# FY3452 - Exercise set 3 spring 2016

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## 1 Exercise 1

The Lagrangian density for a nonrelativistic field $\psi$ is given by

$$\mathcal{L} = -\frac{1}{2}i\hbar(\dot{\psi}^*\psi - \psi^*\dot{\psi}) - \frac{\hbar^2}{2m}(\nabla\psi^*)\cdot(\nabla\psi) \; . \tag{1}$$

1) Find the equation of motion for $\psi$.
2) Find the Hamiltonian density $\mathcal{H}$.
3) Show that the Lagrangian density is invariant under a global phase transformation.
4) Find the conserved current that corresponds to the symmetry and interpret the continuity equation.
5) How can you make the Lagrangian density invariant under *local* phase transformations?

## Exercise 2

The Lagrangian for a free electron-positron field is

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \; . \tag{2}$$

1) Calculate the components of the energy-momentum tensor. Find the energy and momentum densities and show explicitly current conservation.

2) The Lagrangian is invariant under global phase transformations:

$$\psi \to e^{i\alpha}\psi \; , \tag{3}$$
$$\bar{\psi} \to e^{-i\alpha}\bar{\psi} \; . \tag{4}$$

This gives rise to the well-known conservation of charge. Consider instead the transformation

$$\psi \to e^{i\gamma^5\alpha}\psi \; , \tag{5}$$
$$\bar{\psi} \to \bar{\psi}e^{i(\gamma^5)^\dagger\alpha} \; , \tag{6}$$

where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is a *matrix*. The transformation is called a *chiral* transformation. Show that

$$
\begin{aligned}
(\gamma^5)^\dagger &= \gamma^5 \,, &(7)\\
(\gamma^5)^2 &= 1 \,, &(8)\\
\{\gamma^5, \gamma^\mu\} &= 0 \,. &(9)
\end{aligned}
$$

3) For what values of $m$ is the chiral transformation a symmetry of the Dirac Lagrangian? Find the corresponding 4-current (called an axial vector current since it is changes sign under parity, $\mathbf{x} \to -\mathbf{x}$).