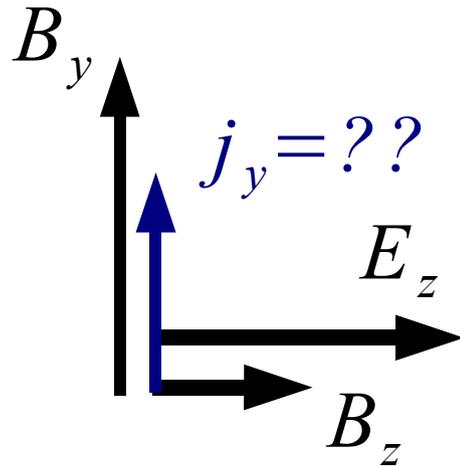


QCD problem

= 2 x QED problem with

$$q E_z = \pm \frac{1}{2} g \mathcal{E}_z \quad q B_z = \pm \frac{1}{2} g \mathcal{B}_z$$

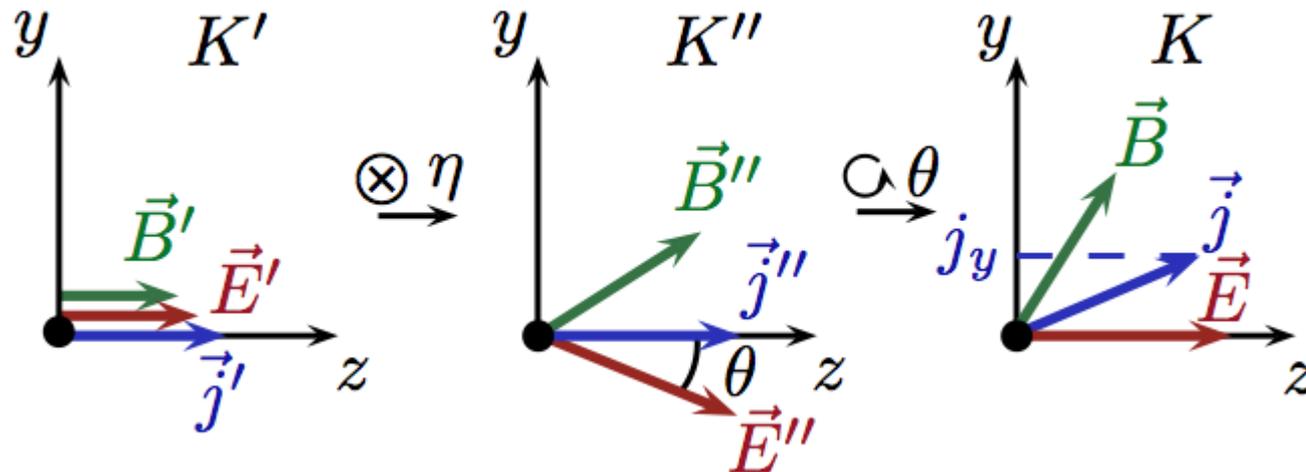
Chiral Magnetic Effect in Color Flux Tube



$$q E_z = \pm \frac{1}{2} g \mathcal{E}_z$$

$$q B_z = \pm \frac{1}{2} g \mathcal{B}_z$$

Solve Dirac equation.

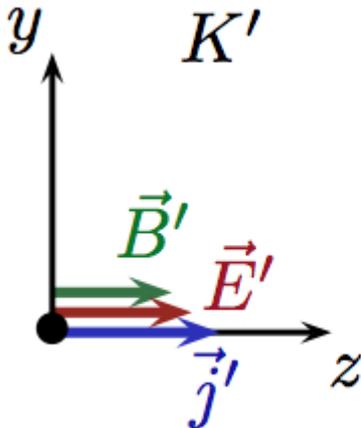


1. Start from frame K' ,
E and B parallel

2. Lorentz boost with
rapidity η in x-dir.

3. Rotation angle
 θ around x-axis.

Chiral Magnetic Effect in Color Flux Tube



In K' particle-anti particle pairs are produced by Schwinger process (Schwinger '51)



Rate per unit volume = (n=1 term in imaginary part effective Lagrangian)

$$\Gamma = \frac{q^2 E_z' B_z'}{4\pi^2} \coth\left(\pi \frac{B_z'}{E_z'}\right) \exp\left(-\frac{m^2 \pi}{|q E_z'|}\right)$$

Nikhishov ('69)
Bunkin and Tugov ('70)

Induced current density: each pair contributes two units

$$\partial_t \vec{j}' = 2q \operatorname{sgn}(q E_z') \Gamma \mathbf{e}_z$$

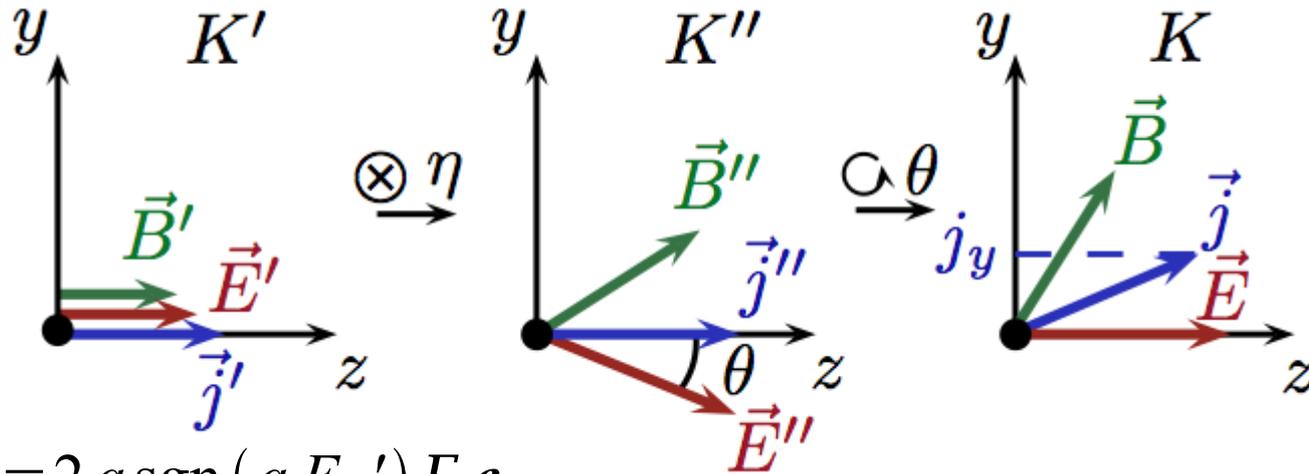
Numerically: Tanji ('09)
Also possible to show analytically
(hard work, Gavrilov and Gitman ('08))

Chirality: Large B' , small E' limit of rate

$$\partial_t n_5 = \frac{q^2 E_z' B_z'}{2\pi^2} \exp\left(-\frac{m^2 \pi}{|q E_z'|}\right)$$

$$m=0 \quad \partial_\mu J_5^\mu = -\frac{e^2}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Chiral Magnetic Effect in Color Flux Tube

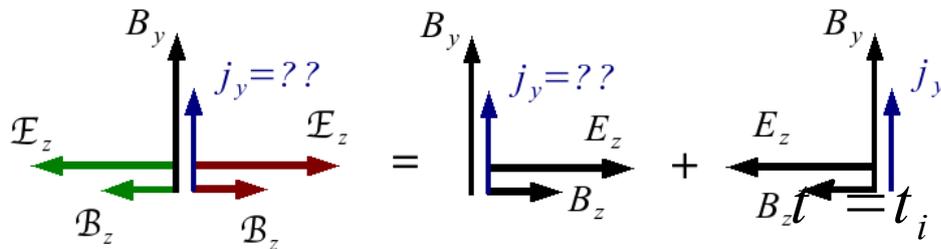


In K' $\partial_t \vec{j}' = 2q \operatorname{sgn}(q E_z') \Gamma \mathbf{e}_z$

In K $\partial_t j_y = 2q \operatorname{sgn}(q E_z') \Gamma \cosh(\eta) \sin(\theta)$

Compute boost and rotation angle in terms of E_z , B_z and B_y

Express in terms of color fields.



$$q E_z = \pm \frac{1}{2} g \mathcal{E}_z$$

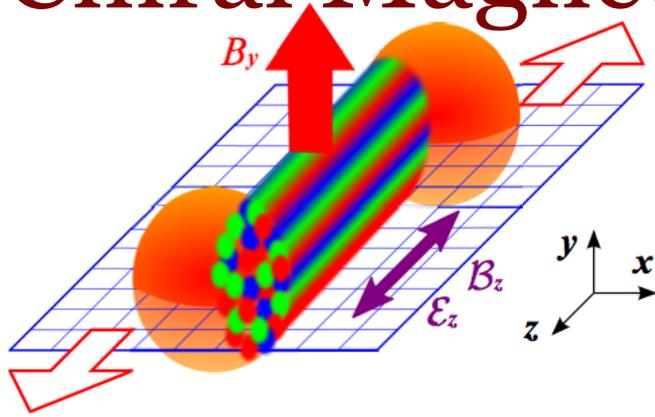
$$q B_z = \pm \frac{1}{2} g \mathcal{B}_z$$

Note: homogeneous switch-on in frame K' becomes inhomogeneous in K

$$t' = t'_i$$

$$t = t'_i / \cosh(\eta) - x \tanh(\eta)$$

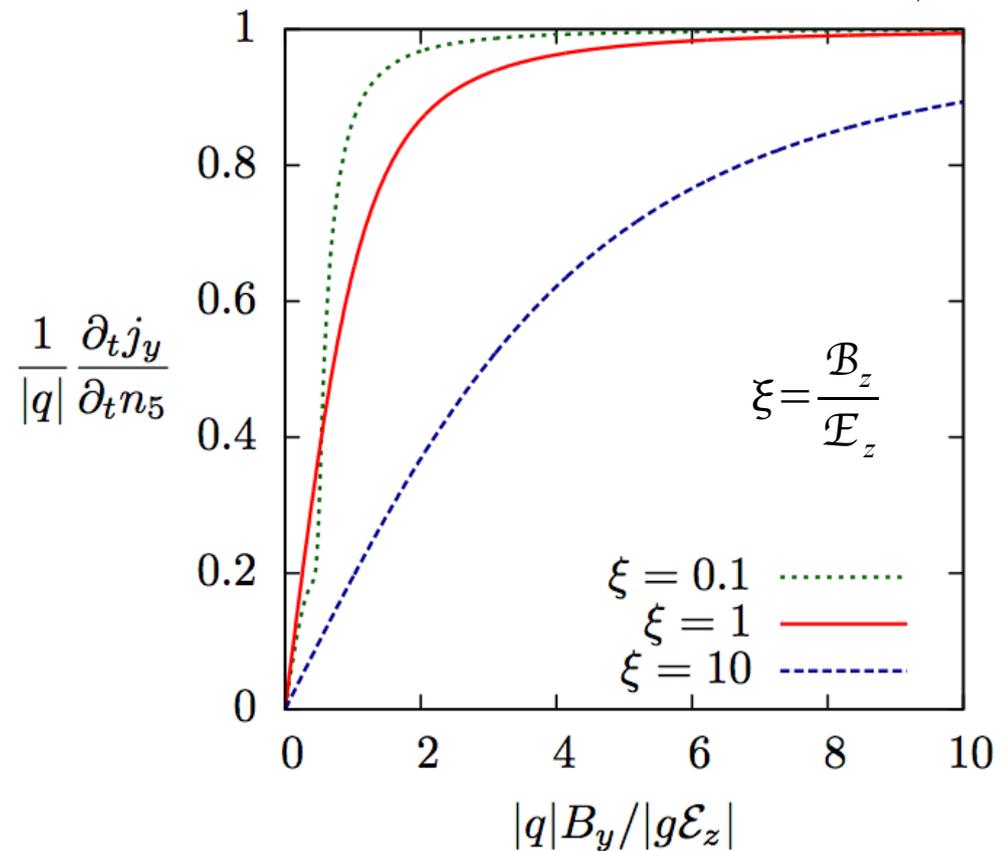
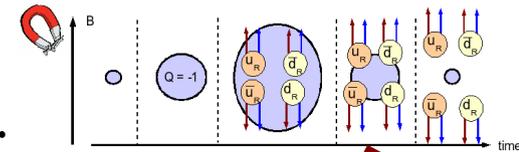
Chiral Magnetic Effect in Color Flux Tube



Dynamical calculation of Chiral Magnetic Effect:

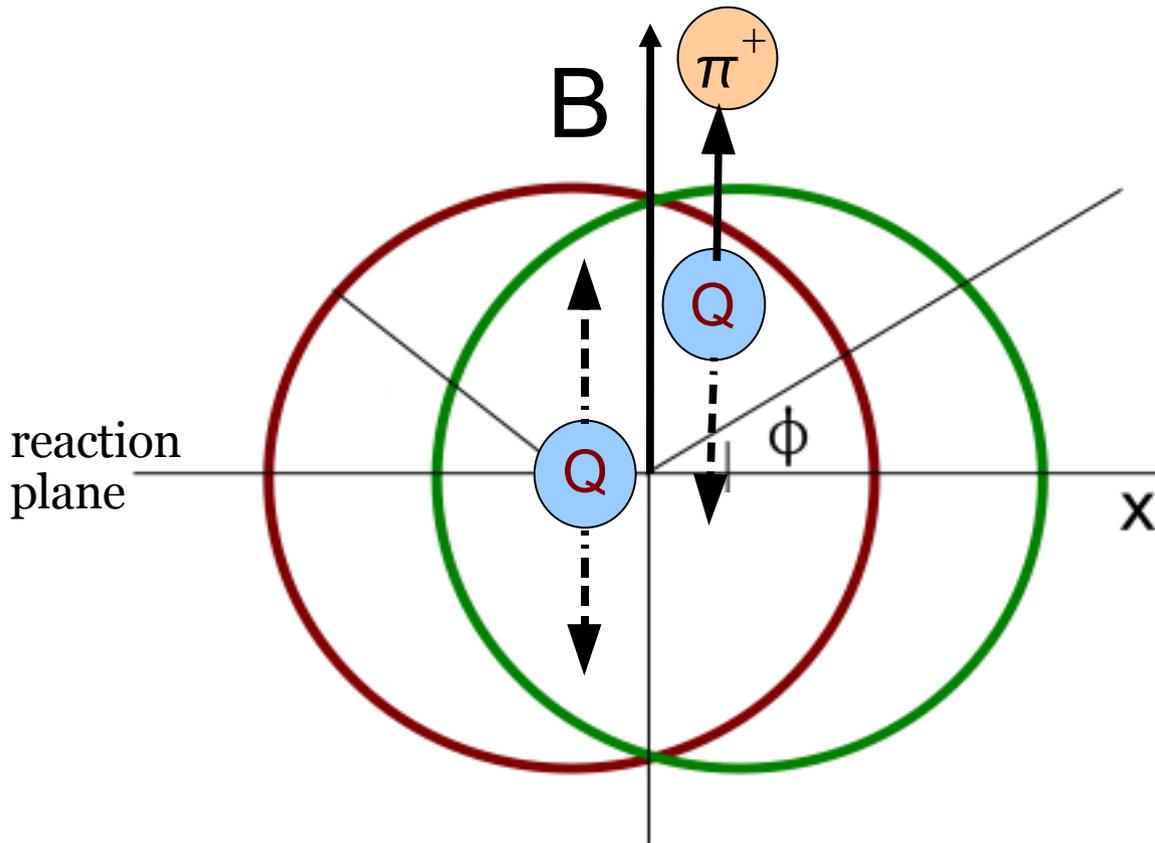
- Completely analytic result for j_y
- Only EM current in y direction
- No B_y , or no chirality: $j_y = 0$
- Large B_y : current=chirality
- Quark mass: reduction in current
- No anomaly: fictional scalar particles completely different behavior.

Current over chirality rate vs. perpendicular magnetic field:



IIIId. Investigating P- and CP-odd effects with heavy ion collisions

Phenomenology



Topological charge Q fluctuates anywhere in the QGP

Measure: variances = nonzero

Medium causes screening

Variance of charge difference between upper and lower side reaction plane:

$$\langle \Delta_{\pm}^2 \rangle = 2 \int_{t_i}^{t_f} dt \int_V d^3x \Gamma [\xi_+^2(x_{\perp}) + \xi_-^2(x_{\perp})] \left(\sum_f \frac{3q_f^2 e B}{\pi^2 T^2} \right)^2$$

Integral over overlap region

Rate of topological charge generation

Screening functions

Amount charge separated by unit top. charge

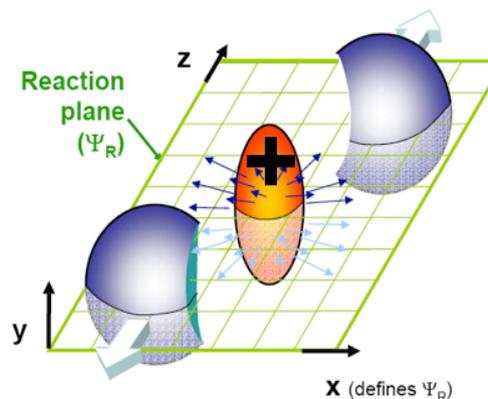
Charge correlations at RHIC

Au-Au and Cu-Cu @ 200 GeV

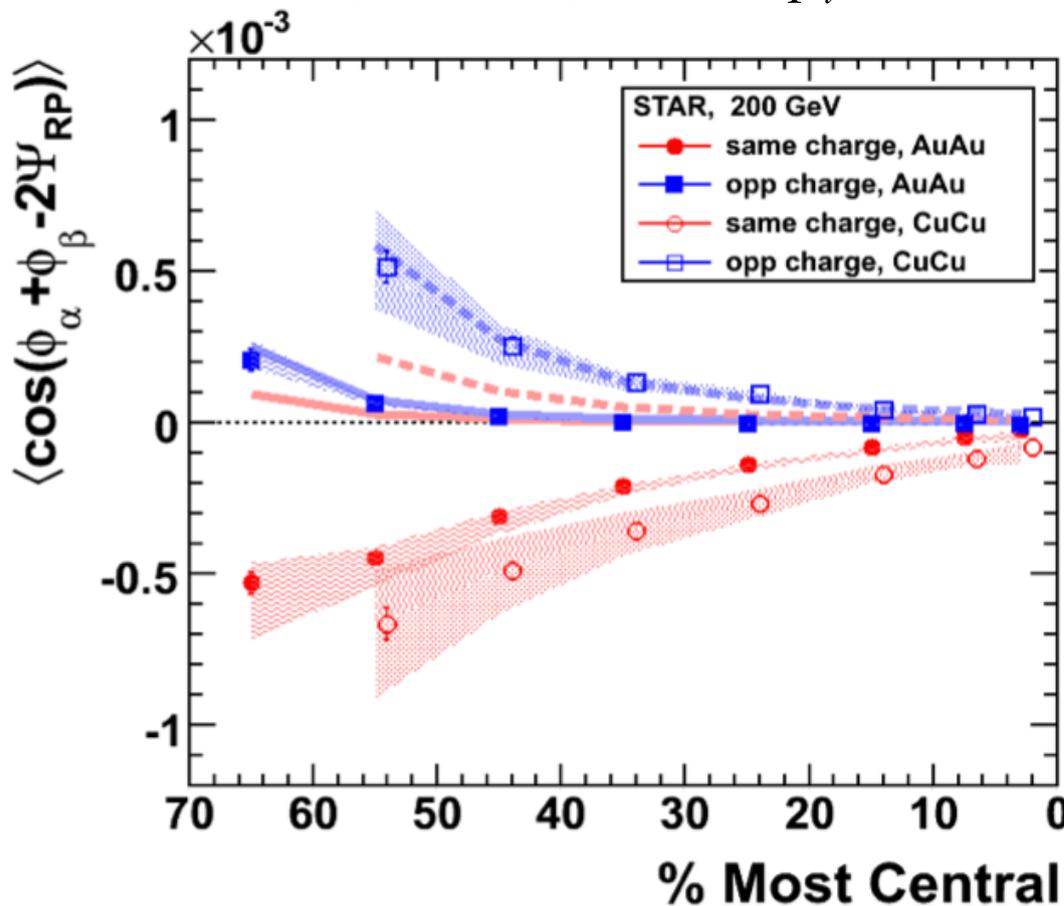
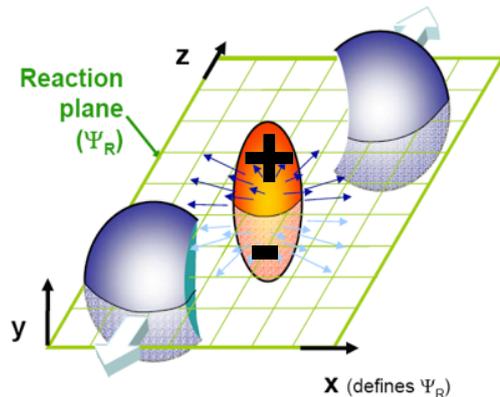
min. bias, $|\eta| < 1.0$, $0.15 < p_t < 2 \text{ GeV}/c$



Red points:



Blue points:



Data cannot be explained by

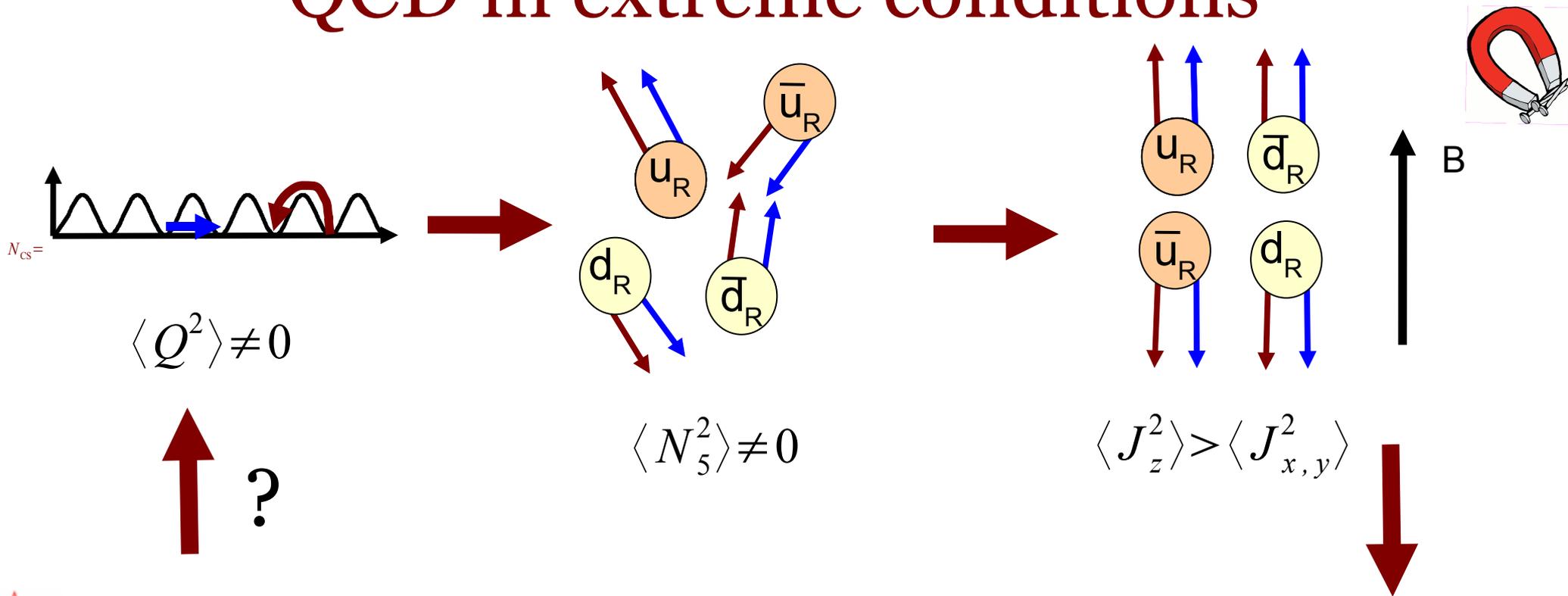
HIJING
HIJING+v2,
MeVSIM,
UrQMD

STAR, Phys.Rev.Lett. **103**, 251601 (2009) and arXiv:0909.1717

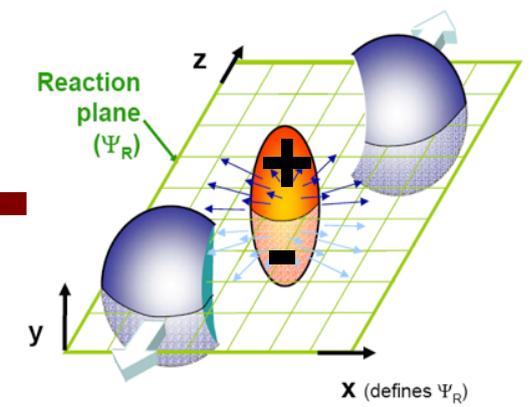
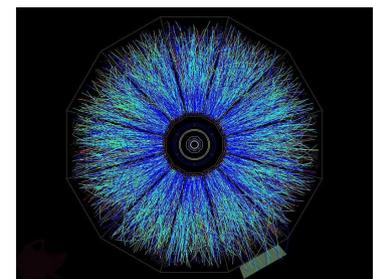
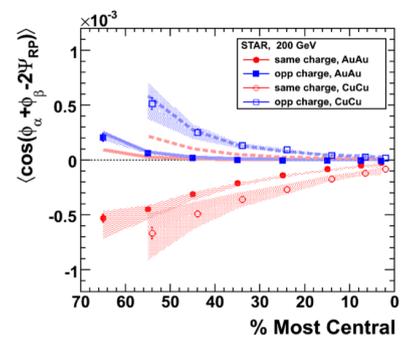
See also B. Müller, Physics 2, 104 (2009)

Strong charge correlations observed at RHIC
is it due to P- and CP-odd effects or something else?

Conclusions: P- and CP-odd effects in QCD in extreme conditions



?



$\langle \cos(\phi_i^\pm + \phi_j^{\pm, \mp} - 2\Psi_{RP}) \rangle \neq 0$

$\langle \Delta_\pm^2 \rangle > 0, \quad \langle \Delta_+ \Delta_- \rangle < 0$

Topological charge + magnetic field naturally leads to charge separation, which is a P- and CP-odd effect.

It could be an explanation for the charge correlations observed by STAR,

But there might exist an alternative explanation we did not think of yet.

Magnitude of the induced current

Fukushima, Kharzeev and HJW ('08)

$$j = \frac{N_c \sum_f q_f^2}{2\pi^2} \mu_5 B$$

Express μ_5 in terms of chirality density cf. baryon chem. potential and density

$$n_5 = \frac{\partial \Omega}{\partial \mu_5}$$

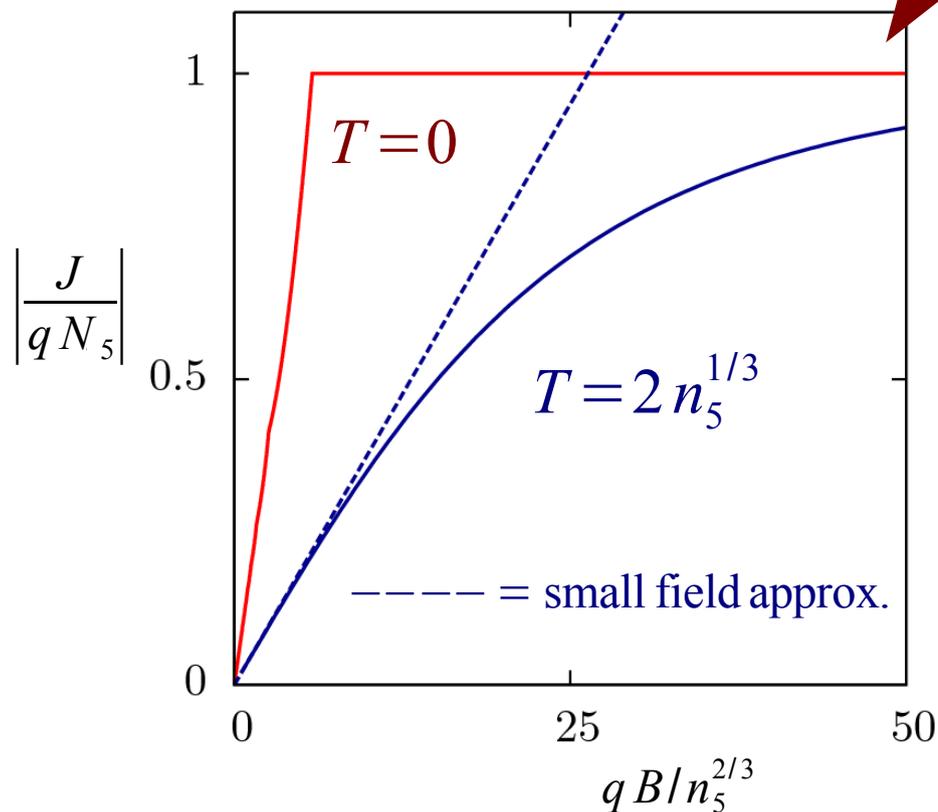
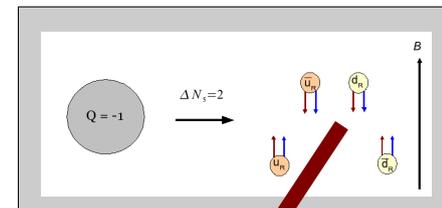
Obtain thermodynamic potential Ω with B at high T using LO. pert. QCD.

$$\mu_5 = f(T, B, \mu, n_5)$$

Relate total chirality to top. charge

$$N_5 = -2Q$$

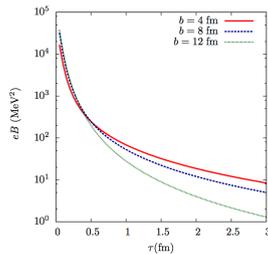
Current over chirality vs. magnetic field



Obtained: estimate induced current in small mag. field: $J \approx -\frac{3}{\pi^2} \frac{Q}{T^2 + \mu^2/\pi^2} B \sum_f q_f^2$

Static chirality plus time-dep. field

Kharzeev and HJW ('09)



Can we have chiral magnetic effect even in the fast changing mag. field of collisions?

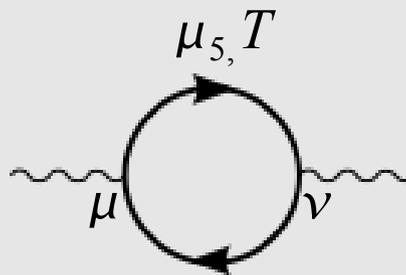
$$\vec{j} = \sigma_E \vec{E} \quad \sigma_E(\omega) = \text{electrical conductivity}$$

$$\vec{j} = \sigma_\chi \vec{B} \quad \sigma_\chi(\omega) = \text{chiral magnetic conductivity}$$

Compute chiral magnetic conductivity as a function of frequency using linear response

Leading order
pert. QCD

Kharzeev and HJW ('09)



$\tilde{\Pi}_R^{jk}$

AdS/CFT
strong coupling:
Ho-Ung Yee ('09)

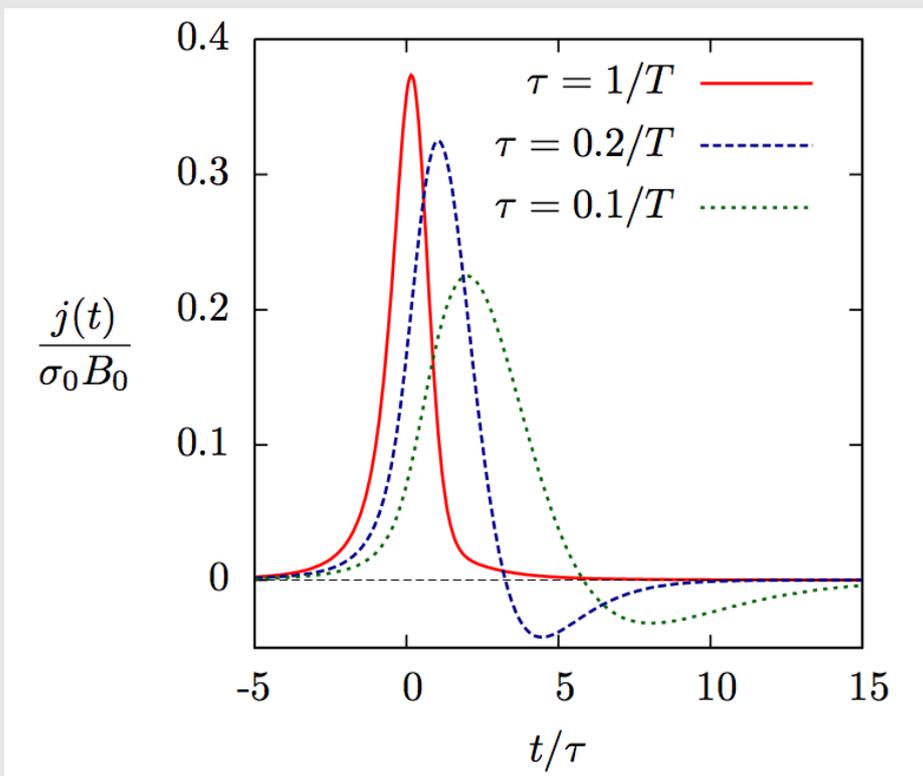
$$\sigma_\chi(\omega=0) = \frac{N_c \sum_f q_f^2}{2\pi^2} \mu_5$$

Static chirality +time-dep. field

Kharzeev and HJW ('09)

$$j(t) = \int_0^\infty \frac{d\omega}{\pi} [\sigma'_x(\omega) \cos(\omega t) + \sigma''_x(\omega) \sin(\omega t)] \tilde{B}(\omega)$$

Normalized current as a function of time



$$B(t) = \frac{B_0}{[1 + (t/\tau)^2]^{3/2}}$$

Red: current in slowly changing fields, adiabatic appr. = ok

Blue and green curves, faster changing mag field, but still induced current.

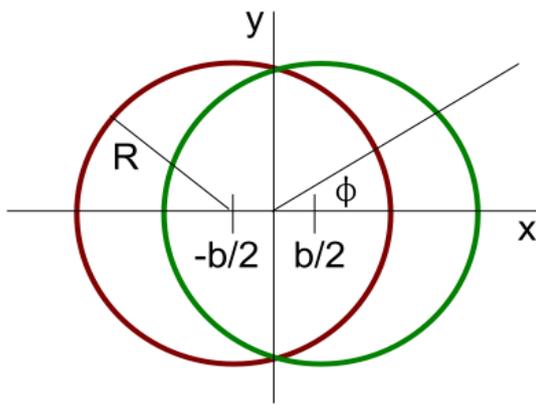
Even stronger response in strongly coupled regime. AdS/CFT: Ho-Ung Yee ('09)

Conclusion: also sizable current in fast changing magnetic field

Experimental observables

Voloshin ('04)

Correlations in azimuthal angle of charged particles



$$a_{++} = \left\langle \frac{1}{N_+ N_+} \sum_{i,j=1}^{N_+, N_+} \cos(\phi_i + \phi_j - 2\Psi_{RP}) \right\rangle$$

$$= \left\langle \frac{1}{N_+^2} \left[\sum_{i=1}^{N_+} \cos(\phi_i - \Psi_{RP}) \right]^2 \right\rangle$$

Charge fluctuations in x-direction

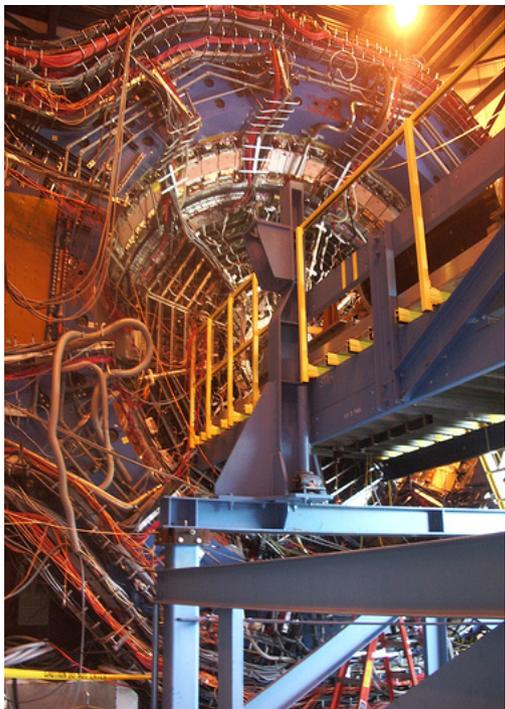
$$- \left\langle \frac{1}{N_+^2} \left[\sum_{i=1}^{N_+} \sin(\phi_i - \Psi_{RP}) \right]^2 \right\rangle$$

Minus fluctuations in y direction

Average is over many similar minimum bias events

Take symmetric interval around zero rapidity

Analysis (and problems) similar to elliptic flow.
See also work of Raimond Snellings group (Nikhef).



STAR detector
Full azimuthal coverage

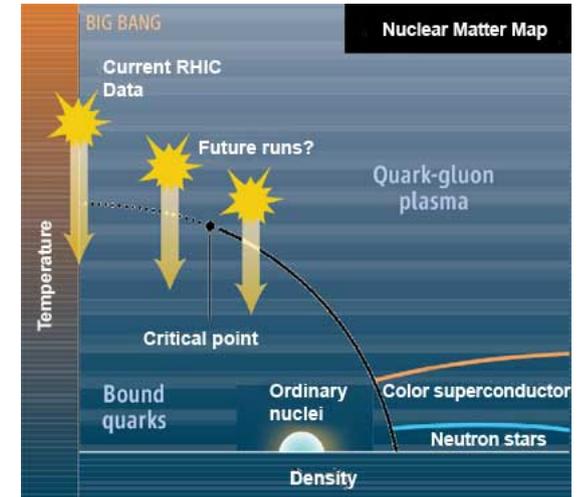
STAR data due to P- and CP-odd effects?

Required:

Deconfinement: to separate quarks

Chiral Symmetry restoration: to induce chirality

Hence no Chiral Magnetic Effect at low energies.
Test energy scan. Also test at LHC



Magnetic field the correlators proportional to Z^2

Test: compare collisions with same A and different Z, isobars
Argon-40 (Z=18), vs. Calcium-40 (Z=20), 23% increase in signal

More quantitative phenomenology really necessary

More data also possible: individual charged particle correlations

Think of other explanations

Cluster model of F. Wang ('09), ???